

## Quantum Field Theory Meets Gravity

- Initial excitement in string theory was not only that it offers a framework for QFT and gravity to meet but also due to its sense of uniqueness.
- Anomaly cancellation [Green, Schwarz, '84]: $\mathrm{E}_{8} \times \mathrm{E}_{8}, \mathrm{SO}(32), \mathrm{U}(1)^{496}$ and $\mathrm{E}_{8} \times \mathrm{U}(1)^{248}$ gauge groups, the former 2 are realized by the heterotic string [Gross, Harvey, Martinec, Rohm,'85].
- Calabi-Yau compactification [Candelas, Horowitz, Strominger, Witten, '85]:

Table 1
Known examples of six (real)-dimensional manifolds with $\mathrm{SU}(3)$ holonomy
together with some of their properties

| Manifold | $\chi$ | $b_{1,1}$ | $b_{2,1}$ | Known holomorphic <br> discrete symmetries <br> that act freely | Number of <br> zero modes |
| :--- | ---: | ---: | ---: | :---: | :---: |
| $\mathrm{Y}_{(4 ; 5)}$ | -200 | 1 | 101 | $\mathrm{Z}_{5} \times \mathrm{Z}_{5}$ | 203 |
| $\mathrm{Y}_{(5 ; 4,2)}$ | -176 | 1 | 89 |  | 179 |
| $\mathrm{Y}_{(5 ; 3,3)}$ | -144 | 1 | 73 | $\mathrm{Z}_{3} \times \mathrm{Z}_{3}$ | 147 |
| $\mathrm{Y}_{(6 ; 3,2,2)}$ | -144 | 1 | 73 |  | 147 |
| $\mathrm{Y}_{(7 ; 2,2,2,2)}$ | -128 | 1 | 65 | $\mathrm{Z}_{2} \times \mathrm{Z}_{2} \times \mathrm{Z}_{2} ; \mathrm{Z}_{8}$ | 131 |
| $\mathrm{Y}^{2}$ | -8 | 1 | 5 | none | 11 |
| Z | +72 | 36 | 0 |  | 36 |

All these manifolds have $b_{0}=1, b_{1}=0, b_{0,2}=b_{2,0}=0, b_{2,1}=b_{1,2}$ and $b_{0,3}=b_{3,0}=1$.

## String Theory Landscape



## String Theory Landscape



## String Theory Landscape



## Landscape vs Swampland

## Landscape

## Landscape vs Swampland

Landscape<br>10500 ||B flux vacua [Douglas, '03]

## Landscape vs Swampland

Landscape<br>10272,000 F-theory vacua [Taylor, Wang, '15]

## Landscape vs Swampland

## Swampland

 [Vafa, '05]
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## Landscape

10272,000 F-theory vacua
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What properties delineate the landscape from the swampland? What are the phenomenological implications?

## Swampland Criteria

There are varying degrees of understanding for different swampland criteria and their interconnections:


These criteria do not follow from purely low-energy EFT considerations. Why are they necessary for consistency of quantum gravitational theories?

## Branes and the Swampland

[Kim, GS, Vafa, '19]

- Completeness of spectrum of charged branes [Polchinski '03], [Banks, Seiberg, '10]: use them to probe consistency of EFTs coupled to gravity.
- Consider e.g., a BPS string:

- The string action is not invariant under a gauge transformation of the 2form $B_{2}$ to which it couples $\rightarrow$ anomaly inflow.
- In a consistent theory, these anomalies must be cancelled by the anomalies coming from the dofs in a unitary worldsheet theory.


