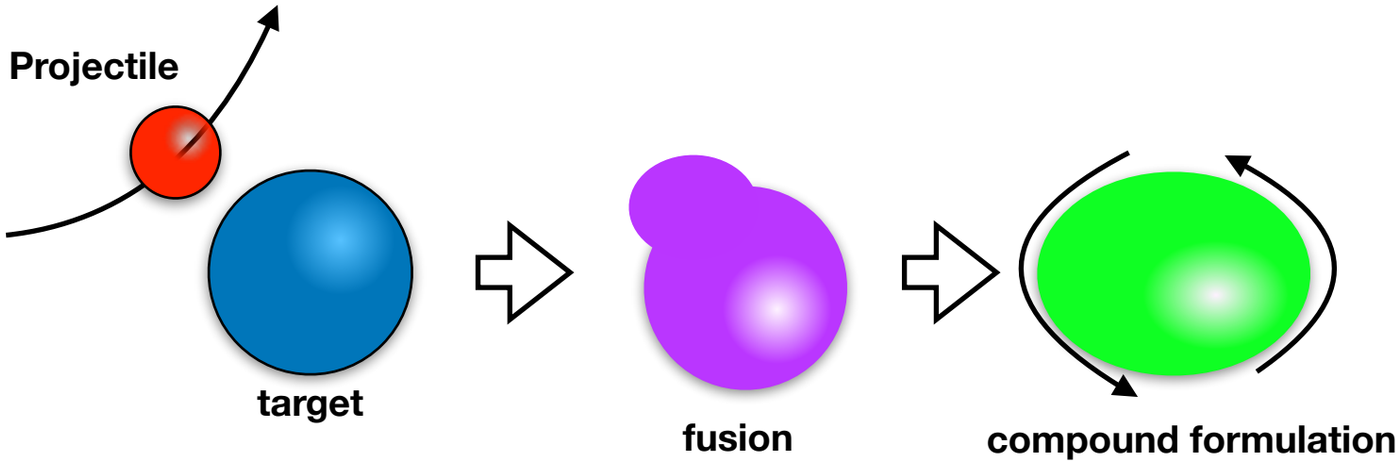
Fusion reaction of a weakly-bound nucleus with a deformed target

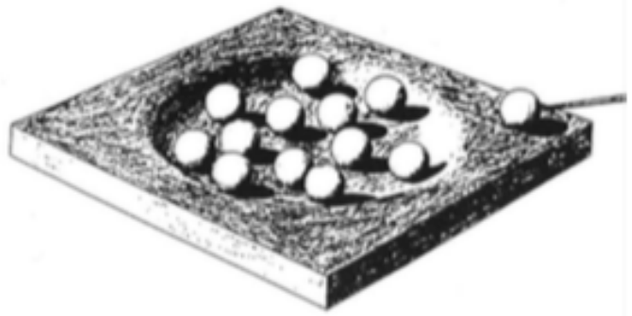
20201114
HaPhy meeting

Ki-Seok Choi

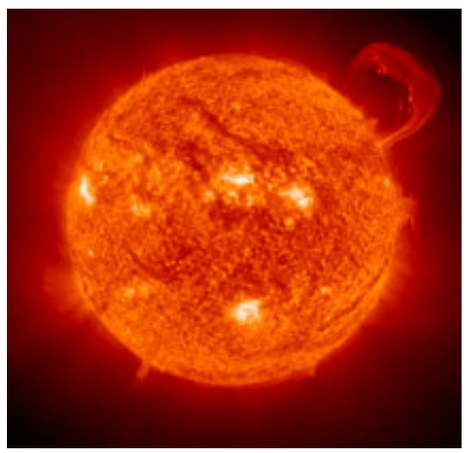
Korea Aerospace University, Korea

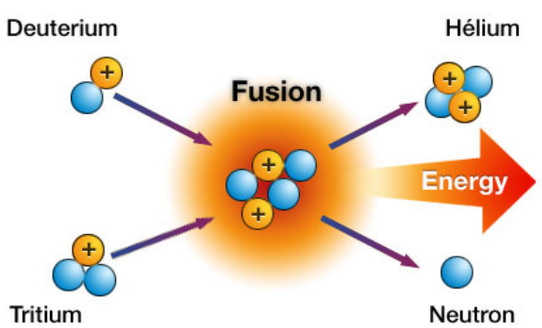
Fusion reaction: compound nucleus formation



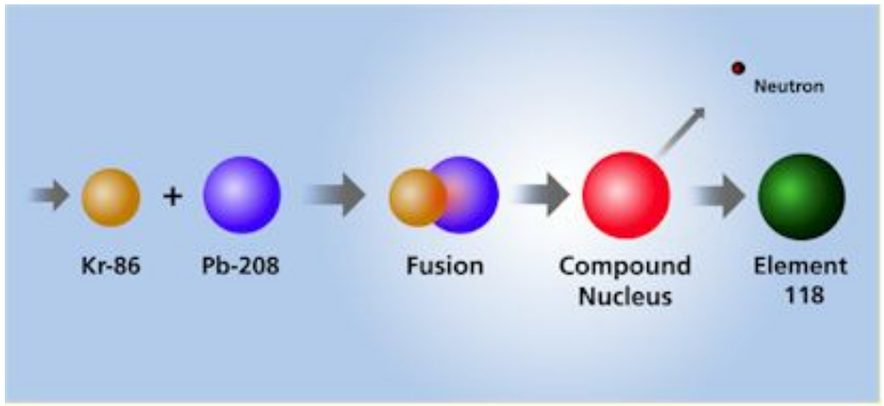


cf. Bohr '36

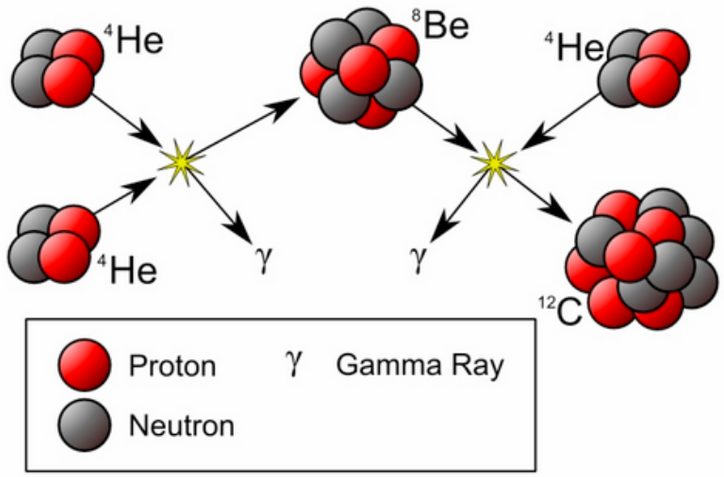




energy production in stars



super heavy elements



nucleosynthesis

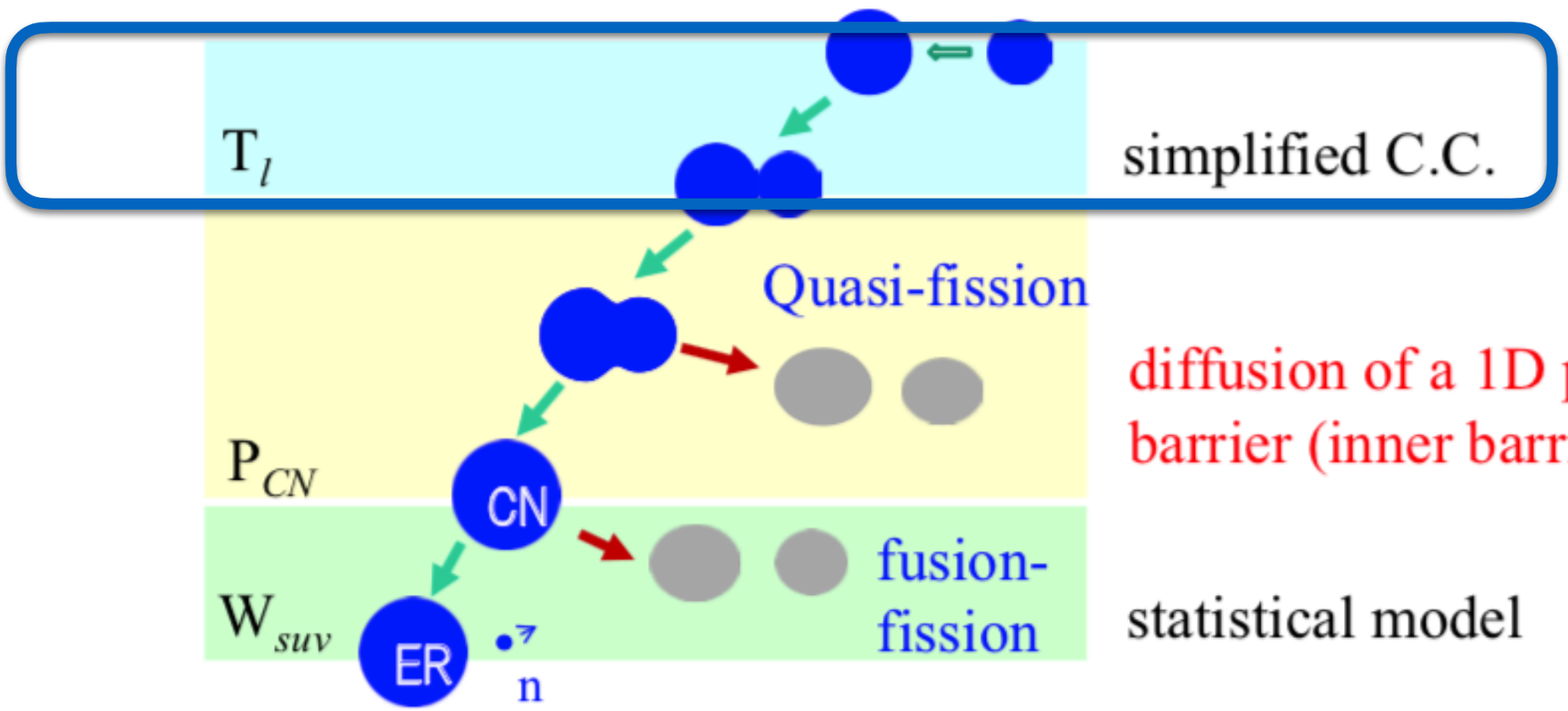
https://fusionforenergy.europa.eu/
https://www.thphys.uni-heidelberg.de

Extension of the fusion-by-diffusion model

K.H., PRC98 ('18) 014607

Fusion-by-diffusion model

W.J. Swiatecki et al., Acta Phys. Pol. B34 ('03) 2049
PRC71 ('05) 014602



Presentation of K.Hagino : 2018. 09. 21

For Fusion

Wong's Formula

penetrability of parabola

$$P(E) = \frac{1}{1 + \exp\left[\frac{2\pi}{\hbar\Omega}(V_b - E)\right]}$$

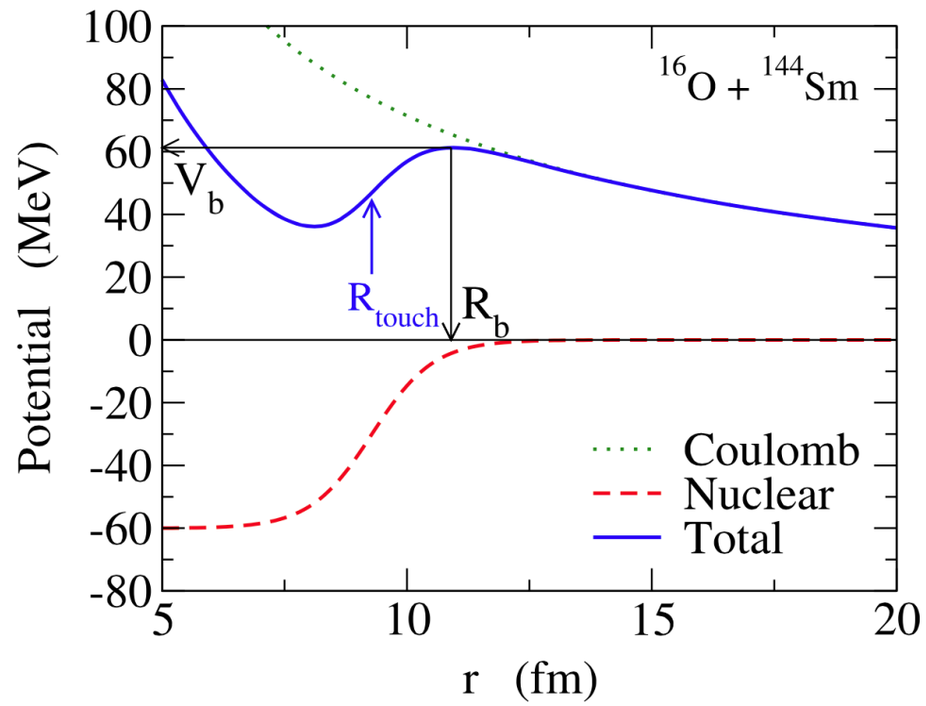
$$\Omega = \sqrt{-\frac{V_N''(R_b) + 2Z_P Z_T e^2/R_b^3}{\mu}}$$:curvature

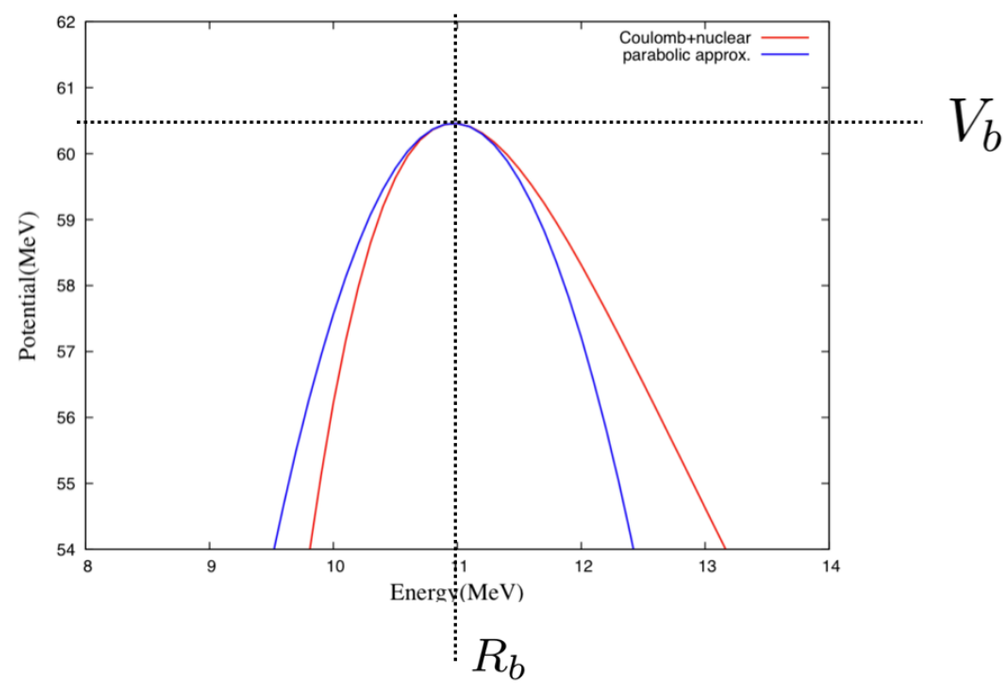
approximated penetrability of parabola with l dependance

$$P_l(E) = P_{l=0}\left(E - \frac{l(l+1)\hbar^2}{2\mu R_b^2}\right)$$

$$\sigma_{\text{fus}}(E) = \frac{\pi}{k^2}\int_0^\infty dl\ (2l+1)P_l(E)$$

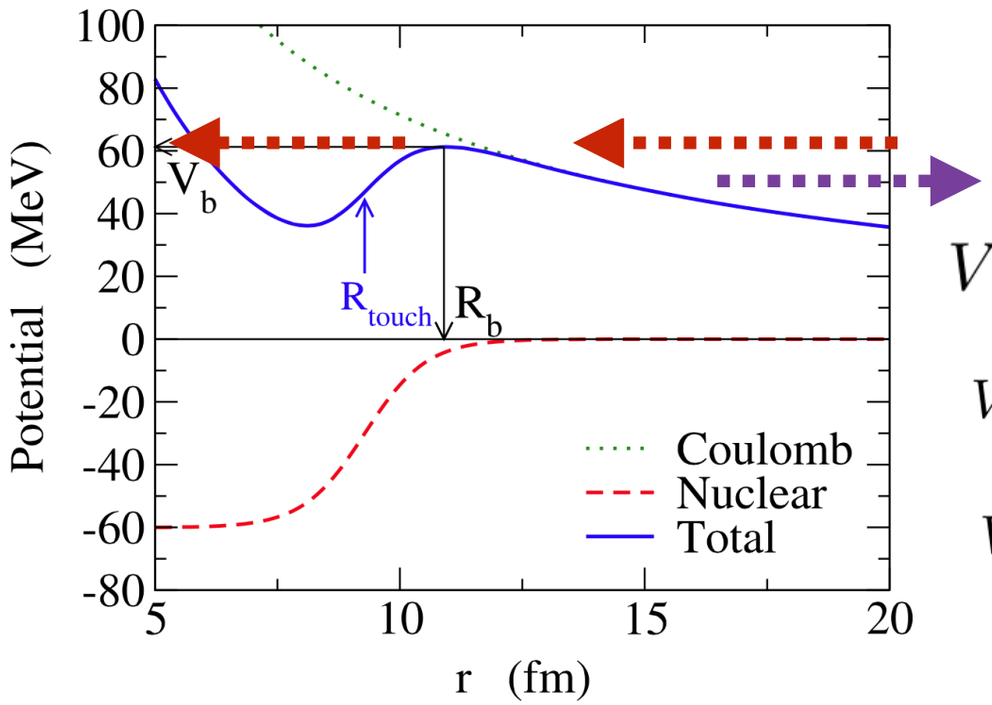
$$\sigma_{\text{fus}}(E) = \frac{\hbar\Omega}{2E}R_b^2 \ln\left[1 + \exp\left(\frac{2\pi}{\hbar\Omega}(E - V_b)\right)\right]$$





C. Y. Wong, Phys. ReV. Lett. 31, 766 (1973)

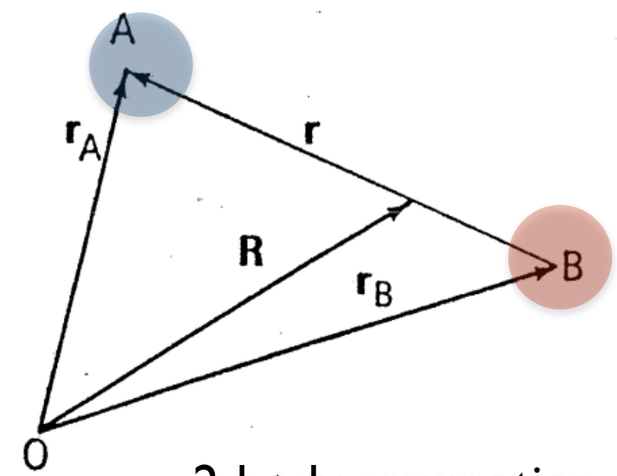
One-dimensional potential model (Barrier penetration model(BPM))



$$V(r) = V_N(r) + V_C(r)$$

$$V_C(r) = \frac{Z_P Z_T e^2}{r}$$

$$V_N(r) = -\frac{V_0}{1+\exp[(r-R_0)/a]}$$



2-body assumption

$$u_l(r) = \sqrt{\frac{k}{k_l(r)}}\,\mathcal{T}_l \exp\left(-i\int_{r_{\rm abs}}^{r} k_l(r')dr'\right),$$

: incoming wave boundary condition (IWBC)

$r \le r_{\rm abs}$

$$k_l(r) = \sqrt{\frac{2\mu}{\hbar^2}\left(E - V(r) - \frac{l(l+1)\hbar^2}{2\mu r^2}\right)}.$$

$$k = \sqrt{2\mu E/\hbar^2}$$

$$H_l^{(-)}(kr) - S_l\, H_l^{(+)}(kr), \qquad r \to \infty$$

$H_l^{(+)}(kr)$ and $H_l^{(-)}(kr)$ are the outgoing and the incoming Coulomb waves

$$P_l(E) = 1 - |S_l|^2 = |\mathcal{T}_l|^2$$

$$\sigma_{\rm fus}(E) = \frac{\pi}{k^2}\sum_l (2l+1)P_l(E).$$

Fusion reaction with Coupled-Channel method in BPM

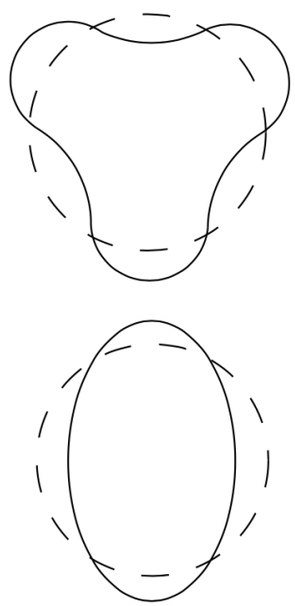
(Barrier penetration model)

