Are We Ready for Precision Cosmology?

General Relativistic Effects and Gauge-Invariant Formalism

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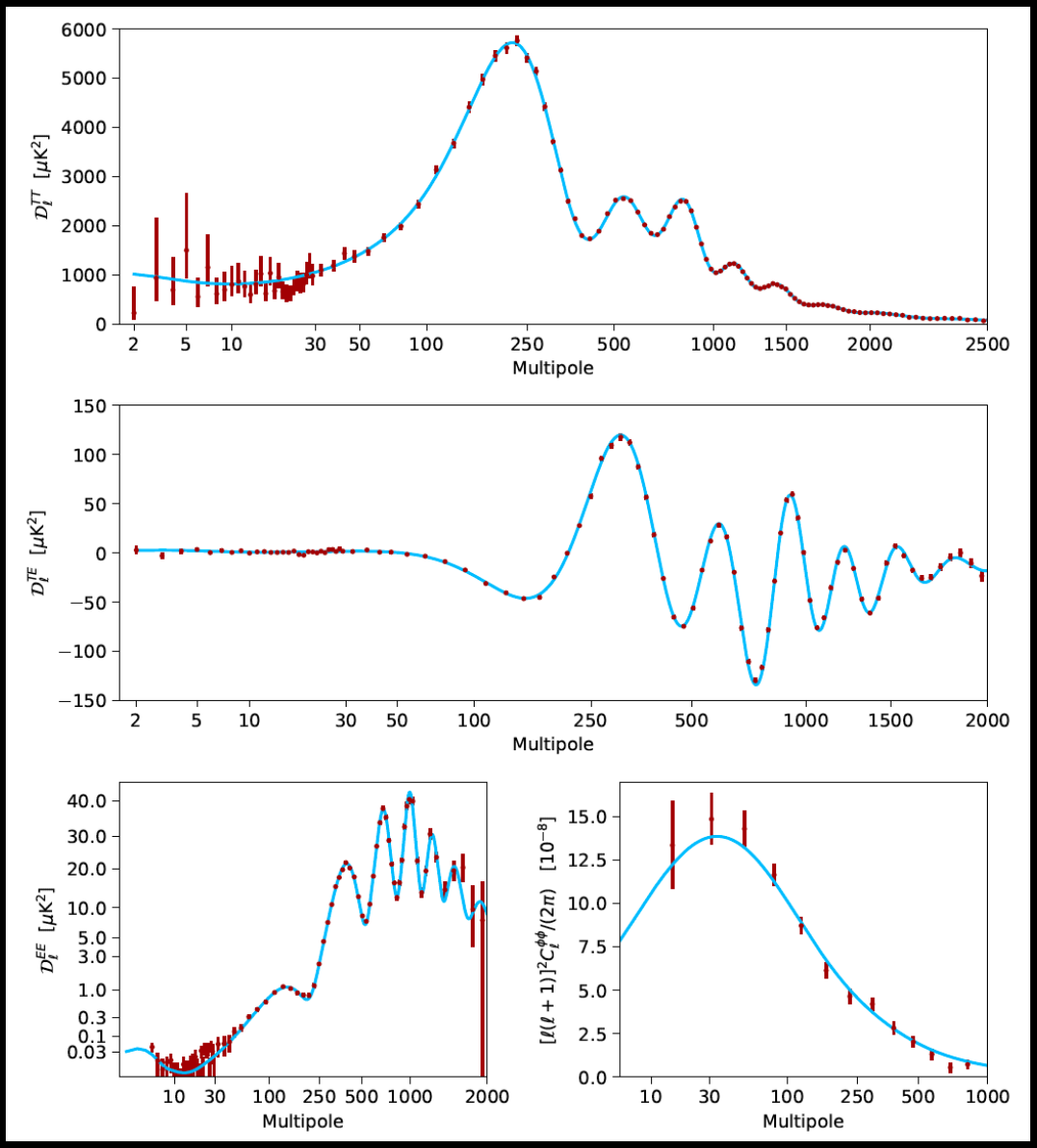
I. PRECISION COSMOLOGY:

Past, Today, and Future

- **Planck collaboration 2018:**

precision measurements!