## Branes and the Swampland

[Kim, GS, Vafa, '19]

- First consider $N=(1,0)$ SUGRA theories in 10 d \& 6d as gauge and gravitational anomaly cancellations severely limit the possibilities.
- We illustrate the power of this approach with just a few examples and with only string probes but we expect this program of using brane probes to understand swampland criteria has wider applicabilities.
- We showed the 10d anomaly-free theories with $E_{8} \times U(1)^{248}$ and $U(1)^{496}$ gauge groups (which have no string realizations) are in the swampland.
- Infinite families of anomaly-free 6d theories [Kumar, Morrison, Taylor, '10] with unbounded gauge group rank, or unbounded number of tensors or matter in exotic representations. We showed that unitarity of current algebra on string probes can rule out some of these infinite families.
- Our method was recently used to bound the \# of abelian gauge group factors in 6d gravitational theories with minimal SUSY [Lee, Weigand] and the rank of gauge groups for N=4 SYM coupled to gravity [Kim, Tarazi, Vafa]


## Quantum Gravity and Global Symmetries

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- Global symmetries are expected to be violated by gravity:

- No hair theorem: Hawking radiation is insensitive to Q .
$\Rightarrow$ Infinite number of states (remnants) with $m \lesssim M_{p}$
$\Rightarrow$ Violation of entropy bounds. At finite temperature (e.g. in Rindler space), the density of states blows up.
- Swampland conjecture: theories with exact global symmetries are not UVcompletable.
- In (perturbative) string theory, all symmetries are gauged [Banks, Dixon, '88]; recently revisited using holography [Harlow, Ooguri, '18].
- Many phenomenological ramifications, e.g., milli-charged DM comes with a new massless gauge boson [GS, Soler, Ye, '13].


## The Weak Gravity Conjecture



Arkani-Hamed, Motl, Nicolis, Vafa ‘06

## The Weak Gravity Conjecture

Arkani-Hamed, Motl, Nicolis, Vafa ‘06

- The conjecture:


## "Gravity is the Weakest Force"

- This is a scale-dependent statement, but as we'll see, the WGC comes with a UV cutoff $\wedge$ (magnetic WGC).
- For every long range gauge field there exists a particle of charge $q$ and mass m, s.t.

$$
\frac{q}{m} M_{P} \geq " 1 " \equiv \frac{Q_{E x t}}{M_{E x t}} M_{P}
$$

- This implies extremal BHs can decay, even though the remnant problem (which applies to global symmetry) does not arise here.
- Applying the WGC to magnetically charged states imply:

$$
q_{\text {mag }} \sim 1 / g, \quad m_{\text {mag }} \sim \Lambda / g^{2} \quad \Rightarrow \quad \Lambda \lesssim g(\Lambda) M_{P}
$$

## WGC for p-form Symmetry

- One can generalize the WGC for 1 -form gauge fields to the WGC for ( $p+1$ )form gauge fields which couple to $p$-branes:

$$
\frac{Q_{p}}{T_{p}} \geq\left(\frac{Q_{p}}{T_{p}}\right)_{\mathrm{Ext}}
$$

- The 0 -form gauge field (axion) case ( -1 form symmetry) is most interesting (axion inflation) but subtle as the "branes" that couple to it are instantons.
- Obtaining an axion by duality [Brown, Cottrell, GS, Soler, ' ${ }^{15}$ ] or dimensional reduction [Heidenreich, Reece, Rudelius, ${ }^{116]}$ suggests that the above inequality can indeed be extrapolated to:

$$
f \cdot S_{\text {inst }} \leq \mathcal{O}(1) M_{P}
$$

- Attempt to give a more direct argument for the -1 form WGC [Andriolo, Huang, Noumi, Ooguri, GS, '20] : the "extremal bound" is set by the action-tocharge ratio of the macroscopic semi-wormhole.