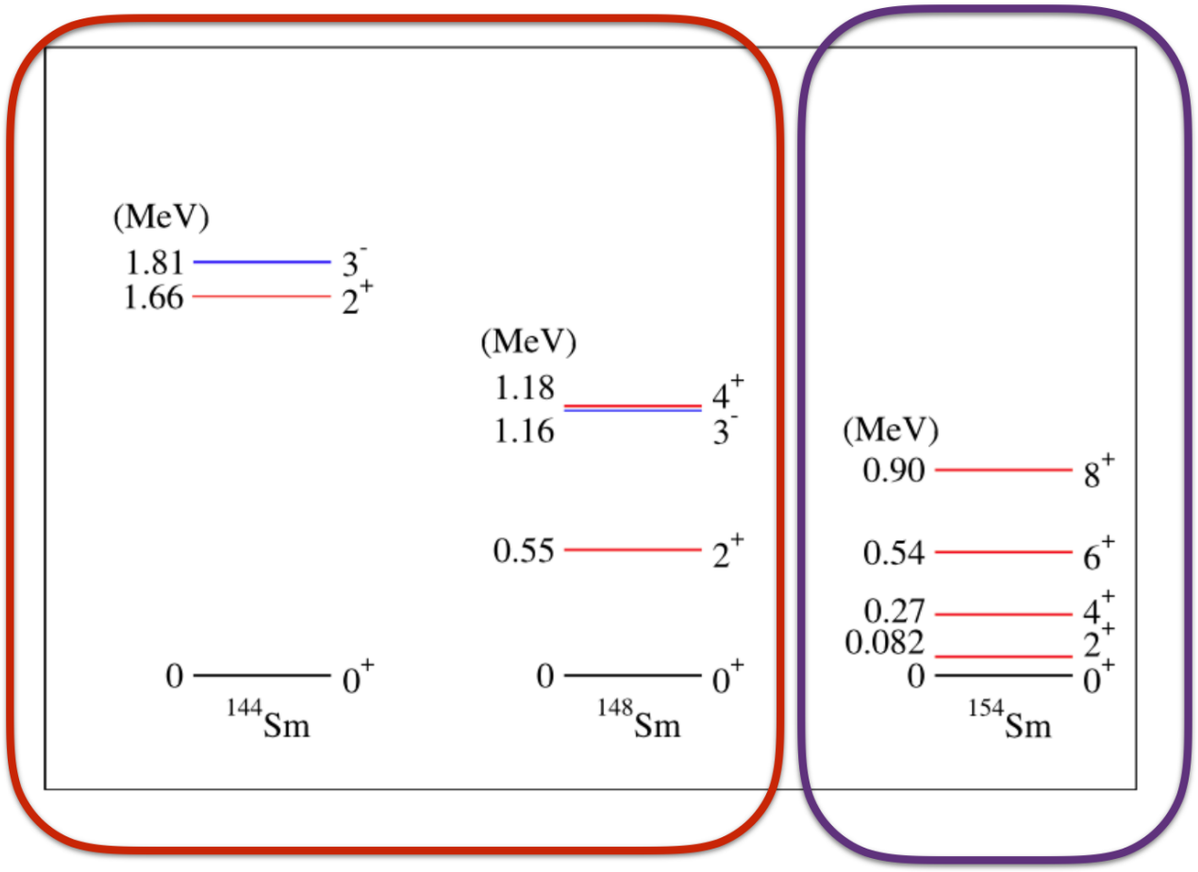
Coupled-channel equations

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2}+\frac{J(J+1)\hbar^2}{2\mu r^2}+V_N^{(0)}(r)+\frac{Z_P Z_T e^2}{r}+\epsilon_n-E\right]\psi_n(r)+\sum_m V_{nm}(r)\psi_m(r)=0,$$

Coupled with excited states

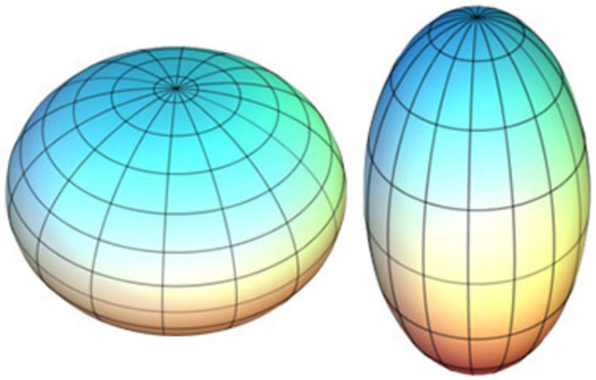
$(2^+ \otimes 3^-)^{J^-}$





vibrational motion

rotational motion



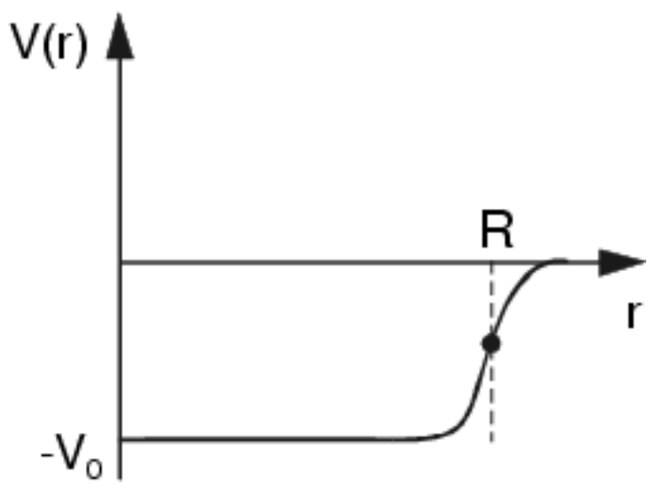
Oblate and Prolate nuclei

K.Hagino et al : CPC 123, 143 (1999)

AKyuz-Winther Potential(Global potential)

$$V_N(r) = -\frac{V_0}{1+\exp[(r-R_0)/a]},$$

(typical Wood-Saxon potential form)

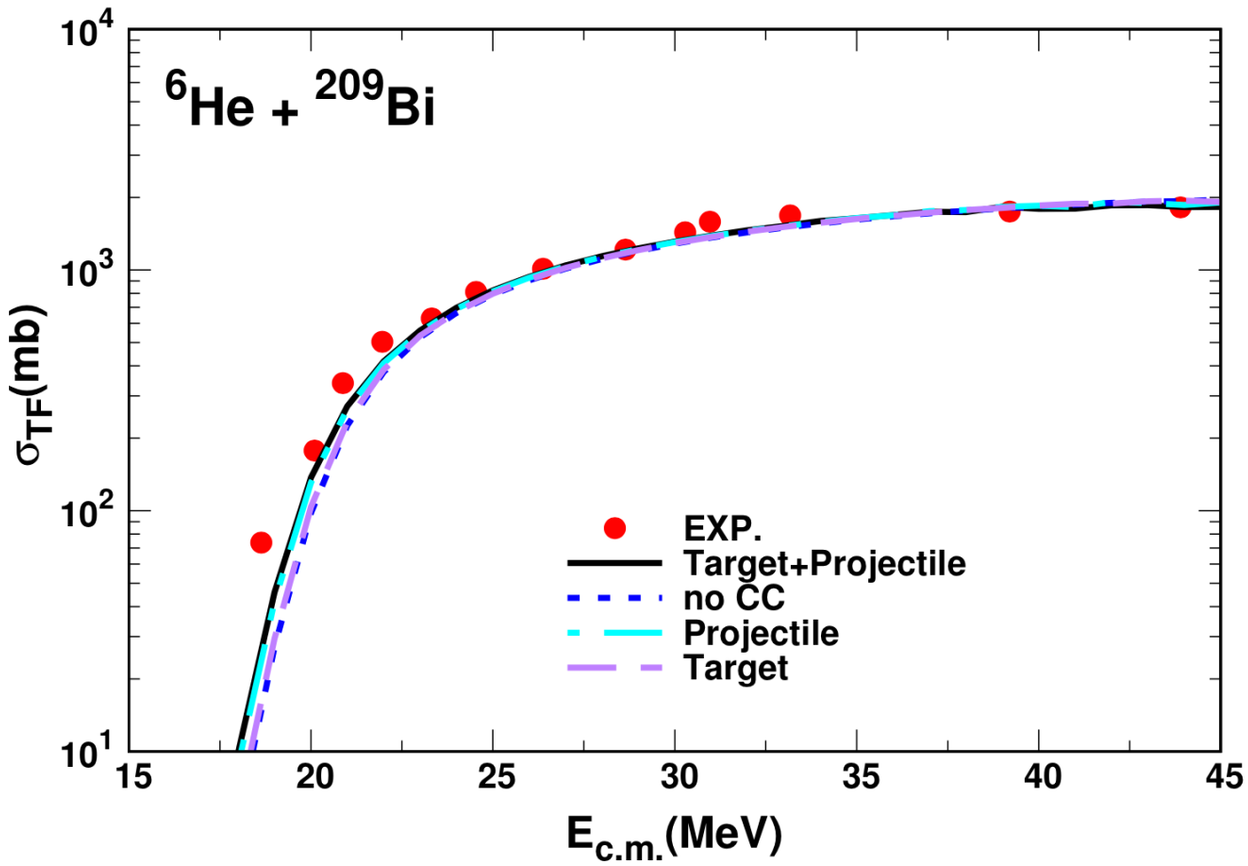


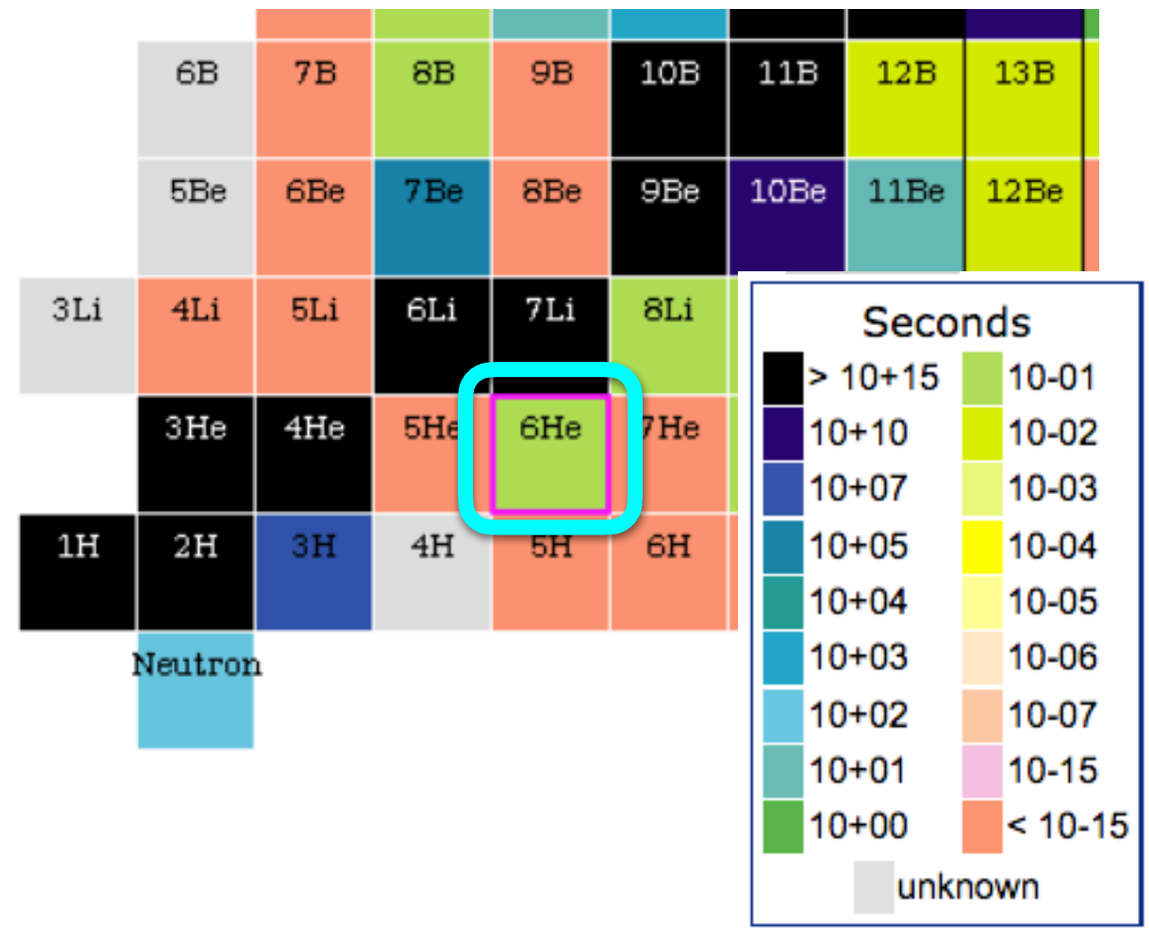
attractive nuclear force

$$V_0 = 16\pi\gamma\bar{R}a,$$
$$R_0 = R_P + R_T,$$
$$R_i = 1.20A_i^{1/3} - 0.09 \text{ fm}, \qquad (i = P, T)$$
$$\bar{R} = R_P R_T/(R_P + R_T),$$
$$\gamma = 0.95\left[1 - 1.8\left(\frac{N_P - Z_P}{A_P}\right)\left(\frac{N_T - Z_T}{A_T}\right)\right] \text{ MeV fm}^{-2},$$
$$1/a = 1.17\left[1 + 0.53\left(A_P^{-1/3} + A_T^{-1/3}\right)\right] \text{ fm}^{-1},$$

R. A. Broglia and A. Winther, Heavy-Ion Reactions (Addison-Wesley, New York, 1991).

Total fusion cross section of $^{6}He + ^{209}Bi$





A. A. Hassan et al : Bull, Rus. Acad. Sci. Phys. 70, 1558 (2006)

System	$V_N^{(0)}$			Target excitation		Projectile excitation	
	V_0 (MeV)	r_0 (fm)	a_0 (fm)	ε_x(Eλ; Band) (MeV)	β_λ	ε_x(Eλ; Band) (MeV)	β_2
$^{6}He+^{209}Bi$	41.746	1.177	0.619	0.9(E2; Vi.),2.6(E3; Vi.)	0.0056, 0.0937	1.8(E2; Vi.)	0.70

Ki-Seok Choi et. al. :JKPS. 70, 42 (2017)
M. Martin : Nuclear Data Sheets 63, 723 (1991)
E. Crema, P. R. S. Gomes, and L. C. Chamon : Phys. Rev. C 75, 037601 (2007)

NATURE 2984—22/9/2004—VBICKNELL—121305

No enhancement of fusion probability by the neutron halo of ^{6}He

R. Raabe[2,1], J. L. Sida[1*], J. L. Charvet[1], N. Alamanos[1], C. Angulo[3], J. M. Casandjian[4], S. Courtin[5], A. Drouart[1], D. J. C. Durand[1], P. Figuera[6], A. Gillibert[1], S. Heinrich[1], C. Jouanne[1], V. Lapoux[1], A. Lepine-Szily[7], A. Musumarra[6], L. Nalpas[1], D. Pierroutsakou[8], M. Romoli[8], K. Rusek[9] & M. Trotta[8]

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[4]*GANIL, BP 55027, F-14076 Caen Cedex 5, France*
[5]*Institut de Recherches Subatomiques, IN2P3-CNRS, F-67037 Strasbourg, Cedex 2, France*
[6]*INFN, Laboratori Nazionali del Sud, Via Santa Sofia 44, 95123 Catania, Italy*
[7]*Instituto de Fisica, Universidade de São Paulo, CP 66318, 05389-970, São Paulo, Brazil*
[8]*University of Napoli and INFN Sezione di Napoli, I-80125, Napoli, Italy*
[9]*Department of Nuclear Reactions, The Andrzej Soltan Institute for Nuclear Studies, Hoza 69, PL-00-681 Warsaw, Poland*

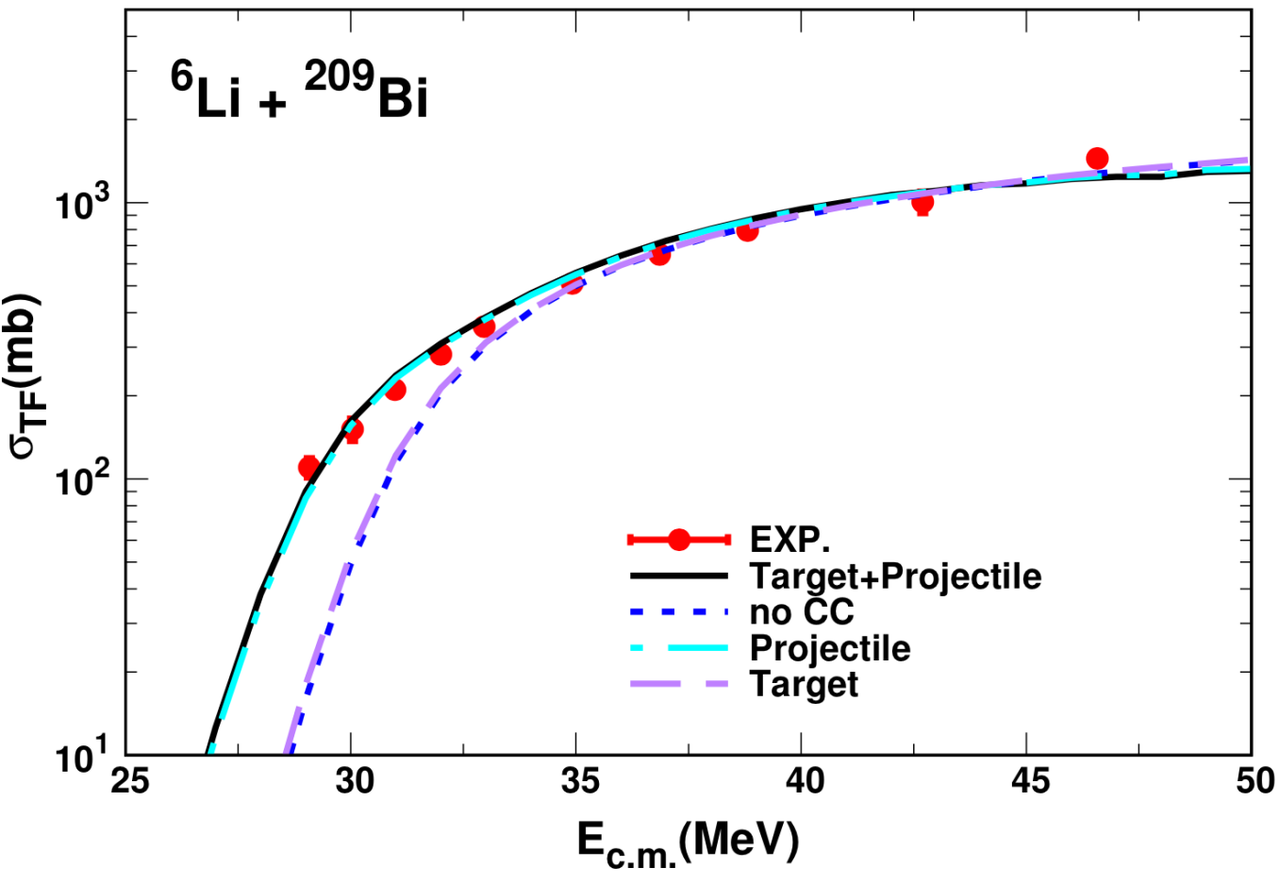
* Present address: DAM/DPTA/ Service de Physique Nucléaire, CEA DIF, F-91680 Bruyères-le-Châtel, France

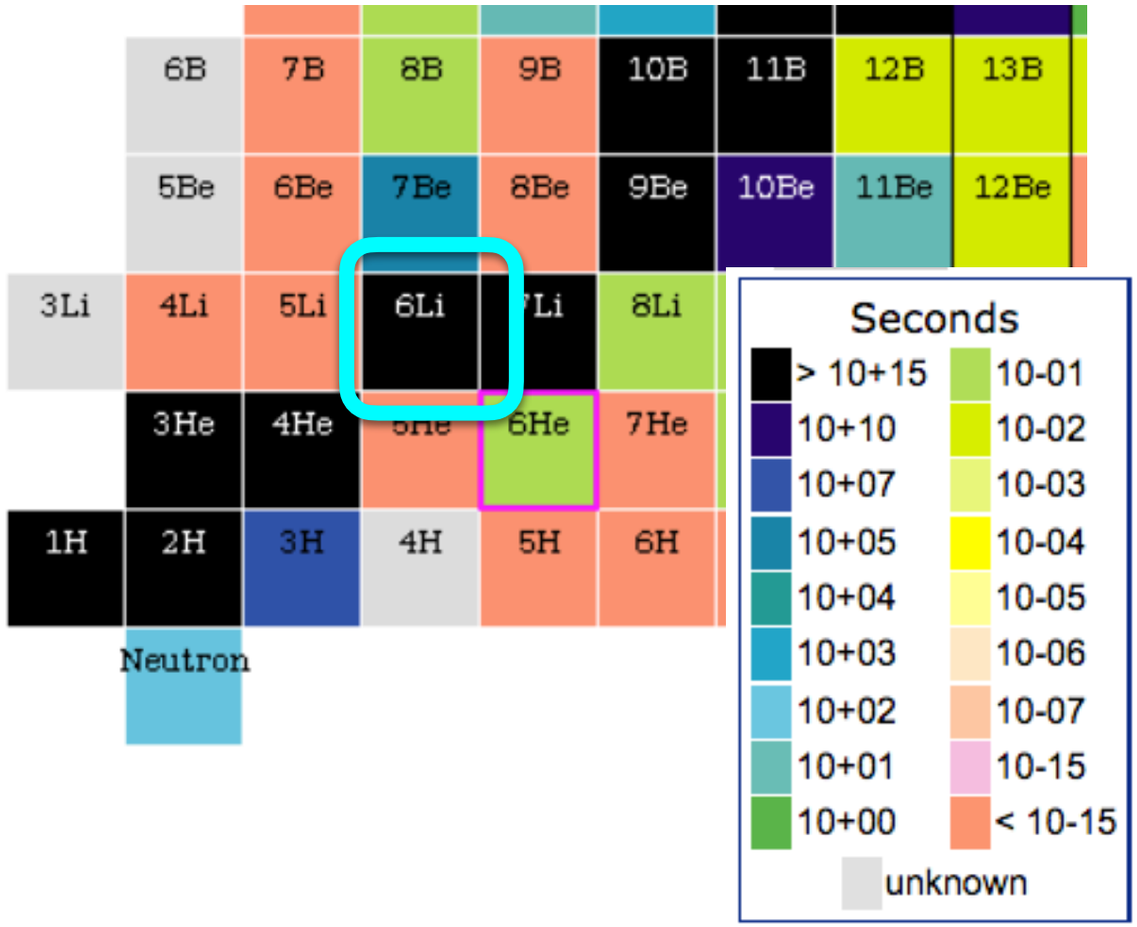
the projectile in the field of the target); however, its effect on fusion is strongly disputed. If considered as any other channel, it should enhance the sub-barrier cross-section[5,13,14]; on the other hand, the breakup process could just prevent the capture of the whole projectile, thereby inhibiting fusion[6,15–17]. Here we use 'fusion' in the sense of complete fusion[14,17–19] (the whole projectile and target fuse together; evaporation of fragments may follow) as distinct from incomplete fusion (only a fragment of the projectile is transferred to the target; in light nuclei this is not distinguishable from a direct transfer).