Parameter	*Planck* alone	*Planck* + BAO
$\Omega_b h^2$	0.02237 ± 0.00015	0.02242 ± 0.00014
$\Omega_c h^2$	0.1200 ± 0.0012	0.11933 ± 0.00091
$100\theta_{MC}$	1.04092 ± 0.00031	1.04101 ± 0.00029
τ	0.0544 ± 0.0073	0.0561 ± 0.0071
$\ln(10^{10} A_s)$	3.044 ± 0.014	3.047 ± 0.014
n_s	0.9649 ± 0.0042	0.9665 ± 0.0038
H_0	67.36 ± 0.54	67.66 ± 0.42
Ω_Λ	0.6847 ± 0.0073	0.6889 ± 0.0056
Ω_m	0.3153 ± 0.0073	0.3111 ± 0.0056
$\Omega_m h^2$	0.1430 ± 0.0011	0.14240 ± 0.00087
$\Omega_m h^3$	0.09633 ± 0.00030	0.09635 ± 0.00030
σ_8	0.8111 ± 0.0060	0.8102 ± 0.0060
$\sigma_8(\Omega_m/0.3)^{0.5}$	0.832 ± 0.013	0.825 ± 0.011
z_{re}	7.67 ± 0.73	7.82 ± 0.71
Age[Gyr]	13.797 ± 0.023	13.787 ± 0.020
r_*[Mpc]	144.43 ± 0.26	144.57 ± 0.22
$100\theta_*$	1.04110 ± 0.00031	1.04119 ± 0.00029
r_{drag}[Mpc]	147.09 ± 0.26	147.57 ± 0.22
z_{eq}	3402 ± 26	3387 ± 21
k_{eq}[Mpc^{-1}]	0.010384 ± 0.000081	0.010339 ± 0.000063
Ω_K	-0.0096 ± 0.0061	0.0007 ± 0.0019
Σm_ν [eV]	< 0.241	< 0.120
N_{eff}	$2.89^{+0.36}_{-0.38}$	$2.99^{+0.34}_{-0.33}$
$r_{0.002}$	< 0.101	< 0.106

Standard Model of Cosmology

- I. *inflationary epoch* in early Universe
  - **seed** fluctuations for formation of galaxies & life

- II. **matter & energy content** of Universe today
  - exotic particles: *dark matter* (22%)
  - repulsive gravity: *dark energy* (74%)
  - ordinary matter: **only 4%!**

- III. *general relativity*: Einstein
  - describe evolution of matter & energy

Problems in Cosmology

- **I. inflationary epoch: *Not Understood***
  - **what generates initial perturbations?**

- **II. dark sector (96%): *Not Understood***
  - **dark matter (*22%*) and dark energy (*74%*)**
  - **what are the nature of dark sector?**
  - **ordinary matter (*4%*): *Understood, check!***

- **III. Einstein's general relativity: *Not Sure***
  - ***valid* on cosmological scales*?* modified gravity*?***
  - ***well tested* in Solar System**

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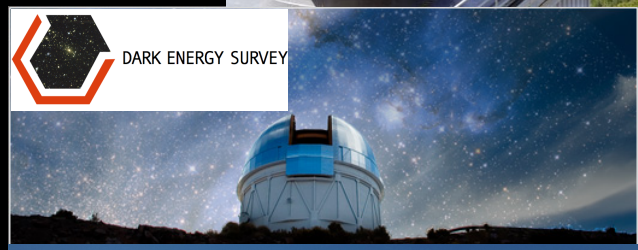
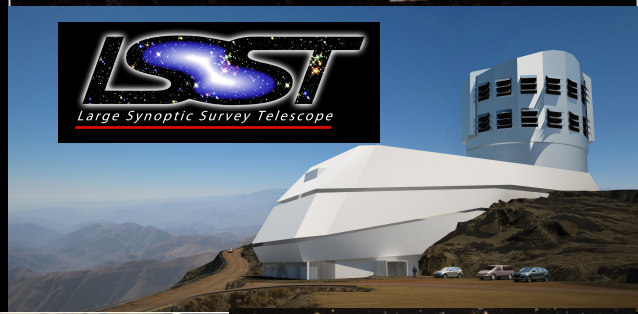
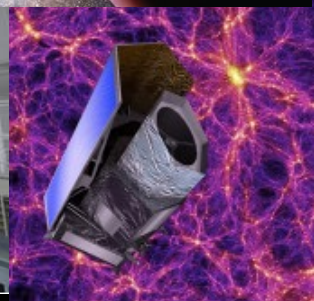
Large-Scale Surveys