## Axions in String Theory

- Consider a 5d particle with mass ' $m_{5}$ ' and charge ' $\mathrm{q}_{5}$ ' whose (Euclidean) worldline wraps the compact dimension


$$
\begin{aligned}
S_{\psi} & =m_{\psi} \oint d \ell+i \oint A \\
& =2 \pi R m_{\psi}+i \phi(x)
\end{aligned}
$$



- This particle sources the axion and is localized to a point in 4 d spacetime, i.e. it is an instanton:

$$
V(\phi) \sim e^{-S_{i n s t}} \cos \left(\frac{\phi}{f}\right)
$$

$$
\begin{aligned}
S_{\text {inst }} & =2 \pi R m_{\psi} \\
f^{-1} & =g_{5} \sqrt{2 \pi R}
\end{aligned}
$$

- The 5d WGC for charged particles $m_{\psi}<g_{5} M_{p, 5 d}^{3}$ translates into:

$$
f \cdot S_{\text {inst }} \leq M_{p}
$$

## WGC and Inflation

## Primordial Gravitational Waves



Ongoing experiments can potentially detect primordial B-mode with a tensor-to-scalar ratio r as small as $\sim 10^{-2}$.

Further experiments, such as CMB-S4 and LiteBIRD, .. may improve further the sensitivity to $r$ as small as $\sim 10^{-3}$.

## B-mode and UV Sensitivity

A detection at the targeted level implies that the inflaton potential is nearly flat over a super-Planckian field range:

$$
\Delta \phi \gtrsim\left(\frac{r}{0.01}\right)^{1 / 2} M_{\mathrm{Pl}}
$$

Syth '96

"Large field inflation" are highly sensitive to UV physics


## Axions \& Large Field Inflation

## Natural Inflation [Freese, Frieman, Olinto]

Pseudo-Nambu-Goldstone bosons are natural inflaton candidates.


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Pseudo-Nambu-Goldstone bosons are natural inflaton candidates.


They satisfy a shift symmetry that is only broken by non-perturbative effects:


Slow roll: $\quad f>M_{P}$
$\uparrow$ decay constant
$V(\phi)=1-\Lambda^{(1)} \cos \left(\frac{\phi}{f}\right)+\sum_{k>1} \Lambda^{(k)}\left[1-\cos \left(\frac{k \phi}{f}\right)\right]$ if $\frac{\Lambda^{(n+1)}}{\Lambda^{(n)}} \sim e^{-S_{\text {inst }}} \ll 1$

## Axions \& Large Field Inflation

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$V(\phi)=1-\Lambda^{(1)} \cos \left(\frac{\phi}{f}\right)+\sum_{k>1} \Lambda^{(k)}\left[1-\cos \left(\frac{k \phi}{f}\right)\right] \quad$ if $\frac{\Lambda^{(n+1)}}{\Lambda^{(n)}} \sim e^{-S_{\text {inst }}} \ll 1$
The WGC implies that these conditions cannot be simultaneously satisfied.

## Loopholes

[Brown, Cottrell, GS, Soler];[Rudelius]

- Naively, the WGC rules out "natural inflation".
- However the WGC requires $\mathrm{f} \cdot \mathrm{m}<1$ for ONE instanton, but not ALL

$$
V=e^{-m}\left[1-\cos \left(\frac{\Phi}{F}\right)\right]+e^{-M}\left[1-\cos \left(\frac{\Phi}{f}\right)\right]
$$

- with $1<m \ll M, \quad F \gg M_{P}>f, \quad M \times f \ll 1$
- The second instanton fulfills the WGC, but is negligible, an "spectator". Inflation is governed by the first term.
- Another loophole is non-periodic axions (aka axion monodromy) as they are not mapped to long-range gauge fields.


## Axion Monodromy

[Silverstein, Westphal];[McAllister, Silverstein, Westphal] and its realization in SUGRA ("F-term axion monodromy"):
[Marchesano, GS, Uranga];[Blumenhagen, Plauschinn];[Hebecker, Kraus, Witowski]


A priori not constrained by the WGC since a monodromy axion is mapped to a massive gauge field.