The halo-nucleus ^{6}He presents the peculiar properties mentioned above, and has a relatively small breakup threshold (the two-neutron separation energy is $S_{2n} = 0.973$ MeV). It is a favourable candidate for experimental studies, because it is available as a beam of accelerated ions at various facilities. However, the weak intensity of these radioactive beams sets a limit on the accuracy attainable in cross-section measurements, and the results obtained so far have not been conclusive. An enhancement was reported in the ^{6}He + ^{209}Bi fusion cross-section at energies below the potential barrier[20], and then related to the presence of strong breakup and/or transfer channels[21]. Our previous measurement with ^{6}He on ^{238}U target nuclei[22] also showed a large probability for the sum of all processes leading to the fission of the residual nucleus. The measurement reported here, however, allows the identification of the contribution of incomplete fusion (transfer) channels, which are found to be the most important in the total reaction cross-section.

Raabe et. al. :Nature. 431, 823 (2004)

Total fusion cross section of $^{6}Li + ^{209}Bi$





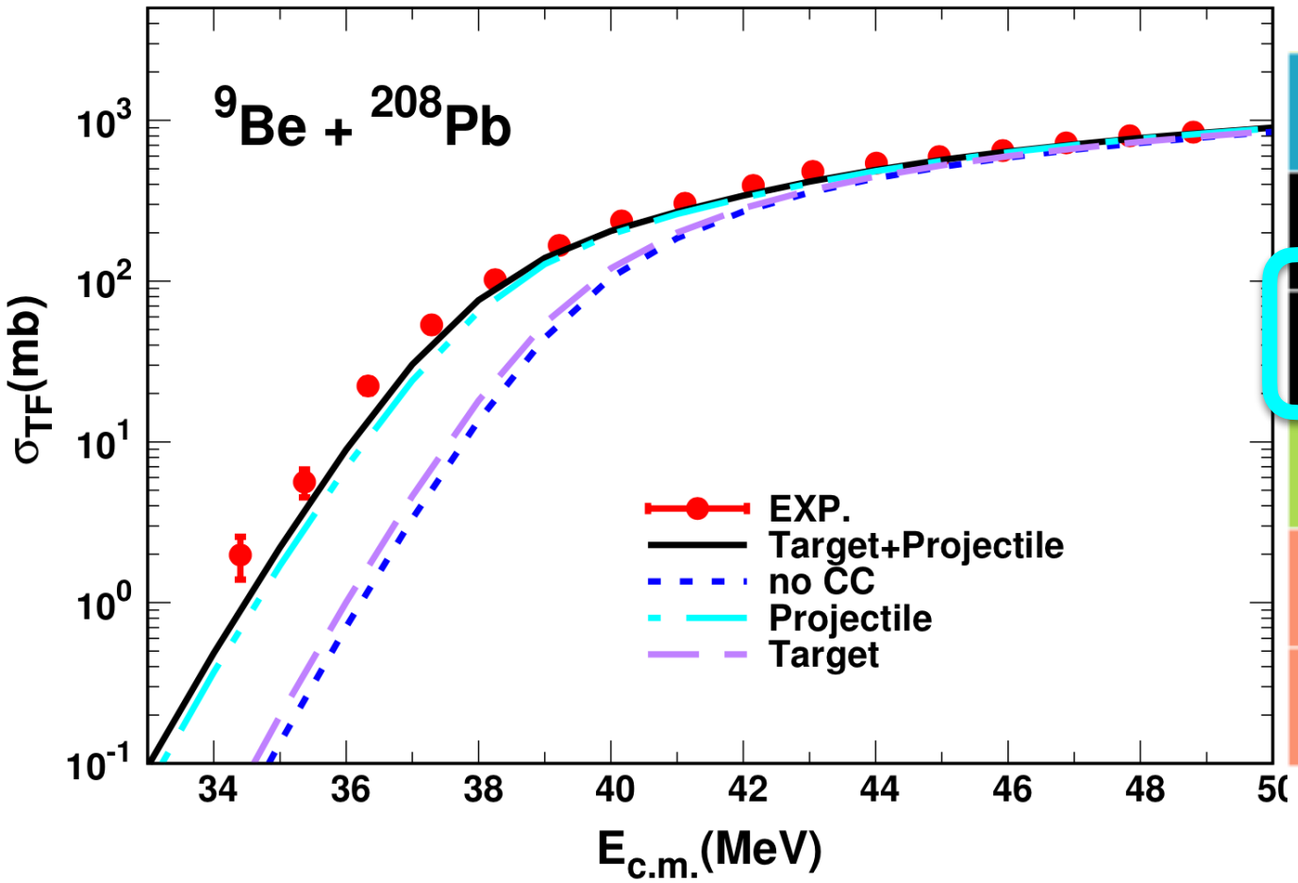
M. Dasgupta et al : Phys. Rev. C 70, 024606 (2004)

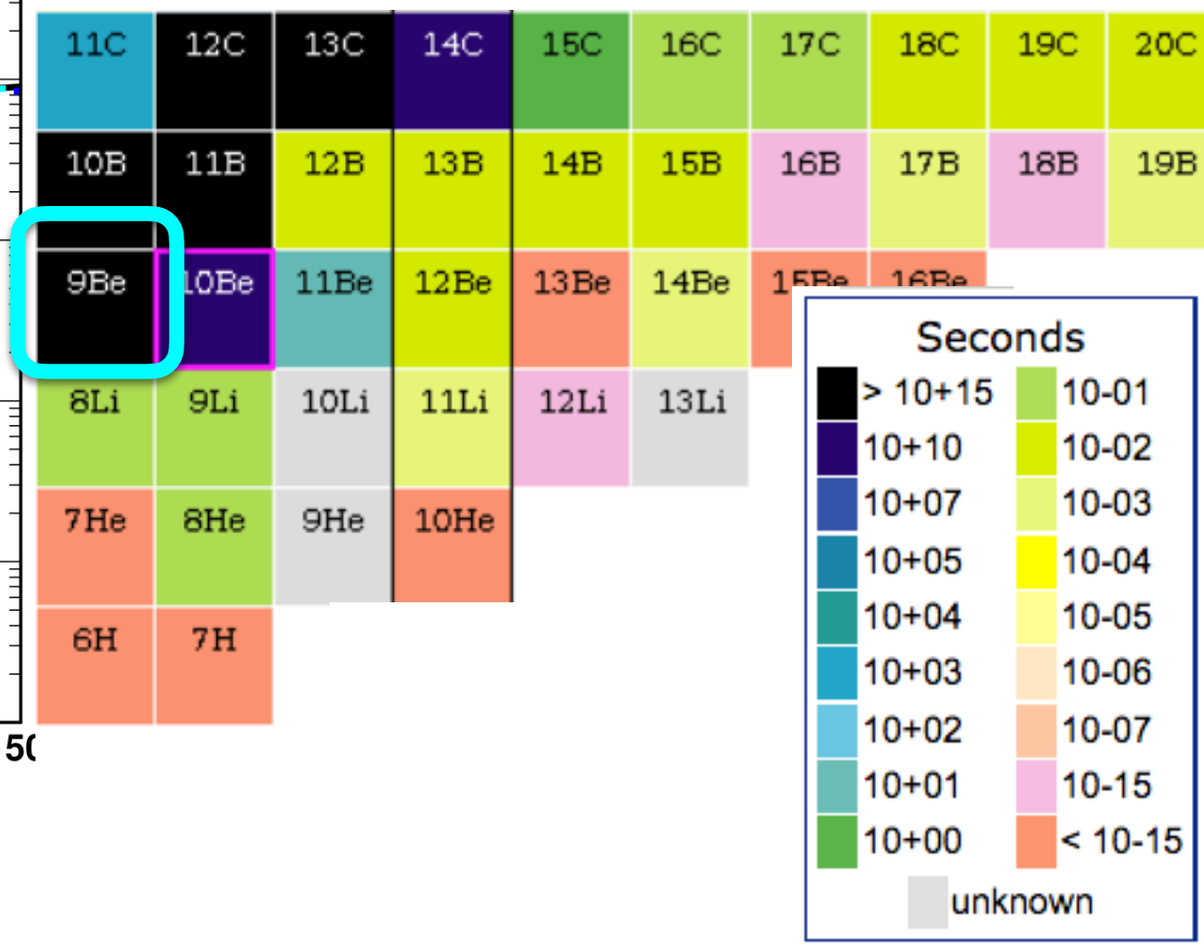
System	$V_N^{(0)}$			Target excitation		Projectile excitation	
	V_0 (MeV)	r_0 (fm)	a_0 (fm)	ε_x(Eλ; Band) (MeV)	β_λ	ε_x(Eλ; Band) (MeV)	β_2
$^{6}Li+^{209}Bi$	47.625	1.177	0.619	0.9(E2; Vi.),2.6(E3; Vi.)	0.0056, 0.0937	2.186(E2; Ro.)	1.37

Ki-Seok Choi et. al. :JKPS. 70, 42 (2017)

M. Martin : Nuclear Data Sheets 63, 723 (1991)
Y.W.Wu et al : Phys. Rev. C 68, 044605 (2007)

Total fusion cross section of $^{9}Be + ^{208}Pb$



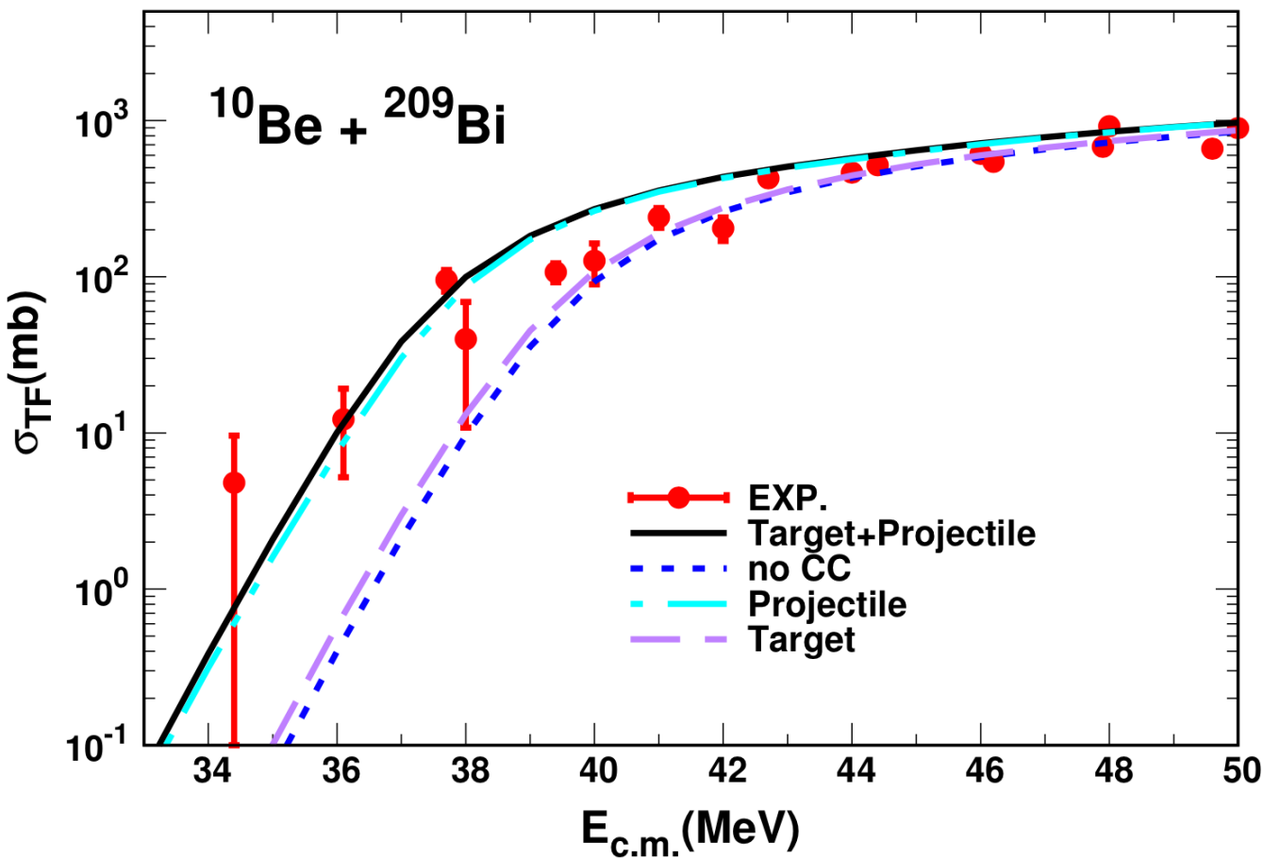


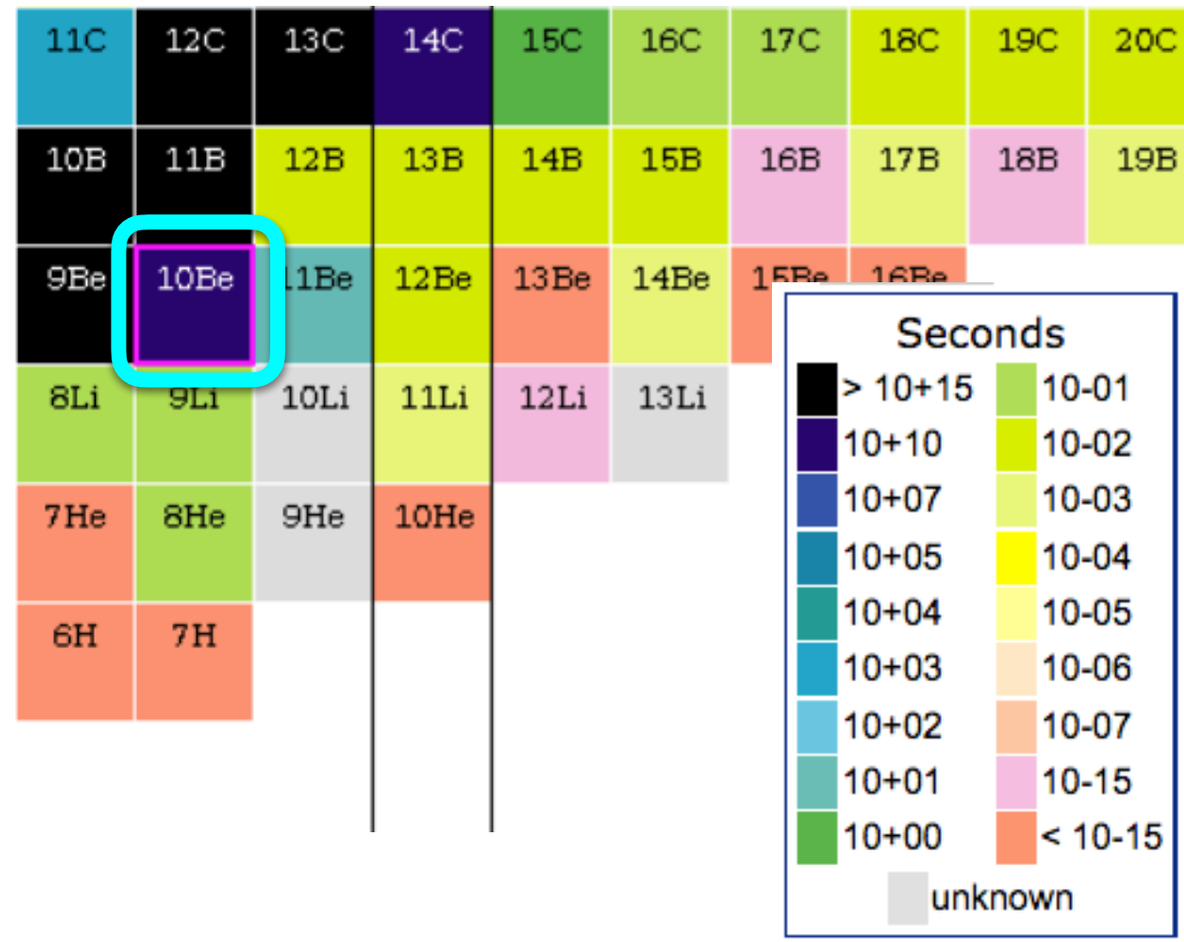
M. Dasgupta et al : Phys. Rev. C 70, 024606 (2004)

System	$V_N^{(0)}$			Target excitation		Projectile excitation	
	V_0 (MeV)	r_0 (fm)	a_0 (fm)	ε_x(Eλ; Band) (MeV)	β_λ	ε_x(Eλ; Band) (MeV)	β_2
$^{9}Be+^{208}Pb$	52.104	1.178	0.636	2.616(E3; Vi.)	0.111	2.43(E2; Ro.)	0.875

Ki-Seok Choi et. al. :JKPS. 70, 42 (2017)
R. H. Spear : At. Data Nucl. Data Tables 42, 55 (1989)
V.V. Parkar et al : Phys. Rev. C 82, 054601 (2010)

Total fusion cross section of ^{10}Be + ^{209}Bi





C. Signorini et al : Nucl. Phys. A 735, 329 (2004)

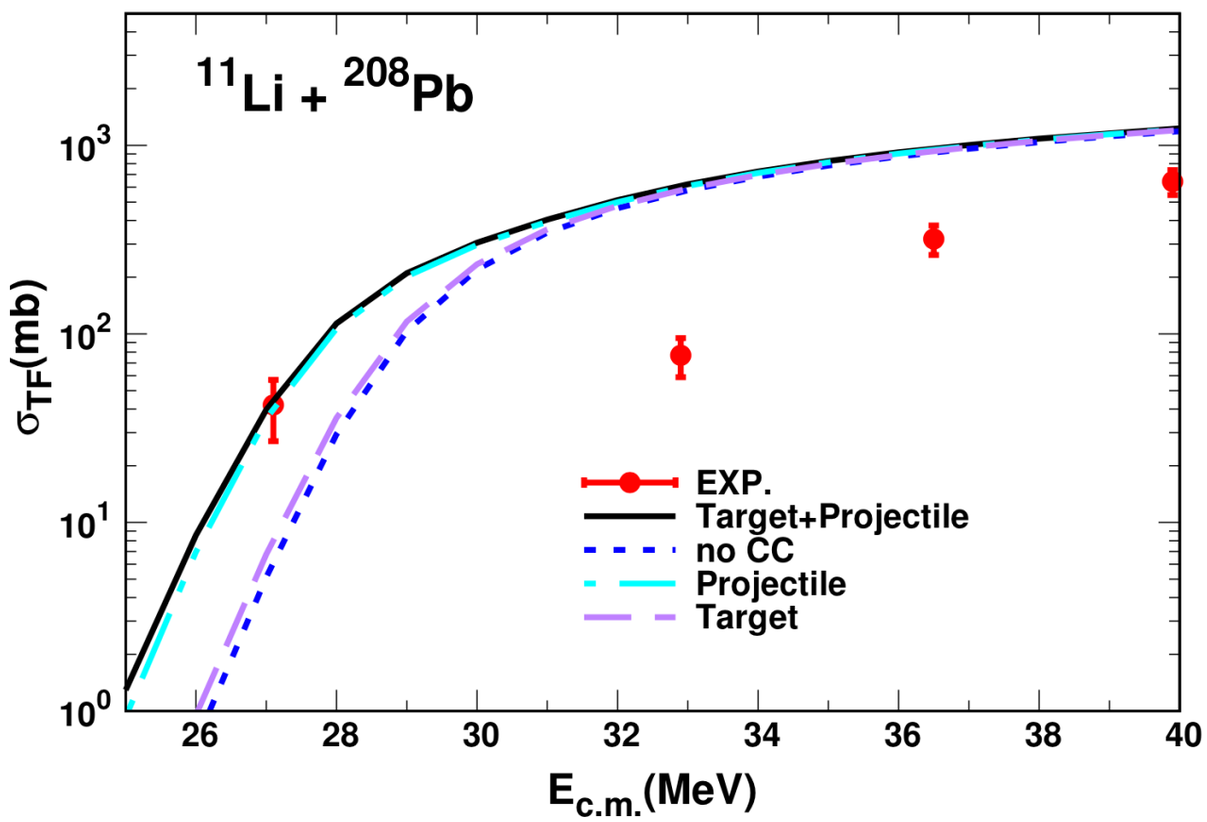
System	$V_N^{(0)}$ V_0 (MeV)	r_0 (fm)	a_0 (fm)	Target excitation ε_x(Eλ; Band) (MeV)	β_λ	Projectile excitation ε_x(Eλ; Band) (MeV)	β_2
$^{10}Be+^{209}Bi$	51.123	1.178	0.640	0.9(E2; Vi.),2.6(E3; Vi.)	0.0056, 0.0937	3.37(E2; Vi.)	1.14