## Convex Hull Condition


[Cheung, Remmen, '14]

## Tower/Sub-Lattice WGC

- Compactifying a theory on a circle gives rise to an additional $\mathrm{U}(1)_{\mathrm{KK}}$. Apply the WGC to black holes with general charges.
- Infinite tower of (super)extremal KK states. Charge-to-mass ratio depends on the radius:


- Convex hull not guaranteed to contain the BH region. This motivates a stronger version of the WGC known as Tower/Sub-Lattice WGC [Andriolo,Junghans, Noumi, GS];[Heidenreich, Reece, Rudelius];[Montero, GS, Soler]


## Multi-Axion Inflation



N -flation [Dimopoulos, Kachru, McGreevy, Wacker, '05]


Alignment/Clockwork
[Kim, Nilles, Peloso, '04];[Choil, Kim, Yun, '14];[Choi, Im, '15];[Kaplan, Rattazzi, '15]

Naively they violate the WGC, but one can come up with loopholes...

## Loopholes



Figure from
[Brown, Cottrell, GS, Soler, '15]

- $\mathrm{Q}=0$. marginally superextremal
- very superextremal


Figure from
[Heidenreich, Long, McAllister, Rudelius, Stout, ‘19]

## Evidence for the WGC

## Evidence for the WGC



## WGC and Black Holes

## Extremality of Black Holes

- The mild form of the WGC requires only some state for an extremal BH to decay to.
- Can an extremal BH satisfy the WGC?
- Higher derivative corrections can
 make extremal BHs lighter than the classical bound $\mathrm{Q}=\mathrm{M}$
- Demonstrated to be the case for 4D heterotic extremal BHs. [Kats, Motl, Padi, '06]
- We showed that this behavior (A) follows from unitarity (at least for some classes of theories). [Hamada, Noumi, GS]


## WGC from Unitarity and Causality

- We assume a weakly coupled UV completion at scale $\Lambda_{\text {QFT. }}$. Our proof for the strict WGC bound applies to at least two classes of theories:

- Theories with light (compared with $\Lambda_{\mathrm{QFT}}$ ), neutral i) parity-even scalars (e.g., dilaton, moduli), or ii) spin $\geq 2$ particles
- UV completion where the photon \& the graviton are accompanied by different sets of Regge states (as in open string theory).


## Higher Derivative Corrections

- In the IR, the BH dynamics is described by an EFT of photon \& graviton.
- In $D=4$, the general effective action up to 4-derivative operators (assume parity invariance for simplicity):

$$
S=\int d^{4} x \sqrt{-g}\left[\frac{2 M_{\mathrm{Pl}}^{2}}{4} R-\frac{1}{4} F_{\mu \nu} F^{\mu \nu}+\Delta \mathcal{L}\right]
$$

where

$$
\begin{aligned}
\Delta \mathcal{L}= & c_{1} R^{2}+c_{2} R_{\mu \nu} R^{\mu \nu}+c_{3} R_{\mu \nu \rho \sigma} R^{\mu \nu \rho \sigma} \\
& +c_{4} R F_{\mu \nu} F^{\mu \nu}+c_{5} R_{\mu \nu} F^{\mu \rho} F_{\rho}^{\nu}+c_{6} R_{\mu \nu \rho \sigma} F^{\mu \nu} F^{\rho \sigma} \\
& +c_{7} F_{\mu \nu} F^{\mu \nu} F_{\rho \sigma} F^{\rho \sigma}+c_{8} F_{\mu \nu} F^{\nu \rho} F_{\rho \sigma} F^{\sigma \mu} .
\end{aligned}
$$

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\begin{aligned}
S=\int & d^{4} x \sqrt{-g}\left[\frac{2 M_{\mathrm{Pl}}^{2}}{4} R-\frac{1}{4} F_{\mu \nu} F^{\mu \nu}+\frac{\alpha_{1}}{4 M_{\mathrm{Pl}}^{4}}\left(F_{\mu \nu} F^{\mu \nu}\right)^{2}\right. \\
& \left.+\frac{\alpha_{2}}{4 M_{\mathrm{Pl}}^{4}}\left(F_{\mu \nu} \widetilde{F}^{\mu \nu}\right)^{2}+\frac{\alpha_{3}}{2 M_{\mathrm{Pl}}^{2}} F_{\mu \nu} F_{\rho \sigma} W^{\mu \nu \rho \sigma}\right]
\end{aligned}
$$

by field redefinition. Here, $W^{\mu v \rho \sigma}$ is the Weyl tensor:

$$
R_{\mu \nu \rho \sigma}=W_{\mu \nu \rho \sigma}+\frac{1}{2}\left(g_{\mu[\rho} R_{\sigma] \nu}-g_{\nu[\rho} R_{\sigma] \mu}\right)-\frac{1}{3} R g_{\mu[\rho} g_{\sigma] \nu}
$$

## Extremality Condition

- The higher derivative operators modify the BH solutions, so the charge-to-mass ratio of an extremal BH is corrected:

$$
z=\frac{\sqrt{2} M_{\mathrm{Pl}}|Q|}{M}=1+\frac{2}{5} \frac{(4 \pi)^{2}}{Q^{2}}\left(2 \alpha_{1}-\alpha_{3}\right) \quad[\text { Kats, Mlotl, Padi, '06] }
$$

applicable when the BH is sufficiently heavy: $M^{2} \sim Q^{2} M_{\mathrm{Pl}}^{2} \gg \alpha_{i} M_{\mathrm{Pl}}^{2}$ because extremal BHs in Einstein-Maxwell theory satisfy:

$$
R \sim M_{\mathrm{Pl}}^{4} / M^{2} \text { and } F^{2} \sim M_{\mathrm{Pl}}^{6} / M^{2}
$$

- Proving the WGC (mild form) amounts to showing:

$$
2 \alpha_{1}-\alpha_{3} \geq 0
$$

so large extremal BHs can decay into smaller extremal BHs.

## Sketch of the Proof: Step 1

[Hamada, Noumi, GS]

- We first show that for the aforementioned theories, causality implies

$$
\left|\alpha_{1}\right| \gg\left|\alpha_{3}\right|
$$

because $\alpha_{3}$ leads to causality violation and an infinite tower of massive higher spin states is required to UV complete the EFT at tree-level [Camanho, Edelstein, Maldacena, Zhiboedov].

phase shift of photon propagation:

$$
\begin{aligned}
& \delta \sim s\left(\ln \left(L_{\mathrm{IR}} / b\right) \pm \frac{\left|\alpha_{3}\right|}{b^{2}}+\ldots\right) \\
& \text { time delay in } \mathrm{GR} \\
& \quad \text { helicity dependent phase shit }
\end{aligned}
$$

$b$ : impact parameter $\quad L_{\mathrm{IR}}:$ IR cutoff
fig: Camanho et al '14

## Sketch of the Proof: Step 1

[Hamada, Noumi, GS]

- Time advancement if $b^{2} \ln (L / b) \ll\left|\alpha_{3}\right|$
- Phase shift generated by spin $J$ is $\delta \sim s^{J-1}$. A finite \# of higher spin particles does not help $\rightarrow$ infinite tower of higher spin states.
- Causality violation above energy scale $\Lambda_{\mathrm{QFT}} \lesssim \frac{M_{P}}{\sqrt{\left|\alpha_{3}\right|}}$
- Integrating out light neutral scalars does not give significant contributions to $\alpha_{3}$ and so $\left|\alpha_{1}\right| \gg\left|\alpha_{3}\right|$
- If there are different Regge towers as in theories with open strings:

$$
\alpha_{1,2,3}^{\text {closed }} \sim \frac{M_{\mathrm{Pl}}^{2}}{M_{s}^{2}} \ll \quad \alpha_{1,2}^{\text {open }} \sim \frac{M_{\mathrm{Pl}}^{2}}{g_{s} M_{s}^{2}}, \quad g_{\text {open }} \sim \sqrt{g_{s}} \gg g_{s}
$$

- If there are light fields or different Regge towers, $\alpha_{3}$ is subdominant compared with the causality preserving terms $\alpha_{1}$ and $\alpha_{2}$.