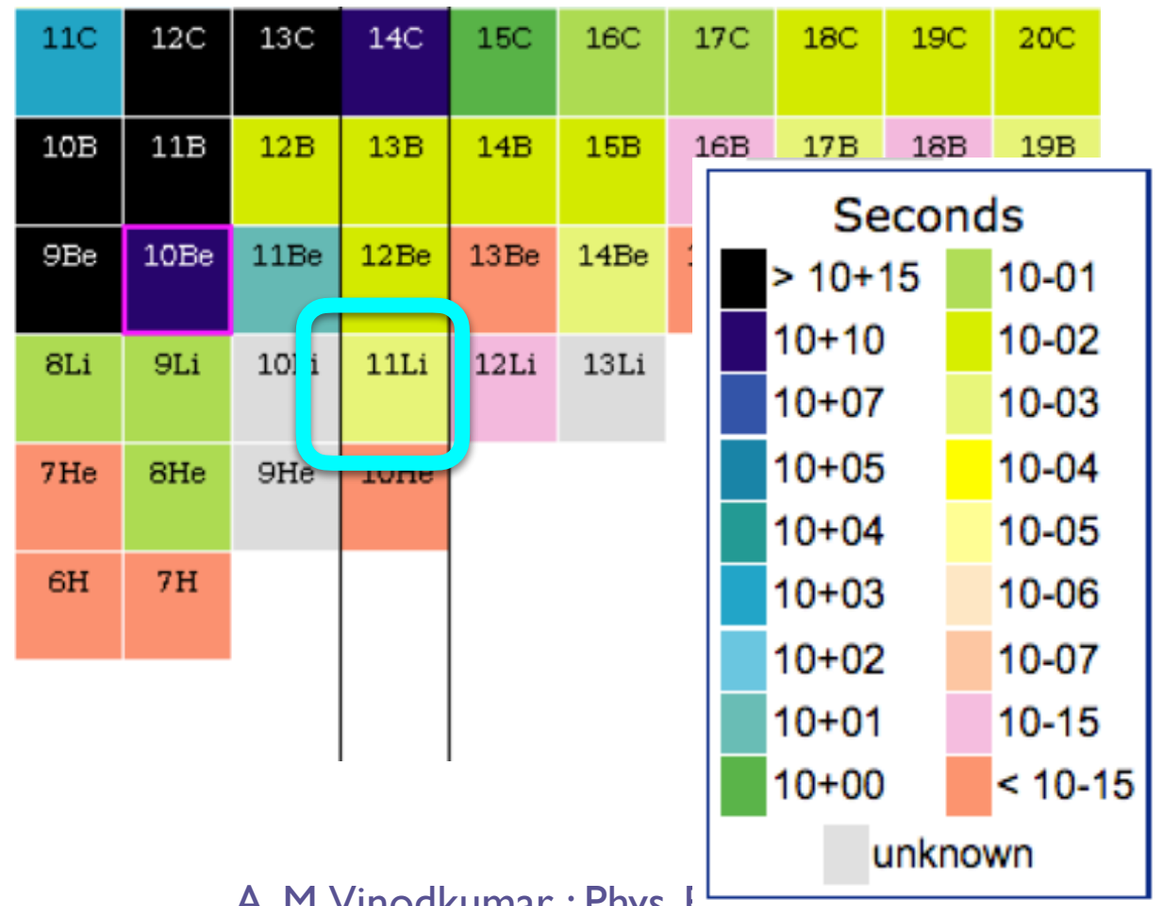
Ki-Seok Choi et. al. :JKPS. 70, 42 (2017)

M. Martin : Nuclear Data Sheets 63, 723 (1991)

C. Signorini et al : Nucl. Phys. A 735, 329 (2004)

Total fusion cross section of ^{11}Li + ^{208}Pb





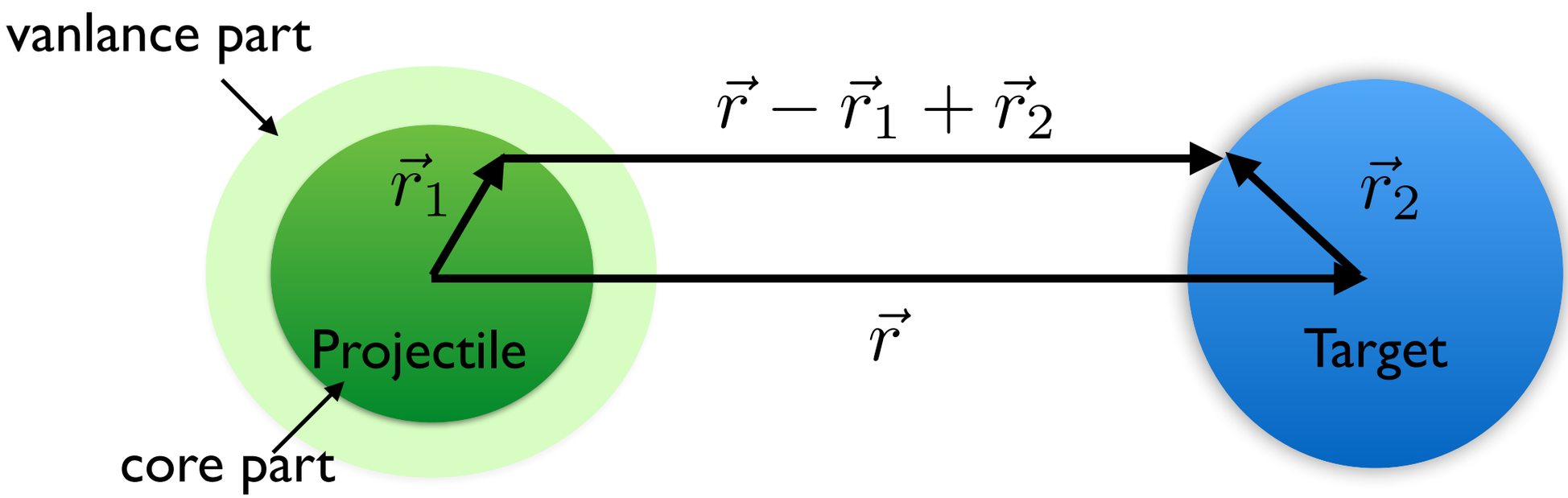
A. M.Vinodkumar : Phys. Rev. C 87, 044603 (2006)

System	$V_N^{(0)}$ V_0 (MeV)	r_0 (fm)	a_0 (fm)	Target excitation ε_x(Eλ; Band) (MeV)	β_λ	Projectile excitation ε_x(Eλ; Band) (MeV)	β_2
$^{11}Li+^{208}Pb$	47.938	1.178	0.644	2.616(E3; Vi.),	0.111	1.266(E2; Vi.)	1.0

Ki-Seok Choi et. al. :JKPS. 70, 42 (2017)

A. M.Vinodkumar : Phys. Rev. C 87, 044603 (2006)

Folding Potential assumption for ^{11}Li + ^{208}Pb

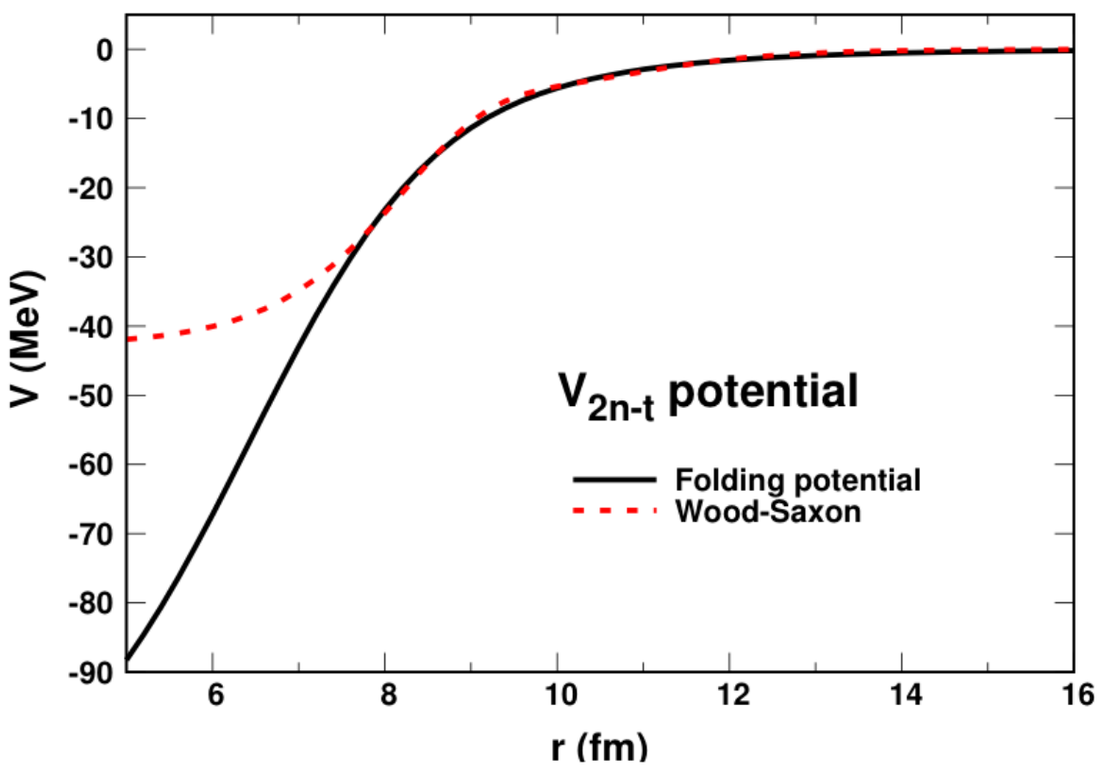


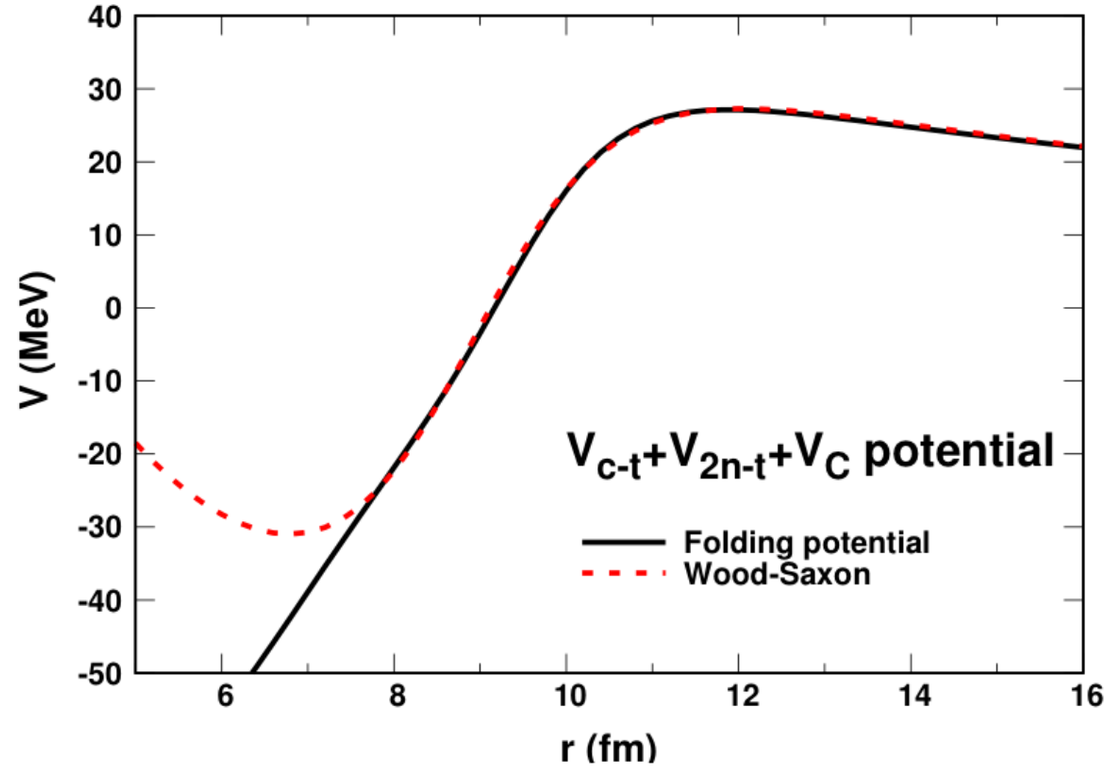
$$V(r) = \int d\mathbf{r}_1 \int d\mathbf{r}_2 \rho_p(r_1)\rho_t(r_2) V_{NN}\left(r_{12} = |\mathbf{r} - \mathbf{r}_1 + \mathbf{r}_2|\right)$$

$$\sim \quad = \int d\mathbf{r}_1 \int d\mathbf{r}_2 \left(\rho_c(r) + \rho_{2n}(r)\right) \rho_t(r_2) V_{NN}\left(r_{12}\right)$$

$$= V_{ct}(r) + V_{2nt}(r),$$

Folding Potential assumption for $^{11}Li + ^{208}Pb$





$$V(r) = \int d\mathbf{r}_1 \int d\mathbf{r}_2 \left(\rho_c(r) + \rho_{2n}(r)\right) \rho_t(r_2) V_{NN}(r_{12})$$

$$= V_{ct}(r) + V_{2nt}(r), \sim V_{WS}^{ct}(\vec{R}) + V_{WS}^{2nt}(\vec{R})$$

Modification of coupled channel equation

$$\begin{pmatrix} K+V_1-E & F_{1\to2}(r) & 0 \\ F_{1\to2}(r) & K+V_2-(E-Q_2) & F_{2\to3}(r) \\ 0 & F_{2\to3}(r) & K+V_2-(E-Q_2-Q_{23}) \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} = 0$$

$^{11}\mathrm{Li}+^{208}\mathrm{Pb}$			$^{9}\mathrm{Li}+^{210}\mathrm{Pb}$			$^{7}\mathrm{Li}+^{212}\mathrm{Pb}$		
Ch 1			Ch 2			Ch 3		
V_0	r_0	a_0	V_{02}	r_{02}	a_{02}	V_{03}	r_{03}	a_{03}
90.309	1.090	0.852	47.304	1.178	0.636	47.298	1.177	0.626
WS by folding potential			AW potential			AW potential		

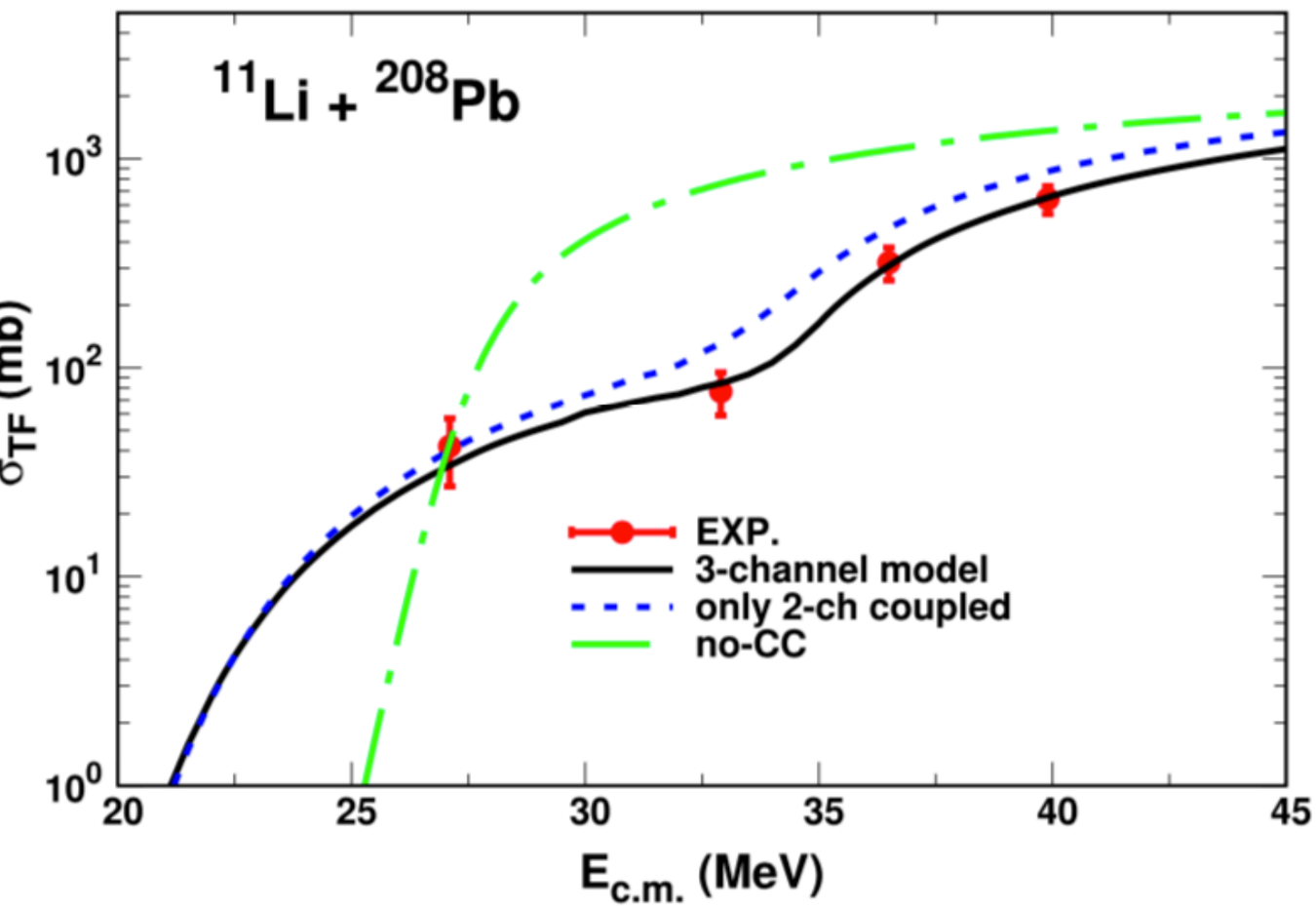
$F_{2\to3}(r)$ form factor

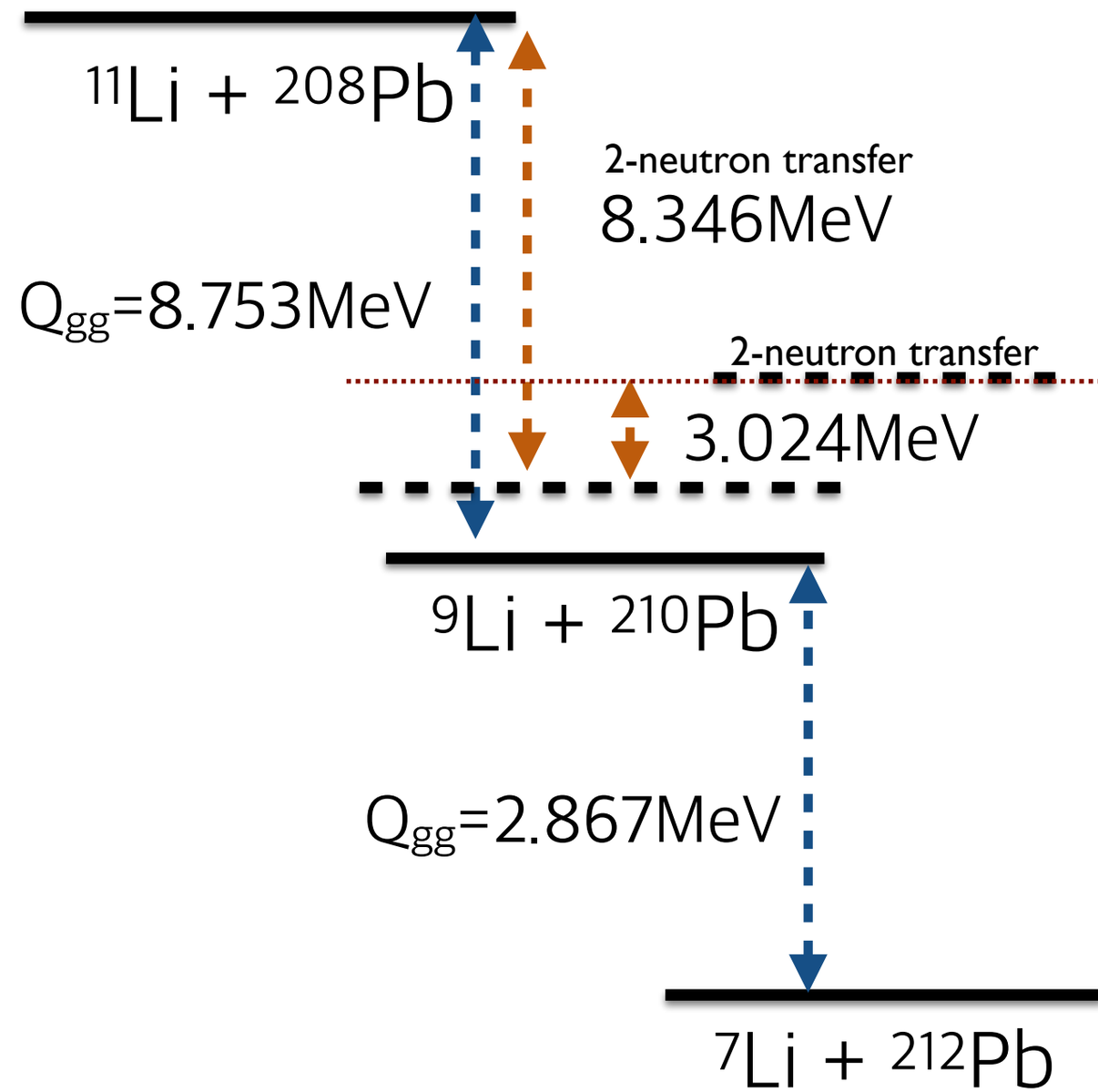
Q-value	F_t(MeVfm)	r_{coup}(fm)	a_{coup}(fm)
-3.204	51.367	1.357	0.264

$F_{1\to2}(r)$ 4 fitting parameters

$F_{2\to3}(r)$ 4 fitting parameters

^{11}Li + ^{208}Pb fusion reaction with transfer contribution





channel potential

V_0	r_0	a_0	V_{02}	r_{02}	a_{02}	V_{03}	r_{03}	a_{03}
90.309	1.090	0.852	47.304	1.178	0.636	47.298	1.177	0.626
^{11}Li + ^{208} Pb			^{9}Li + ^{210} Pb			^{7}Li + ^{212} Pb		

form factor fitting parameters

Channel	Q (MeV)	F_t (MeV fm)	r_{coup} (fm)	a_{coup} (fm)
$1 \rightarrow 2$	+8.346	40.227	1.666	0.857
$2 \rightarrow 3$	−3.204	51.367	1.357	0.264

Ki-Seok Choi et. al. :PLB. 780, 455 (2018)
A. M.Vinodkumar : Phys. Rev. C 87, 044603 (2006)

PRL **106,** 172701 (2011) PHYSICAL REVIEW LETTERS week ending 29 APRIL 2011

Fusion Reactions with the One-Neutron Halo Nucleus ^{15}C

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The structure of ^{15}C, with an $s_{1/2}$ neutron weakly bound to a closed-neutron shell nucleus ^{14}C, makes it a prime candidate for a one-neutron halo nucleus. We have for the first time studied the cross section for the fusion-fission reaction ^{15}C + ^{232}Th at energies in the vicinity of the Coulomb barrier and compared it to the yield of the neighboring ^{14}C + ^{232}Th system measured in the same experiment. At sub-barrier energies, an enhancement of the fusion yield by factors of 2–5 was observed for ^{15}C, while the cross sections for ^{14}C match the trends measured for ^{12,13}C.