## Sketch of the Proof: Step 2

[Hamada, Noumi, GS]

- The forward limit $t \rightarrow 0$ of YY scattering for the aforementioned theories:
- The higher derivative operator parametrized by $\alpha_{1}$ leads to:

$$
\alpha_{1}\left(F_{\mu \nu} F^{\mu \mu}\right)^{2} \quad \Rightarrow \quad \mathscr{M} \sim \alpha_{1} s^{2} \quad \text { Unitarity } \Rightarrow \alpha_{1}>0
$$



- a state $q \geq m$ can be an extremal BH !


## More General Black Holes

- We found an entropy-extremality relation [Hamada, Noumi, GS] which implies that in theories satisfying the $W G C, Z_{\text {ext }}>1 \Leftrightarrow \Delta S>0$.
- However, for Einstein-Maxwell-dilaton theory, positivity bounds alone do not ensure that $Z_{\text {ext }}>1$ [Loges, Noumi, GS, '19].

$$
I=\int \mathrm{d}^{4} x \sqrt{-g}\left[\frac{M_{\mathrm{P1}}^{2}}{2} R-\frac{M_{\mathrm{P1}}^{2}}{2}(\partial \phi)^{2}-\frac{1}{4} e^{-2 \lambda \phi}\left(F^{2}\right)\right]
$$

- The leading 4-derivative operators:

$$
\begin{gathered}
\int \mathrm{d}^{4} x \sqrt{-g}\left[\frac{\alpha_{1}}{4} e^{-6 \lambda \phi}\left(F^{2}\right)^{2}+\frac{\alpha_{2}}{4} e^{-6 \lambda \phi}(F \widetilde{F})^{2}+\frac{\alpha_{3}}{2} e^{-4 \lambda \phi}(F F W)+\frac{\alpha_{4}}{2} e^{-2 \lambda \phi}\left(R_{\mathrm{GB}}\right)\right. \\
\left.+\frac{\alpha_{5}}{4} e^{-2 \lambda \phi}(\partial \phi)^{4}+\frac{\alpha_{6}}{4} e^{-4 \lambda \phi}(\partial \phi)^{2}\left(F^{2}\right)+\frac{\alpha_{7}}{4} e^{-4 \lambda \phi}(\partial \phi \partial \phi F F)\right]
\end{gathered}
$$

modifies the extremal bound for a general dyonic black hole:

$$
z_{\mathrm{ext}}=1+\frac{2}{5 q_{e} q_{m}} \alpha_{i} \mathcal{M}_{i}(\zeta) \quad \zeta \rightarrow 1 \text { extremal }
$$

## The Role of Symmetries

[Loges, Noumi, GS, '20] (also [Andriolo, Noumi, Huang, Ooguri, GS])

- Unitarity requires $\alpha_{7}>0$ but $\mathrm{M}_{7}(\zeta)<0$ for all $\zeta$ so $\Delta \mathrm{Z}_{\text {ext }}<0$.
- Such operator does not appear in isolation. In some well motivated UV complications, the set of $\alpha_{\mathrm{i}}$ combine to give an overall $\Delta z_{\text {ext }}>0$.
- In [Loges, Noumi, GS, '20], we examine how symmetries can impose additional structure on the EFT to ensure that $\Delta z_{\text {ext }}>0$.
- Consider Einstein-Maxwell-dilaton-axion theory. We found that extra symmetries (e.g. SL(2,R) (S-duality) and O(d,d) (T-duality)) when combined with either scattering positivity bounds or null energy condition, are strong enough to ensure that $\Delta z_{\text {ext }}>0$.
- We also explore implications of $\mathrm{N} \geq 2$ SUSY. We found that the puzzling terms which give $\Delta z_{\text {ext }}<0$ are needed to make corrections to extremality identically zero, as expected for BPS states.