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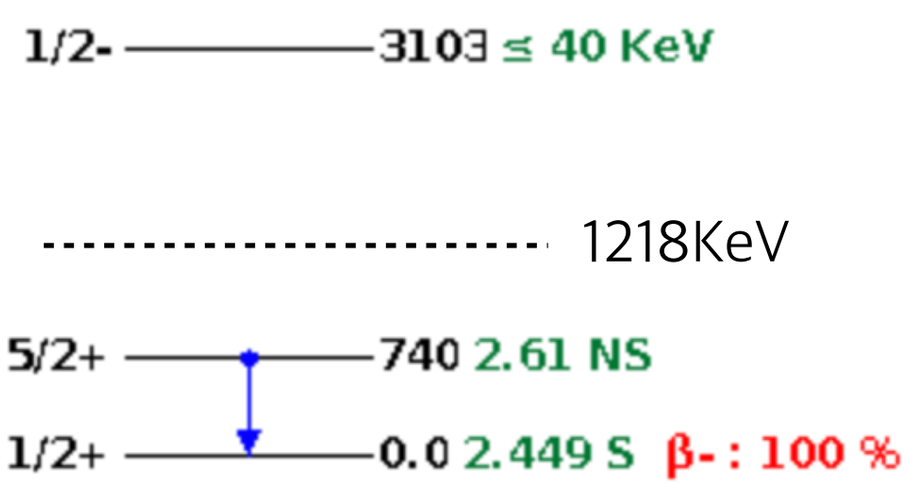
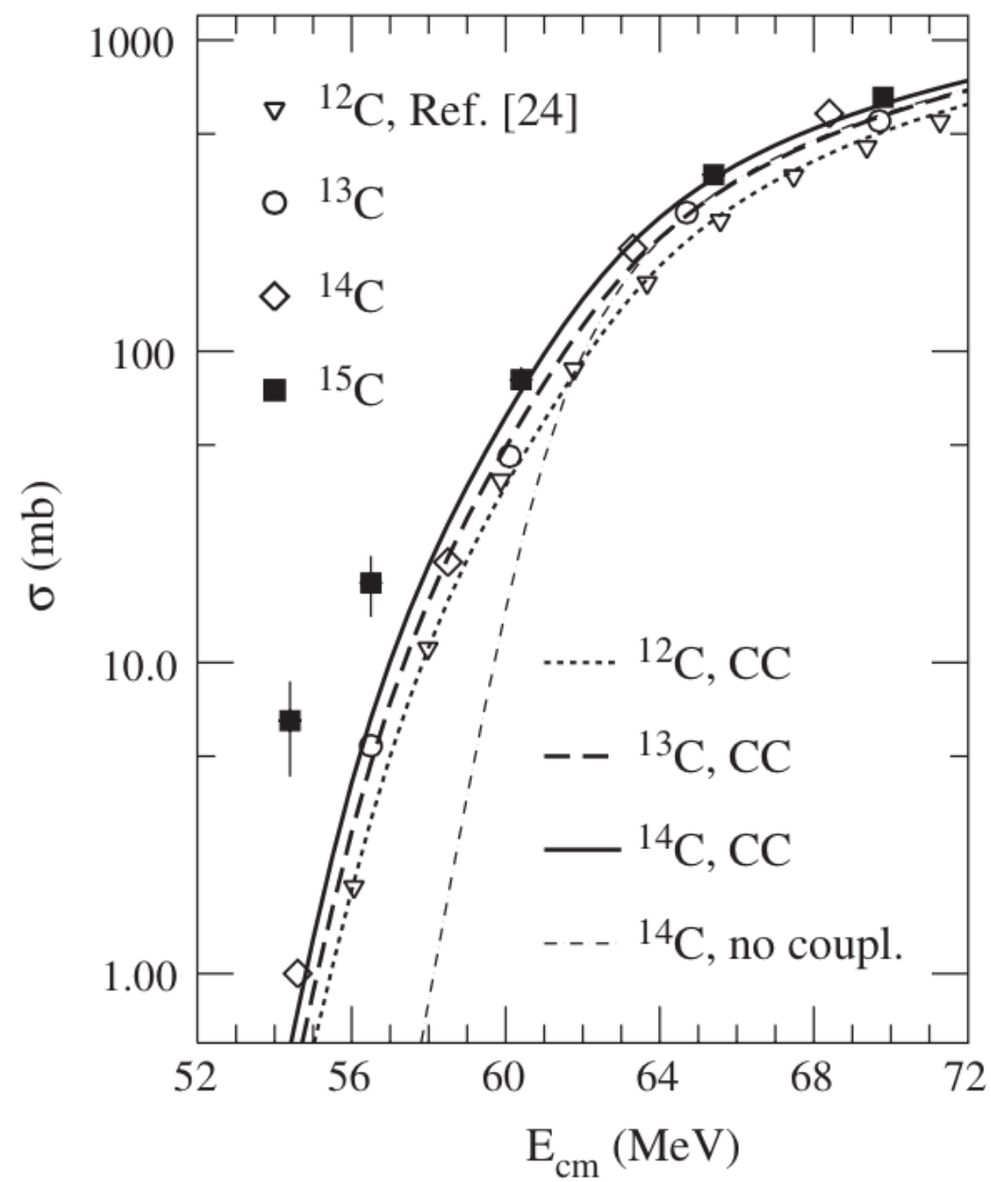
Q(β-)=9771.7 keV *8* S_n= 1218.1 keV *8*

S_p= 21080 keV 22 Q_α= -12728.9 keV *9*

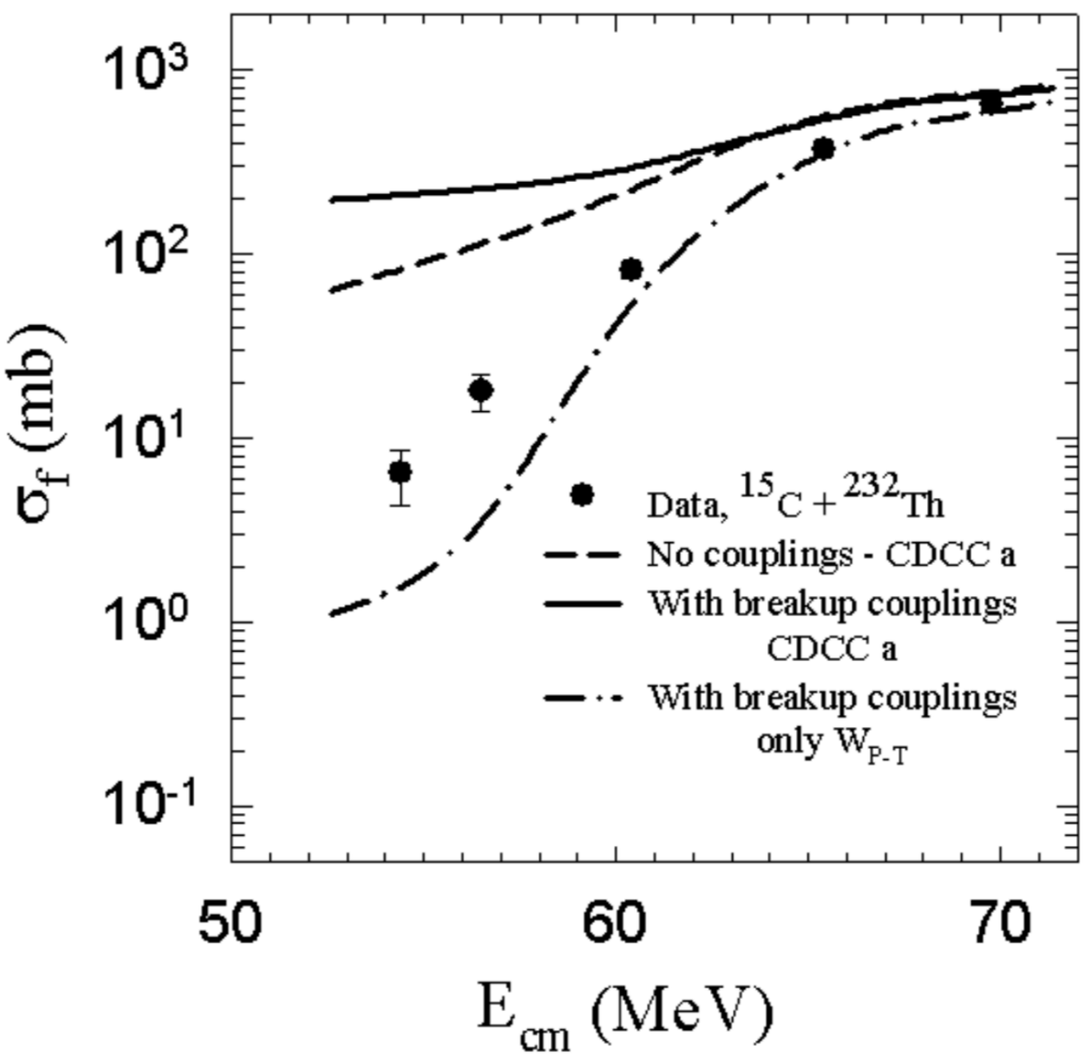


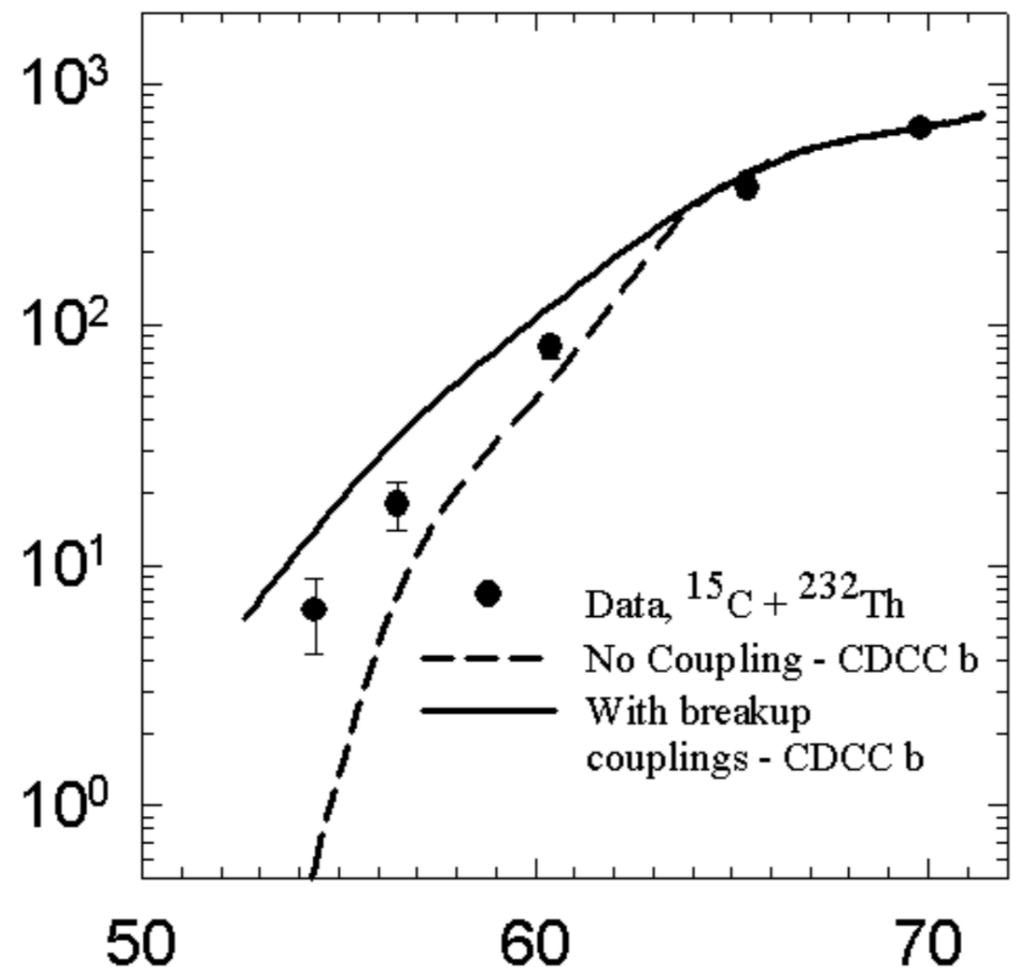


Proceedings of the DAE-BRNS Symp. on Nucl. Phys. 61 (2016) 636

Role of continuum couplings in the fusion of ^{15}C with ^{232}Th nucleus

V. Jha[1*] and A. Mitra[1]
[1] *Nuclear Physics Division, Bhabha Atomic Research Centre, Mumbai - 400085, INDIA*





Coupled channel equation

$$\begin{pmatrix} K + V_1 - E & F_{1\to 2}(r) \\ F_{1\to 2}(r) & K + V_2 - (E - Q_2) \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = 0.$$

Ch1 : $^{15}\mathrm{C} + ^{232}\mathrm{Th}$

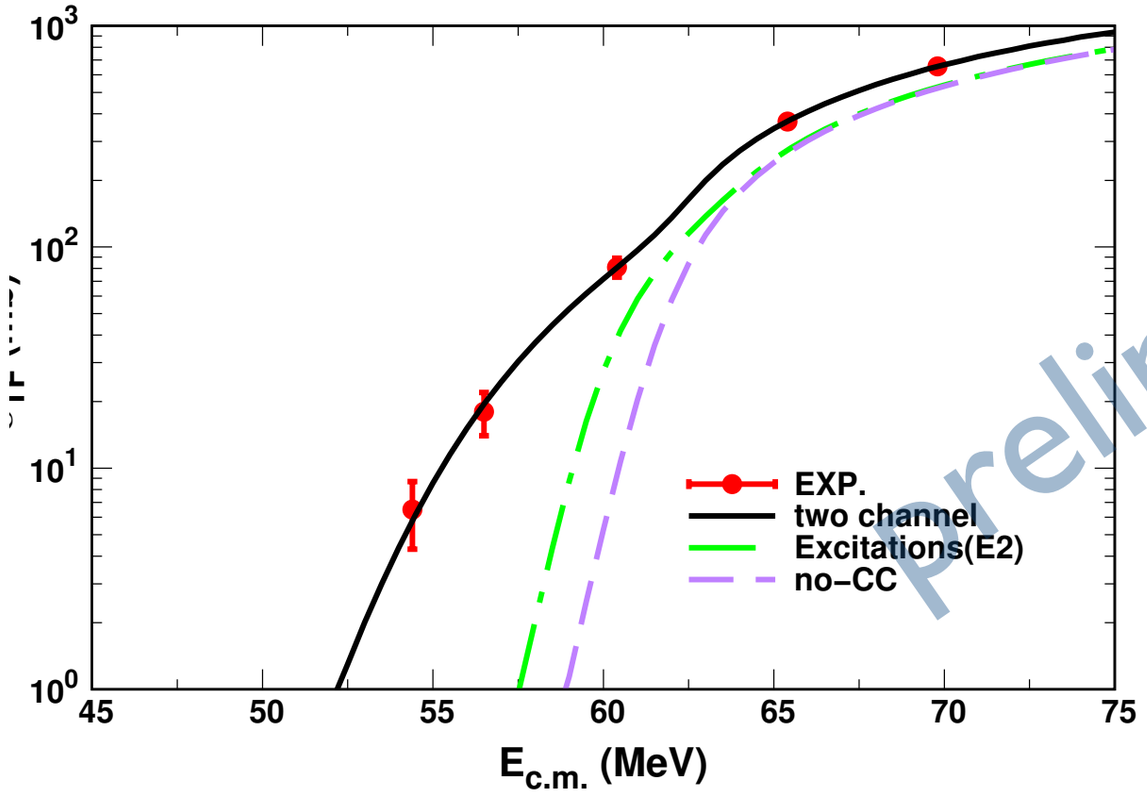
Ch2 : $^{14}\mathrm{C} + ^{233}\mathrm{Th}$

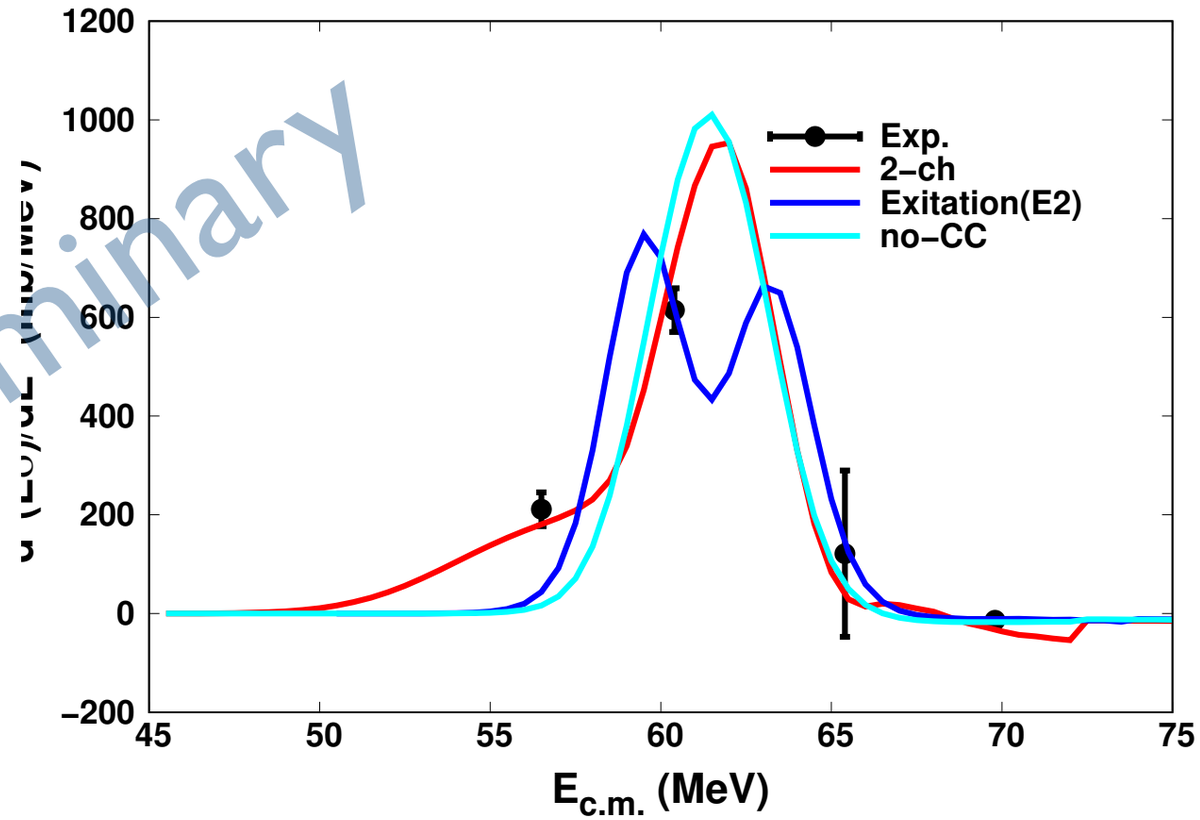
1-neutron transfer

$$F_{i\to j}(r) = \beta \frac{d}{dr}\left(\frac{1}{1 + \exp((r - R_{coup})/a_{coup})}\right)$$

:transition form factor

$^{15}C + ^{232}Th$ fusion reaction with transfer channel





preliminary

potentials

V_0	r_0	a_0	V_{02}	r_{02}	a_{02}
59.359	1.179	0.657	59.564	1.179	0.654

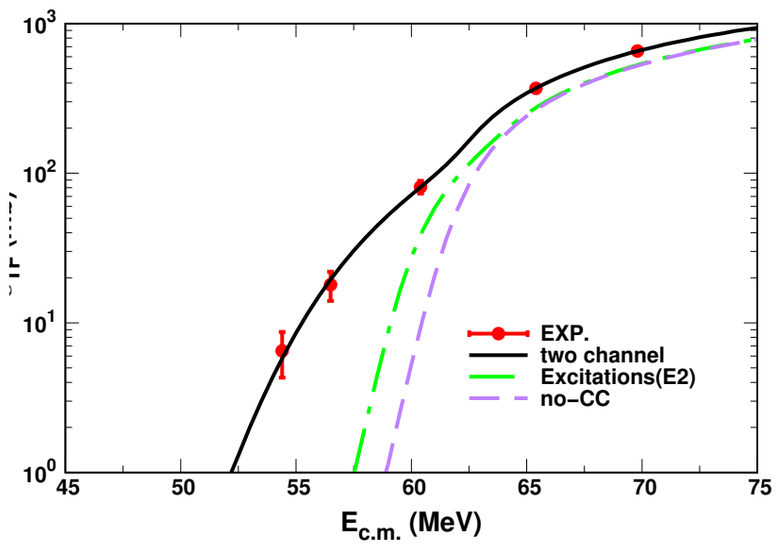
$^{15}C + ^{232}Th$ $^{14}C + ^{233}Th$

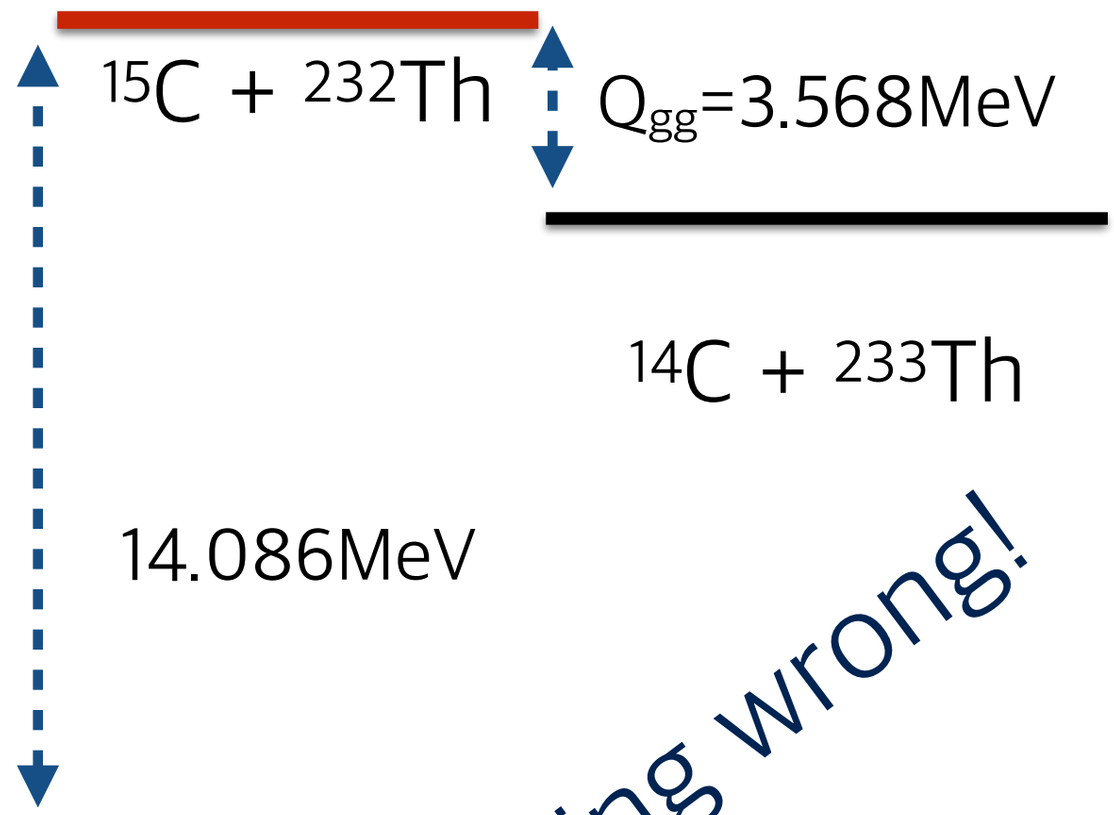
form factor

Ω	β	r_{coup}	a_{coup}
-14.086	9.348	1.543	0.268

χ
0.047

$^{15}C + ^{232}Th$ fusion reaction with transfer channel





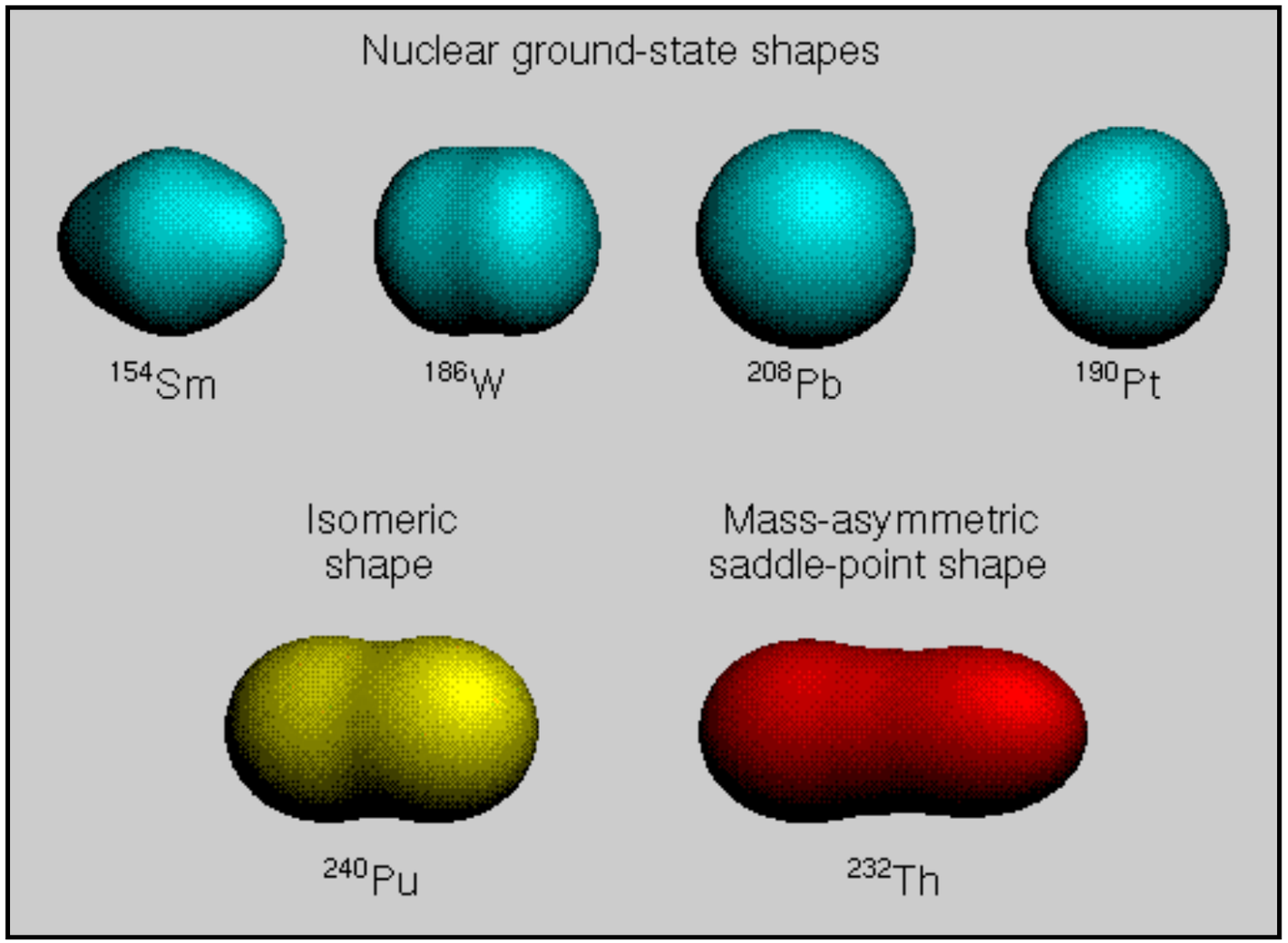
Something wrong!

fitting set.

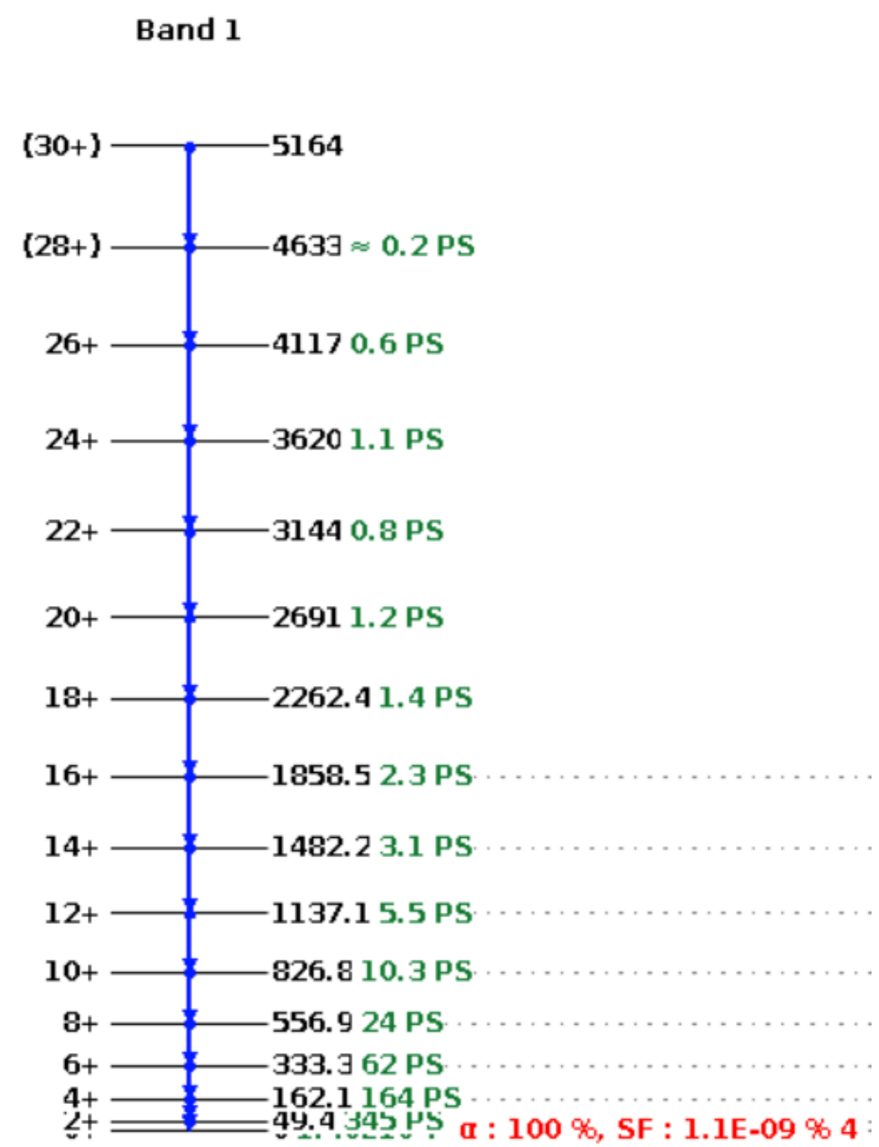
V_0	r_0	a_0	V_{02}	r_{02}	a_{02}
59.359	1.179	0.657	59.564	1.179	0.654

Ω	β	r_{coup}	a_{coup}
-14.086	9.348	1.543	0.268

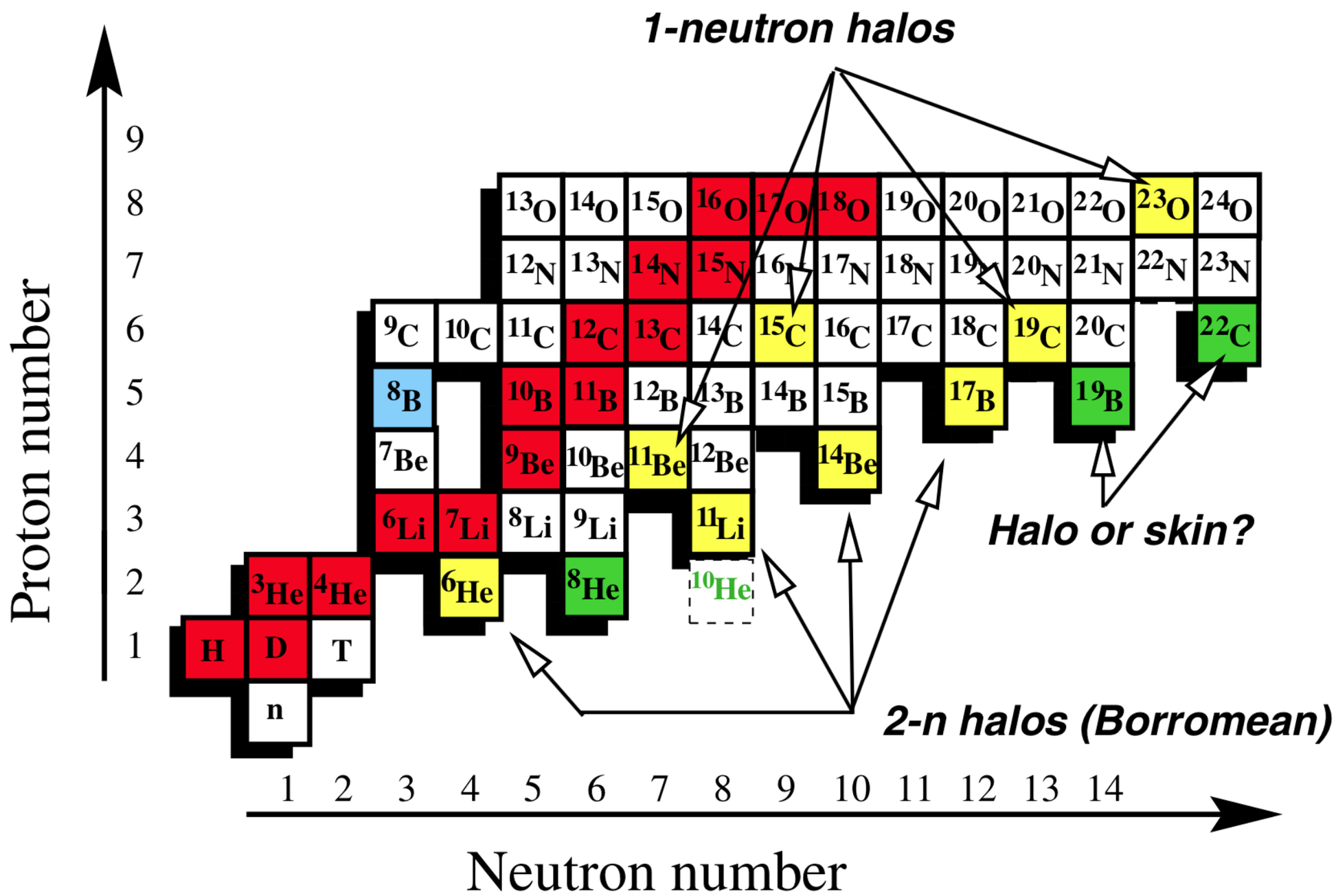
Nuclear reaction with exotic nuclei



^{232}Th level



^{15}C is halo nuclei

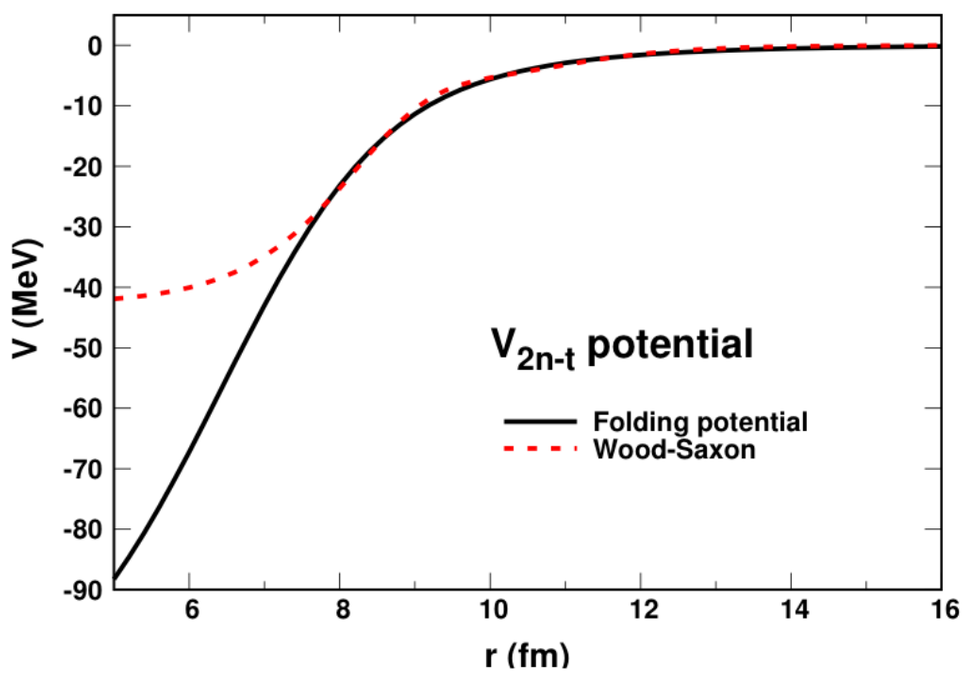


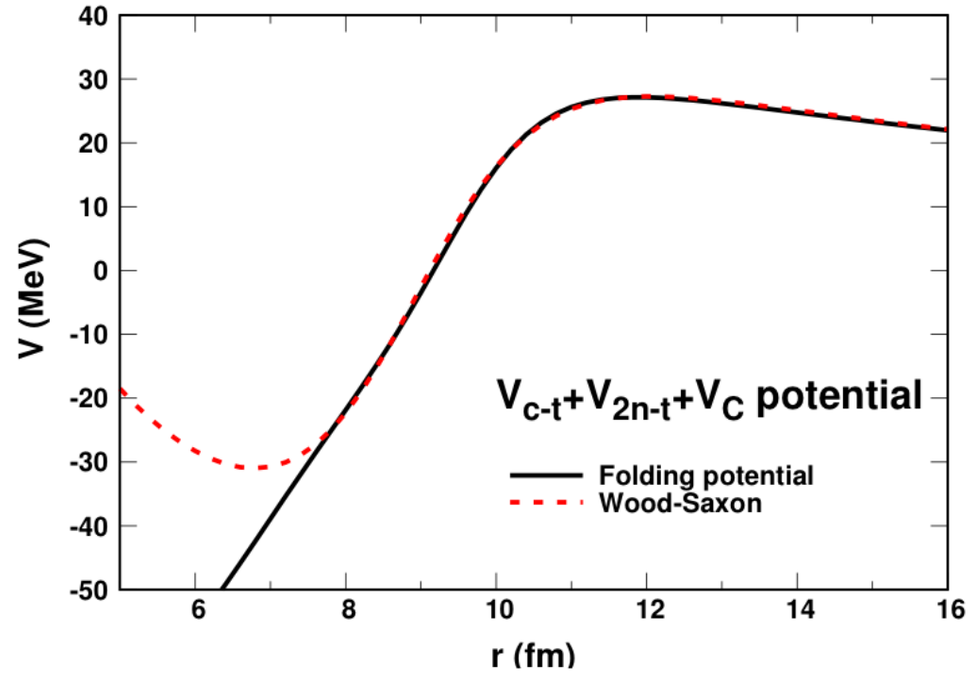
1. I. Tanihata et al : Phys. Lett . B 206, 592 (1998)
2. J.S. AL-Khalili et al : Phys. Rev. C 54 1843 (1996)

$^{15}C + ^{232}Th$

Halo nuclei + deformed Target…….