## A New Spin on the WGC

[Aalsma, Cole, Loges, GS, '20]

- We reformulate the WGC as an integrated condition:

$$
\int_{\Sigma} \mathrm{d}^{d-1} x \sqrt{h} \delta T_{a b}^{\mathrm{eff}} \xi^{a} n^{b} \leq 0
$$

- For BTZ black hole, a spinning WGC follows from the c-theorem:

- Corrected extremality bound: $\quad \frac{J_{3}}{\ell M_{3}} \leq 1+\frac{48 \pi G_{3}\left(3 \alpha_{1}+\alpha_{2}\right)}{\ell}$


## Total Landscaping Principle

- The 5d boosted black string has a BTZ×S2 near-horizon geometry and, after KK reduction, describes a 4D charged black hole.
- Given the entropy-extremality relation [Hamada, Noumi, GS], one may naively think that the spinning WGC implies the charged WGC.
- Note however that the said relation is between the extremality bound (fixed Q/J, T) \& the microcanonical entropy (fixed Q/J, M).
- The extremality bounds for the spinning WGC and charged WGC do not line up; rather they together strengthen the WGC in 5D:

- WGC-compatible, $\alpha_{1}>0$
- WGC-compatible, $\alpha_{1}<0$
----- $8 \alpha_{1}+3 \alpha_{2}-12 \alpha_{3}=0 \quad$ (near-horizon BTZ $\times S^{2}$ )
----- $8 \alpha_{1}+7 \alpha_{2}+6 \alpha_{3}=0 \quad$ (4D 4-charge black hole)
----- $8 \alpha_{1}-\alpha_{2}-6 \alpha_{3}=0 \quad$ (5D electric black hole)


## Stronger forms of the WGC

## WGC and Modular Invariance

- In [Aalsma, Cole, GS], we argued that for extremal BHs with a near horizon $\mathrm{AdS}_{3}$ geometry, we can use modular invariance and anomalies to infer that there is a tower of superextremal states interpolating between perturbative string states and BHs.



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## Spectral Flow

- The worldsheet partition function enjoys modular invariance:

$$
\mathrm{T}: \quad Z(\tau+1 ; \mu)=Z(\tau ; \mu), \mathbf{S}: Z(-1 / \tau ; \mu / \tau)=e^{\pi i k \frac{\mu^{2}}{\tau}} Z(\tau ; \mu)
$$

- Compactness of the Abelian gauge group:

$$
Z(\tau ; \mu+\rho)=Z(\tau ; \mu) \quad \forall \rho \in \Gamma_{Q}^{*} \quad=\left\{\rho \mid \rho Q \in \mathbb{Z} \forall Q \in \Gamma_{Q}\right\}
$$

- Performing a $\mathrm{U}(1)$ transformation between two S transformations:

$$
L_{0} \rightarrow L_{0}+Q \rho+k \frac{\rho^{2}}{2} \quad ; \quad Q \rightarrow Q+k \rho
$$

- Given a perturbative string state with mass m and charge q :

$$
m=\sqrt{\frac{4}{\alpha^{\prime}} \Delta}=\sqrt{\frac{4}{\alpha^{\prime}} \tilde{\Delta}}
$$

- Spectral flow generates a tower of states with charge-to-mass ratio:

$$
Z \equiv \frac{2}{k \alpha^{\prime}} \frac{q^{2}}{m^{2}} \rightarrow 1
$$

## Spectral Flow \& Black Hole

- Given a Z > 1 state, spectral inflow implies a tower of states monotonically approaching $Z=1$ from above:


Turn on a small string coupling

$$
g_{c} \sim N^{-1 / 4} \ll 1
$$

The excited string state turned into a black hole.