recall folding potential assumption

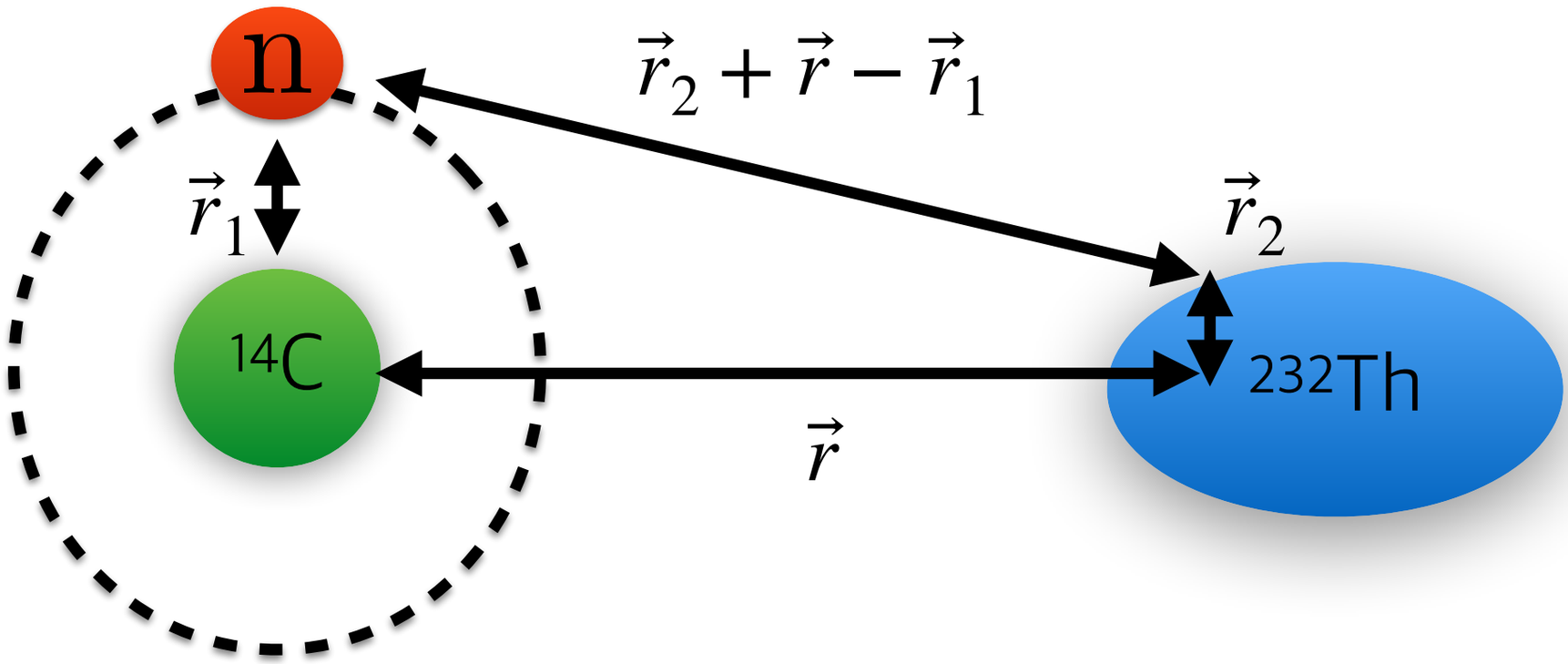




$$V(r) = \int d\mathbf{r}_1 \int d\mathbf{r}_2 \left(\rho_c(r) + \rho_{2n}(r)\right) \rho_t(r_2) V_{NN}(r_{12})$$

$$= V_{ct}(r) + V_{2nt}(r), \sim V_{WS}^{ct}(\vec{R}) + V_{WS}^{2nt}(\vec{R})$$

folding potential including halo and deformation



$$V_{^{15}C-T}(r,\theta) = \int d\mathbf{r}_1 \int d\mathbf{r}_2 \rho_p(r_1)\rho_T(r_2,\theta) V_{NN}\left(r_{12} = |\mathbf{r} - \mathbf{r}_1 + \mathbf{r}_2|\right)$$

$$V_{^{15}C-T}(r,\theta) = V_{^{14}C-T}(r,\theta) + V_{n-T}(r,\theta)$$

$$V_{n-T}(r,\theta) = \sum_{\lambda} V_{\lambda}(r) Y_{\lambda 0}(\theta)$$

folding potential including halo and deformation

$$V_{^{15}C-T}(r,\theta) = V_{^{14}C-T}(r,\theta) + V_{n-T}(r,\theta)$$

$$\frac{V_0}{1 + e^{(r - R_0 - R_0\beta_2 Y_{20}(\theta))/a}}$$

$$\sum_{\lambda} V_{\lambda}(r) Y_{\lambda 0}(\theta)$$

$$V_0(r)Y_{00}(\theta) + V_2(r)Y_{20}(\theta) + V_4(r)Y_{40}(\theta)$$

folding potential including halo and deformation

$$V(\vec{r}) = V_{ct-WS}(r,\theta) + \sum_{\lambda} V_{\lambda}(r) Y_{\lambda 0}(\theta)$$

$$\frac{V_0}{1 + e^{(r - R_0 - R_0\beta_2 Y_{20}(\theta))/a}}$$

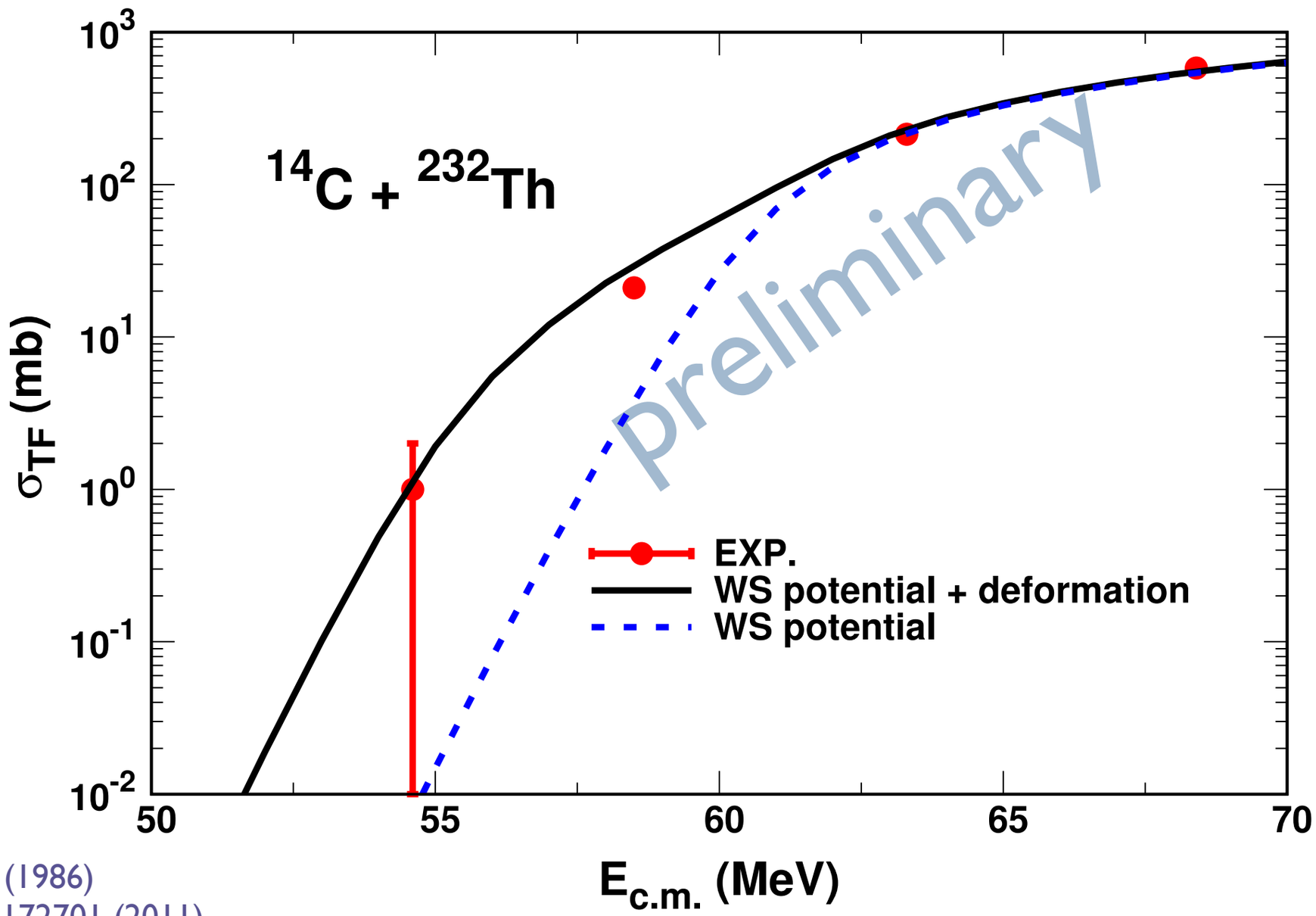
$^{14}C + ^{232}Th$

WS potential

V_0	r_0	a_0
90.0	1.179	0.654

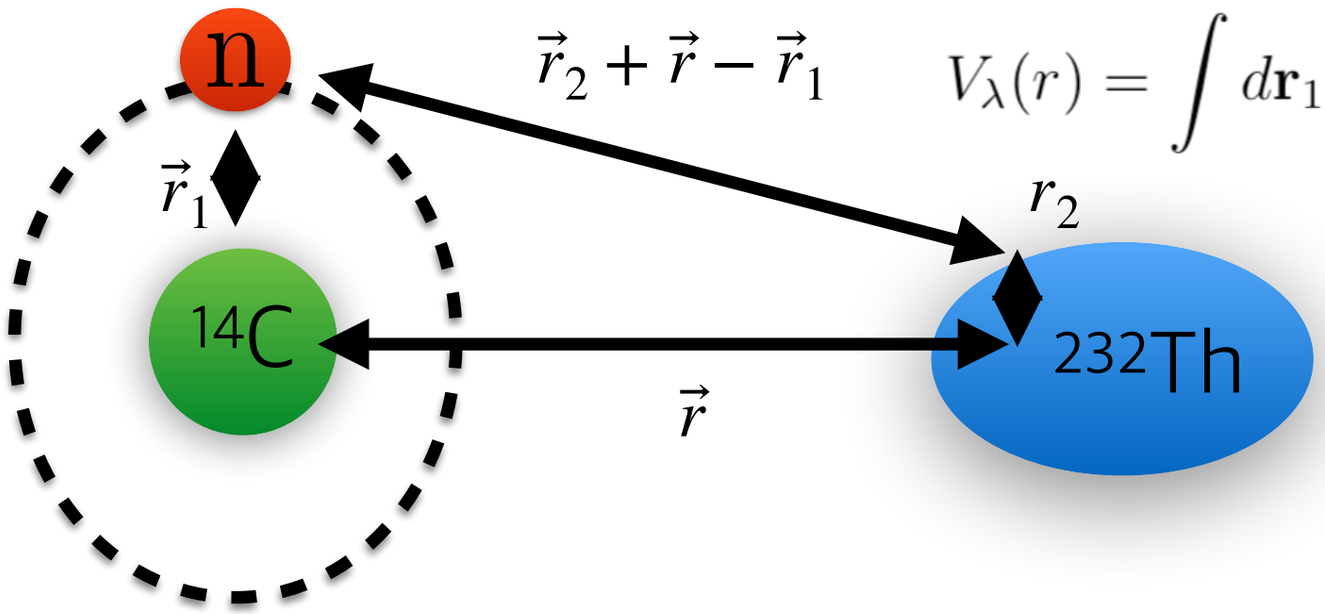
deformation parameter

β_2	β_4
0.233	0.0946



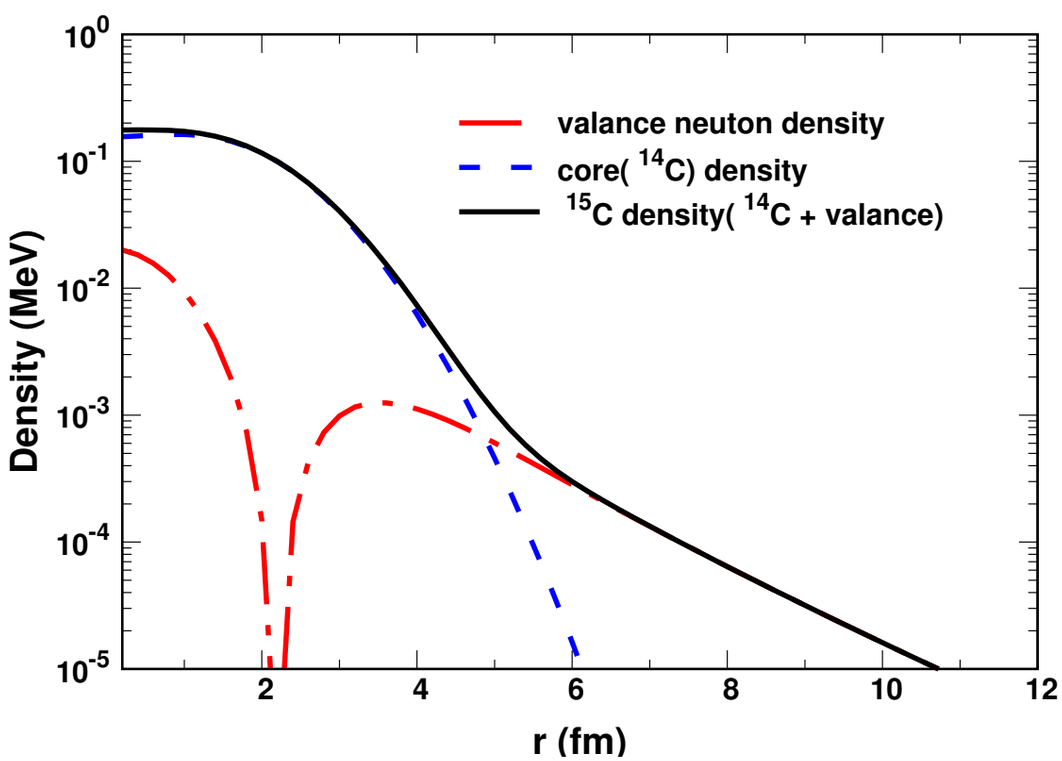
J. Zumbro et. al. : Phys. Lett. B 167, 383 (1986)
M. Alcorta et. al. : Phys. Rev. Lett. 106, 172701 (2011)

folding potential including halo and deformation



$$V_\lambda(r) = \int d\mathbf{r}_1 \int d\mathbf{r}_2 \rho_n(r_1)\rho_{T\lambda}(r_2) V_{NN}\left(r_{12} = |\mathbf{r} - \mathbf{r}_1 + \mathbf{r}_2|\right)$$

J. Zumbro et. al. : Phys. Lett. B 167, 383 (1986)
H. D. Wries et. al. : At. Data Nucl. Data Tables 36,495 (1987)



$$\rho_T(r,\theta) = \frac{\rho_0}{1 + \exp\left[\left(\frac{r - c(1+\sum_\lambda \beta_{\lambda T} Y_{\lambda 0}(\theta))}{z}\right)\right]}$$

$$= \sum_\lambda \rho_{T\lambda}(r) Y_{\lambda 0}(\theta)$$

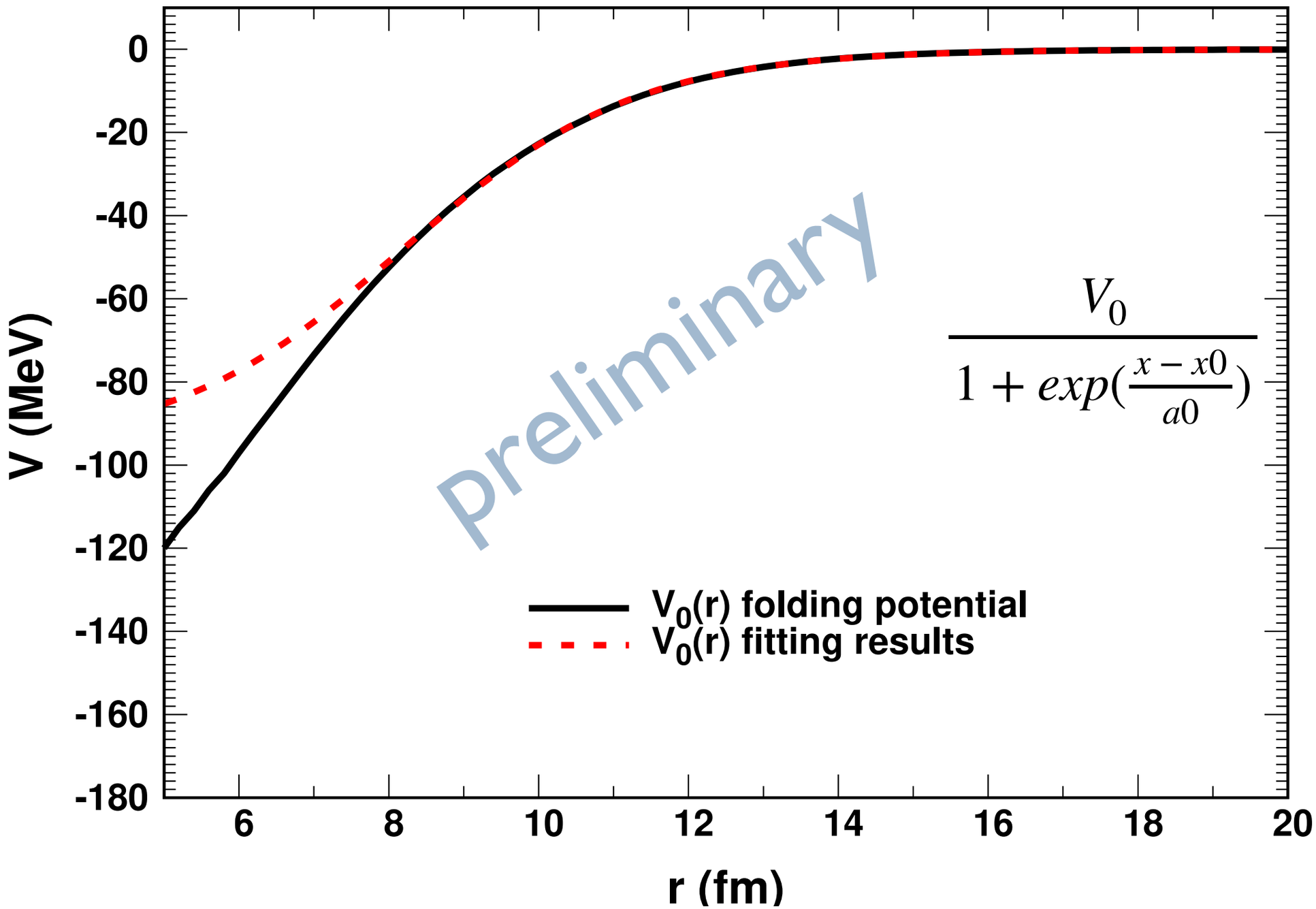
Parameters

β_2	β_4
0.233	0.0946

c	z	ρ_0
6.851	0.518	0.162

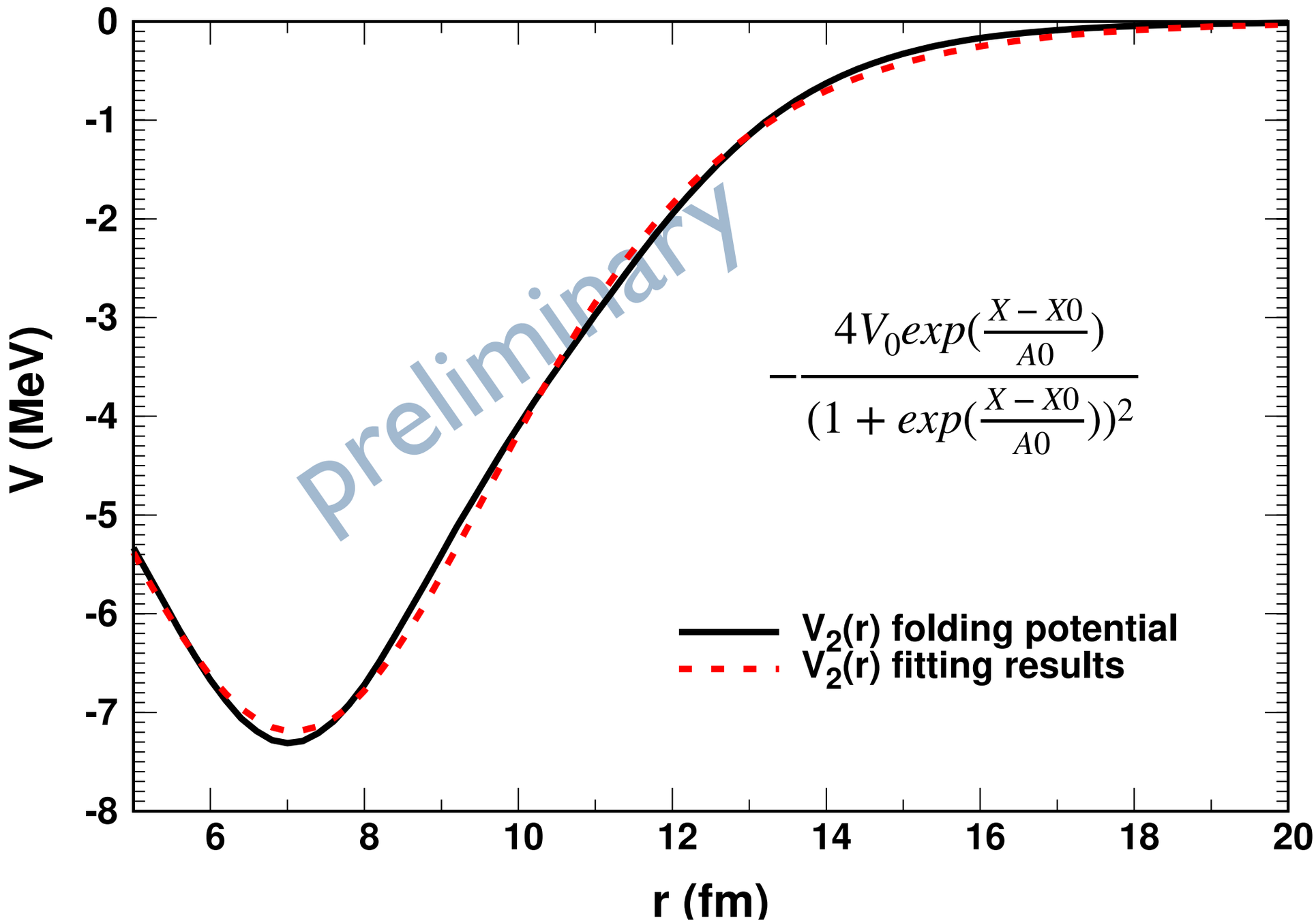
folding potential including halo and deformation

$$V_0(r)Y_{00}(\theta) + V_2(r)Y_{20}(\theta) + V_4(r)Y_{40}(\theta)$$



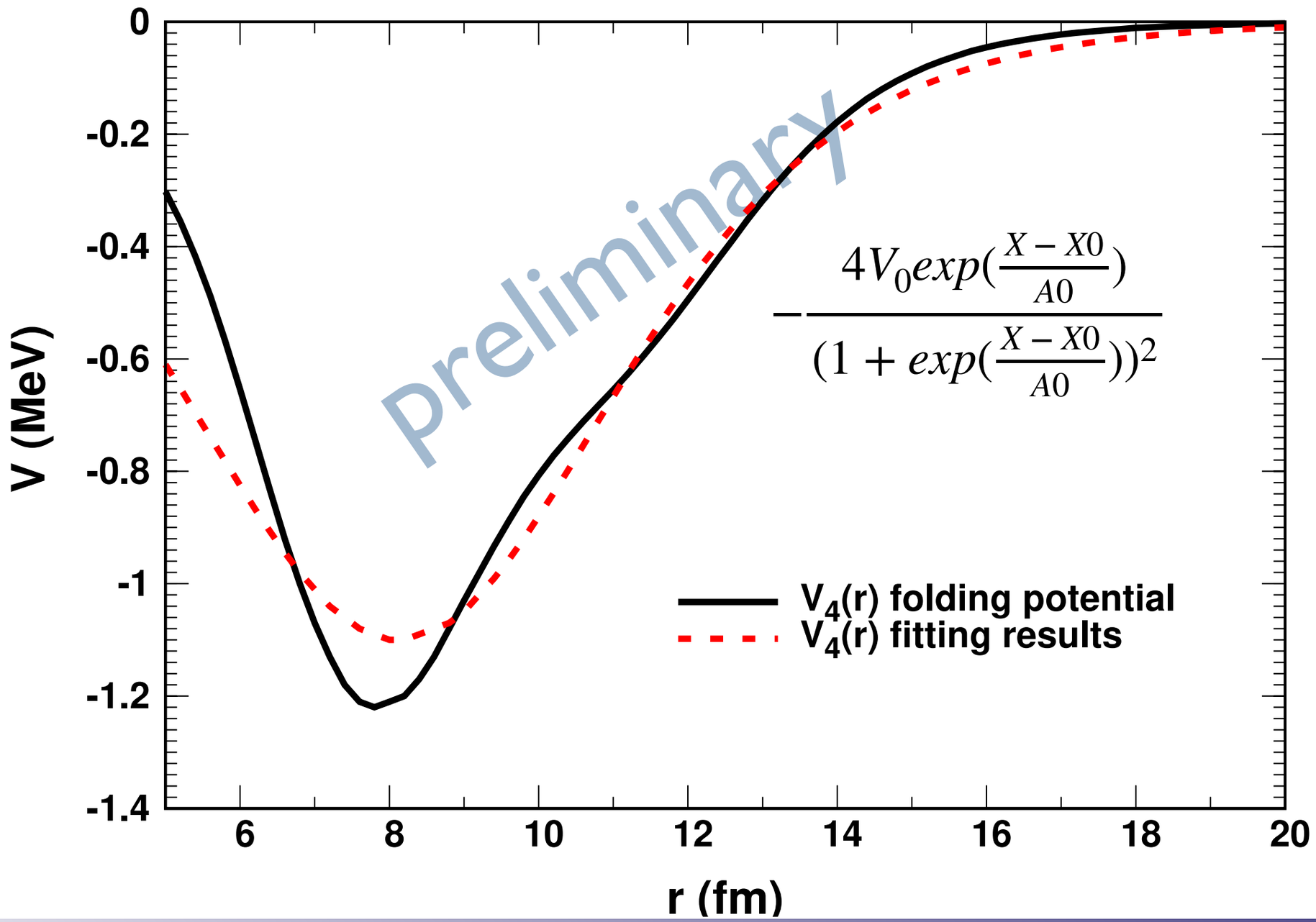
folding potential including halo and deformation

$$V_0(r)Y_{00}(\theta) + V_2(r)Y_{20}(\theta) + V_4(r)Y_{40}(\theta)$$



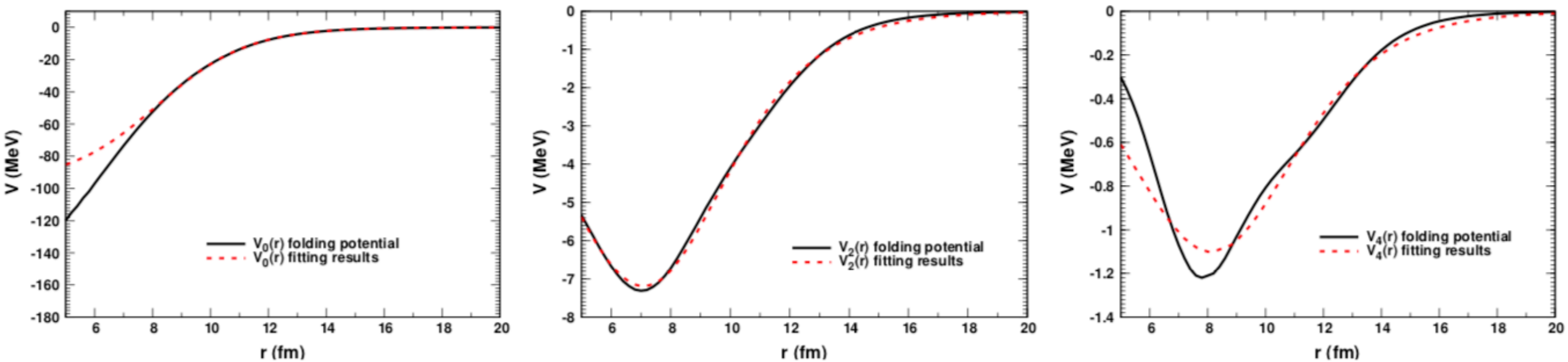
folding potential including halo and deformation

$$V_0(r)Y_{00}(\theta) + V_2(r)Y_{20}(\theta) + V_4(r)Y_{40}(\theta)$$



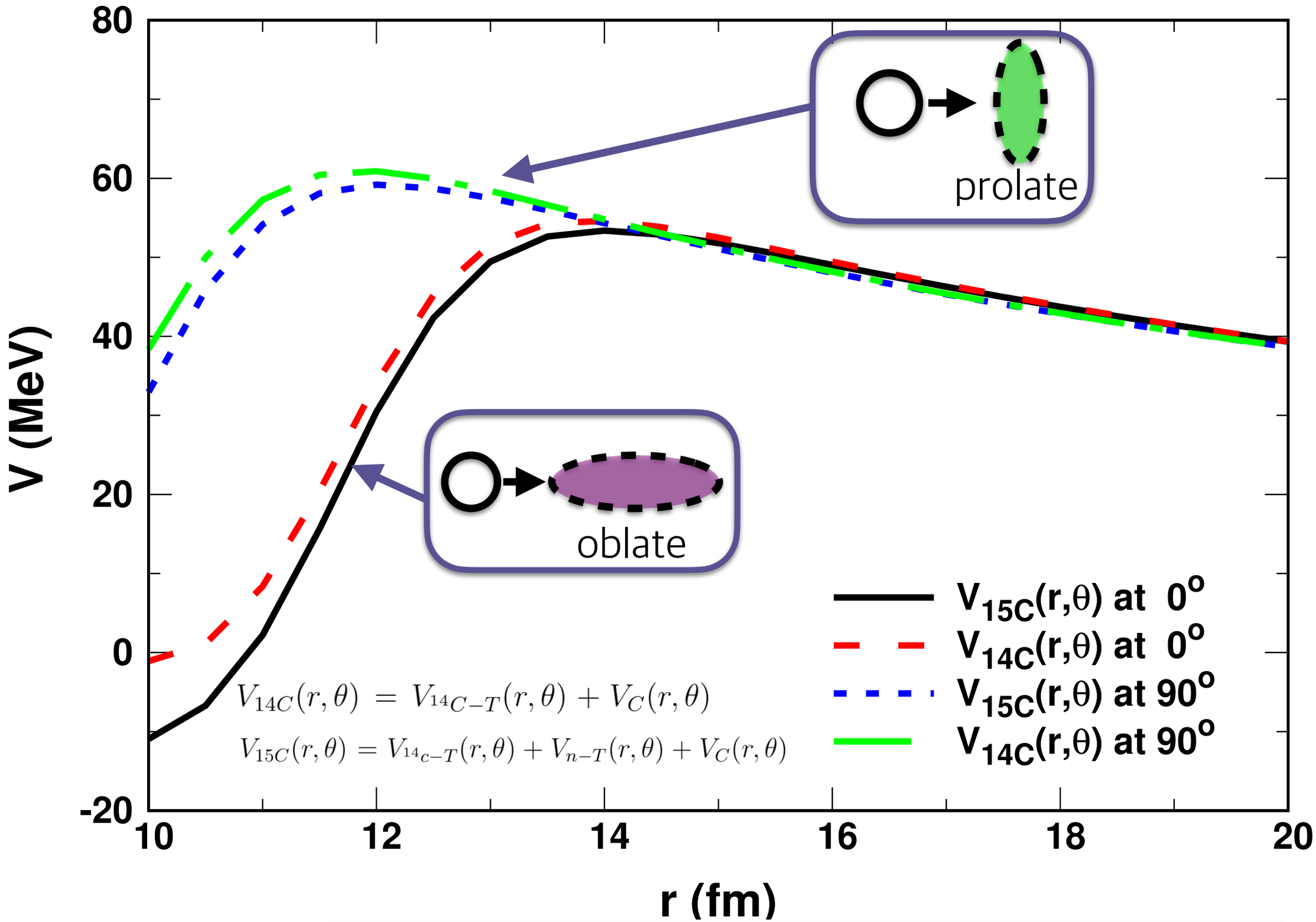
folding potential including halo and deformation

$$V_{n-T}(r,\theta) = \sum_{\lambda} V_{\lambda}(r) Y_{\lambda 0}(\theta) \quad = V_0(r)Y_{00}(\theta) + V_2(r)Y_{20}(\theta) + V_4(r)Y_{40}(\theta)$$

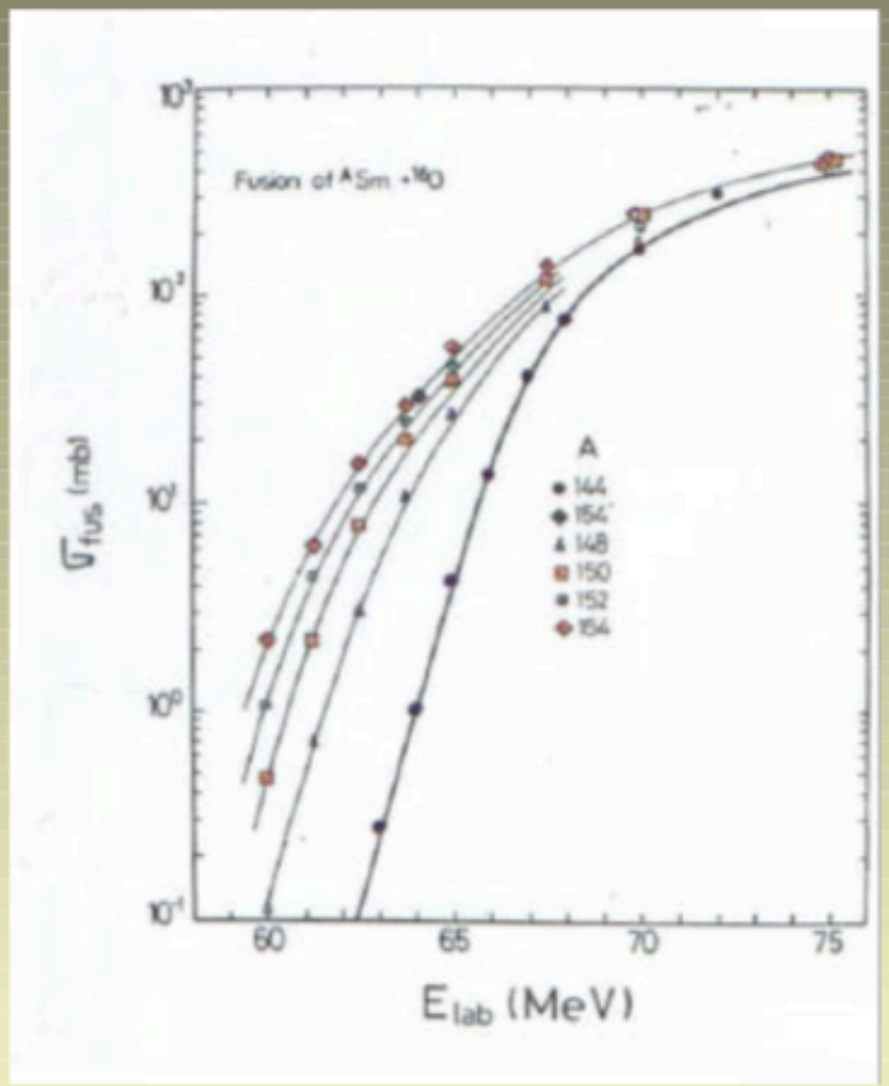


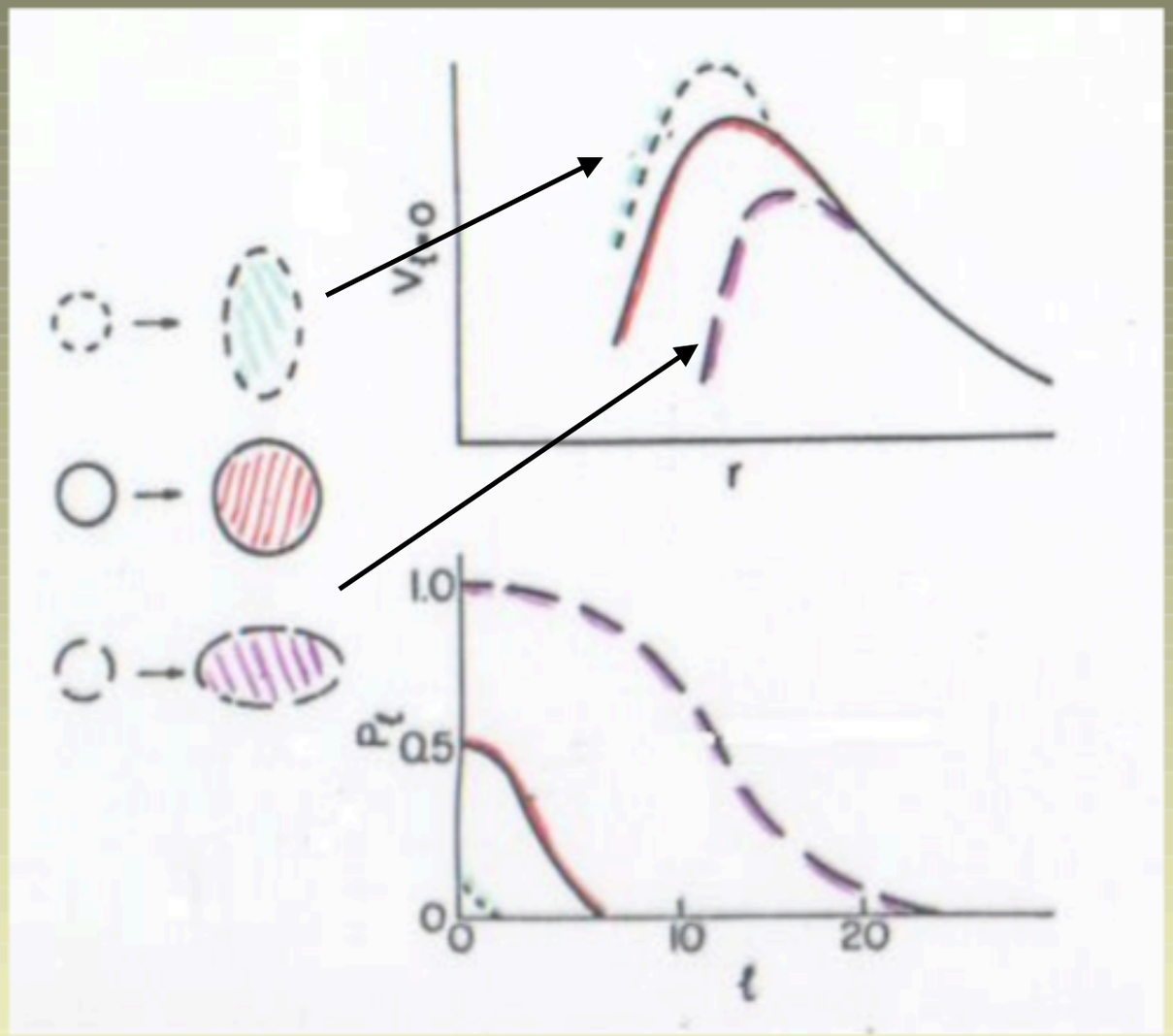
set	V_0(MeV)	r_0(fm)	a_0(fm)
$V_0(r)$	105.923	1.117	1.572
$V_2(r)$	7.189	0.991	1.890
$V_4(r)$	1.099	1.138	1.944

Variations of Coulomb barrier



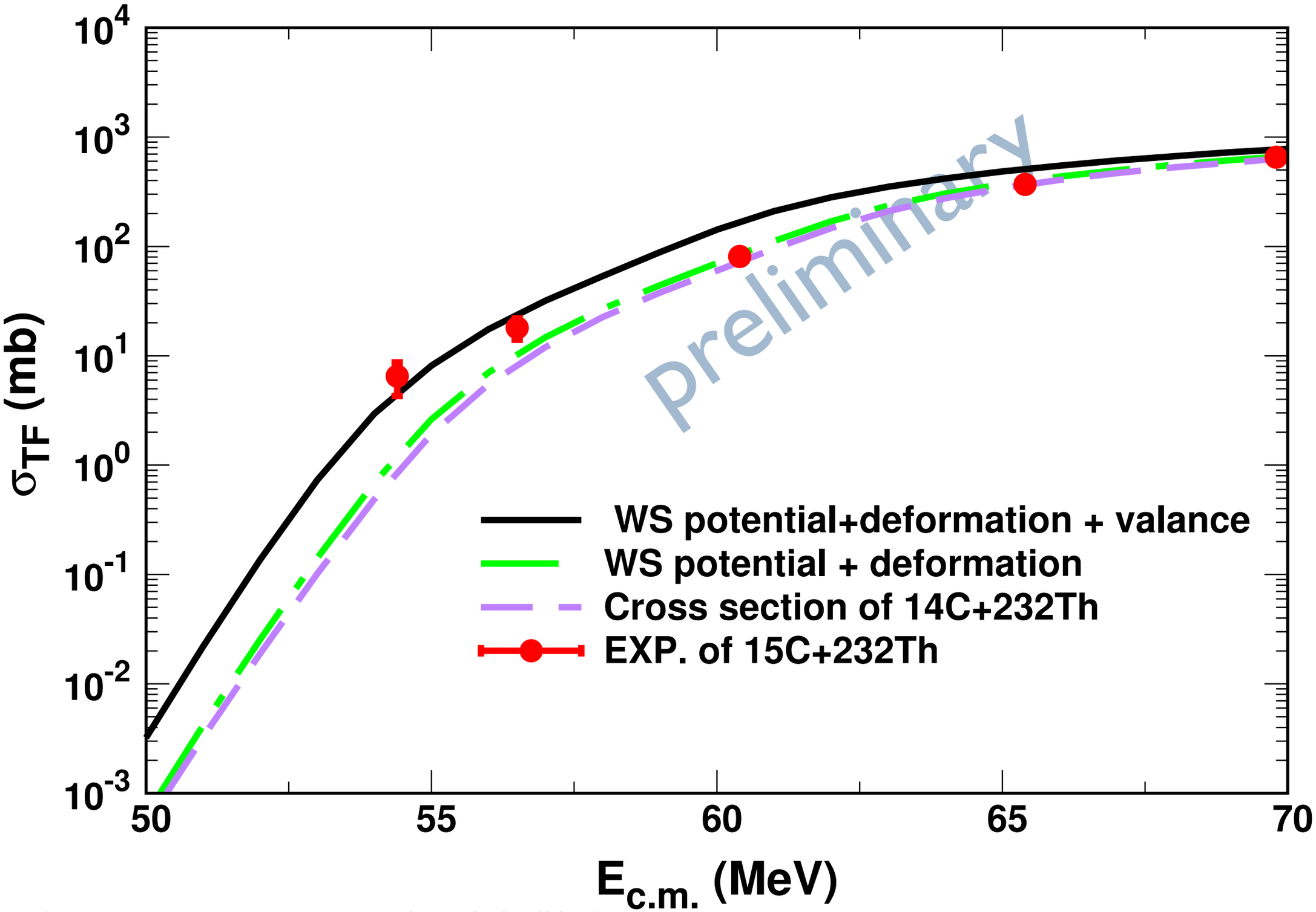
Fusion with tightly bound nuclei below the Coulomb barrier





P. R. S. Gomes presentation (2006)

Fusion cross section of $^{15}C + ^{232}Th$



M. Alcorta et. al. : Phys. Rev. Lett. 106, 172701 (2011)

Summary

- We introduced fusion reaction in the progress of compound nucleus.
- We also explored fusion reaction including halo nuclei.
- For halo nuclei effect, we used coupled channel method.
- Especially, we studied the case with halo and deformation nuclei.
- Finally, we successfully described fusion reaction of $^{15}C + ^{232}Th$.