The correspondence principle [Horowitz, Polchinski]:

$$
S_{\text {string }}=\mathcal{O}(1) S_{B H}
$$

- This however does not suffice to show that the string states stay $Z>1$ when we turn on $\mathrm{g}_{\mathrm{c}} \sim \mathrm{N}^{-1 / 4}$.


## Entropy Matching

- The near horizon geometry of a $4 d$ extremal $B H$ is $A d S_{2}$ but if the BH arises from a higher dim theory with an $\mathrm{S}^{1} \rightarrow \mathrm{AdS}_{3}$ (BTZ).
- The BH entropy is given by the Cardy's formula:

$$
S_{\mathrm{BH}}=2 \pi\left(\sqrt{\frac{c_{L}}{6} h_{L}}+\sqrt{\frac{c_{R}}{6} h_{R}}\right)
$$

- The central charges are fixed by anomalies [Kraus, Larsen, '05]:
- $C_{L}-C_{R}$ : gravitational Chern-Simons term
- $C_{R} \quad: S U(2)$ Chern-Simons term
- This fixes the BH entropy: $\quad S_{\mathrm{BH}}=2 \pi\left(2 \sqrt{h_{L}}+\sqrt{2 h_{R}}\right)$
- Exact match with the entropy of strings at the string/BH transition:

$$
S_{\mathrm{stat}}=2 \pi\left(2 \sqrt{N_{L}}+\sqrt{2 N_{R}}\right)
$$

- Mass correction is negligible when we turn on string coupling.


## WGC from Modular Bootstrap

[Montero, GS, in progress]
Sublattice WGC: a finite index sublattice of the full charge lattice exists with a (super)extremal particle at each site.


Missing WGC lattice sites form a group:

$$
\Gamma \equiv \Lambda / \Lambda^{*}
$$

Modular bootstrap? [Montero, Soler, GS, '16]; [Bae, Lee, Song,'18]; [Lin, Shao, '19]; ...

## Preliminary Results

[Montero, GS, in progress]

- The CFT data can be encoded into a $\Gamma$-valued partition vector:

$$
Z=\sum_{\gamma \in \Gamma} \mathcal{Z}_{\gamma}(\tau) \vartheta(\tau, \gamma), \quad \vartheta(\tau, \gamma) \equiv \sum_{\vec{\lambda} \in \Lambda^{*}} e^{2 \pi i\left(\tau_{1}(\vec{\gamma}+\vec{\lambda}, \vec{\gamma}+\vec{\lambda})-i \tau_{2}\langle\vec{\gamma}+\vec{\lambda}, \vec{\gamma}+\vec{\lambda}\rangle\right.}
$$

- Modular bootstrap turns this into a linear programming problem. For $\Gamma=Z_{2}$, the minimal charged operator with $Q=1$ satisfies the WGC:
$\Delta$

$$
N=2, Q=1
$$

=- = $\quad W G C$ bound : $\Delta<\frac{Q^{2}}{2 N}+\frac{c}{12}$

- modular bootstrap (up to 19 derivatives)

General $\Gamma$ currently
under investigation

## Summary

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- The Swampland program attempts to clarify the formulation, motivation and applications of several consistency criteria:
- No global symmetries $\rightarrow$ Mini-charged DM. [GS, Soler, Ye, '13]
- Completeness conjecture $\rightarrow$ Low energy spectrum [Kim, GS, Vafa, '19]
- Weak Gravity Conjecture $\rightarrow$ Large field (natural) inflation, Fuzzy DM
- Distance Conjecture $\rightarrow$ Axion monodromy inflation.
- Instability of non-SUSY AdS Neutrino physics? [Ooguri, Vafa, '16]; [lbanez, Martin-Lozano, Valenzuela, '17];[Hamada, GS, '17]
- No dS -> Inflation, CC, quintessence [Obied, Ooguri, Spodyneiko, Vafa, '18];[Ooguri, Palti, GS, Vafa, '18];[Bedroya, Vafa, '19]
- Much remains to be done to fully understand the origin and consequences of these conjectures.