- current and future ground-based surveys:
  - Baryonic Oscillation Spectroscopic Survey
  - Dark Energy Survey
  - Dark Energy Spectroscopic Instrument
  - Large Synoptic Survey Telescope





- future space missions:
  - Euclid
  - Wide-Field Infrared Survey Telescope

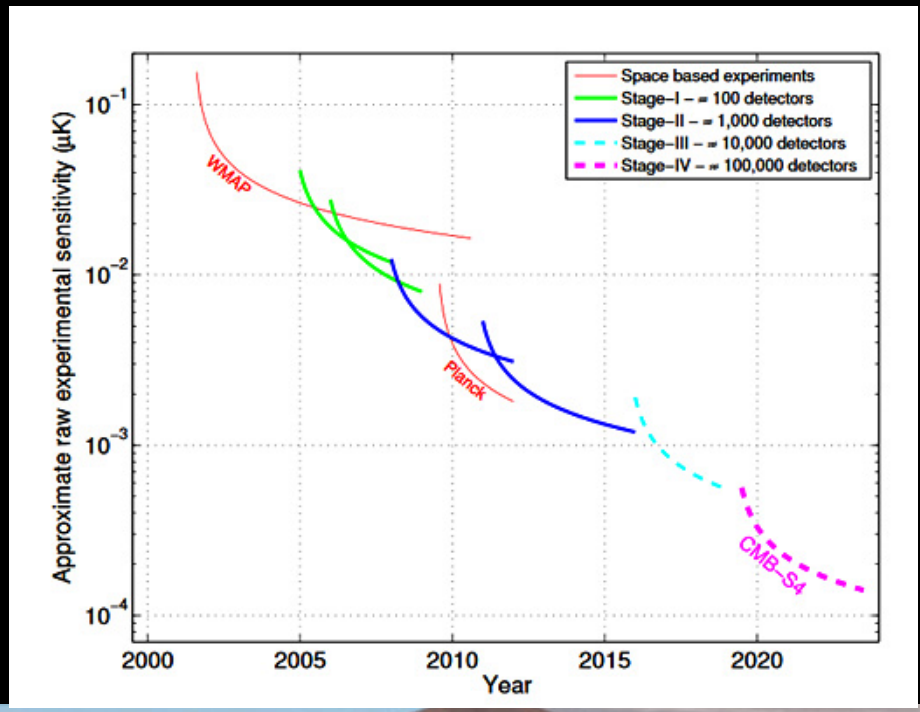


- <u>sub-percent level</u>

precision measurements!

CMB Stage IV (S4)

- **next-generation** CMB experiment:
  - dedicated telescopes
  - South Pole & Chile Atacama
  - and more telescopes?
  - inflation $r < 0.002$
  - neutrino mass $\sum m_\nu$
  - relativistic species $\sigma(N_{\rm eff}) = 0.02$



Challenges

- **precision measurements <u>demand</u>:**
  - ***substantial advances* in theoretical modeling of cosmological observables**

- **standard theoretical descriptions:**
  - **<u>sufficiently accurate</u> to describe precision measurements*?* answer: *No!***
  - **galaxy clustering, weak lensing, Boltzmann eq. etc**
  - **<u>incomplete</u> and <u>limited</u> to linear theory due to *gauge dependence* & *missing observer specification***

Research Program

- **re-write** theoretical descriptions of all cosmological observables:
  - in proper *relativistic framework*
  - check *gauge-invariance*
  - work out *impact of missing physics* on observables
  - relativistic effects as *novel probes* of cosmology
  - work in progress!

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warning:

They work well!

BUT not quite so at the percent level or better

CONTENTS

I. Cosmology: Past & Future

II. Missing Ingredients in Standard Cosmology

III. Improving Standard Cosmology

IV. Do We have to Care at all?

V. Summary and Future Work

II. INCOMPLETENESS in STD COSMOLOGY

General Relativistic Effects & Gauge Invariance

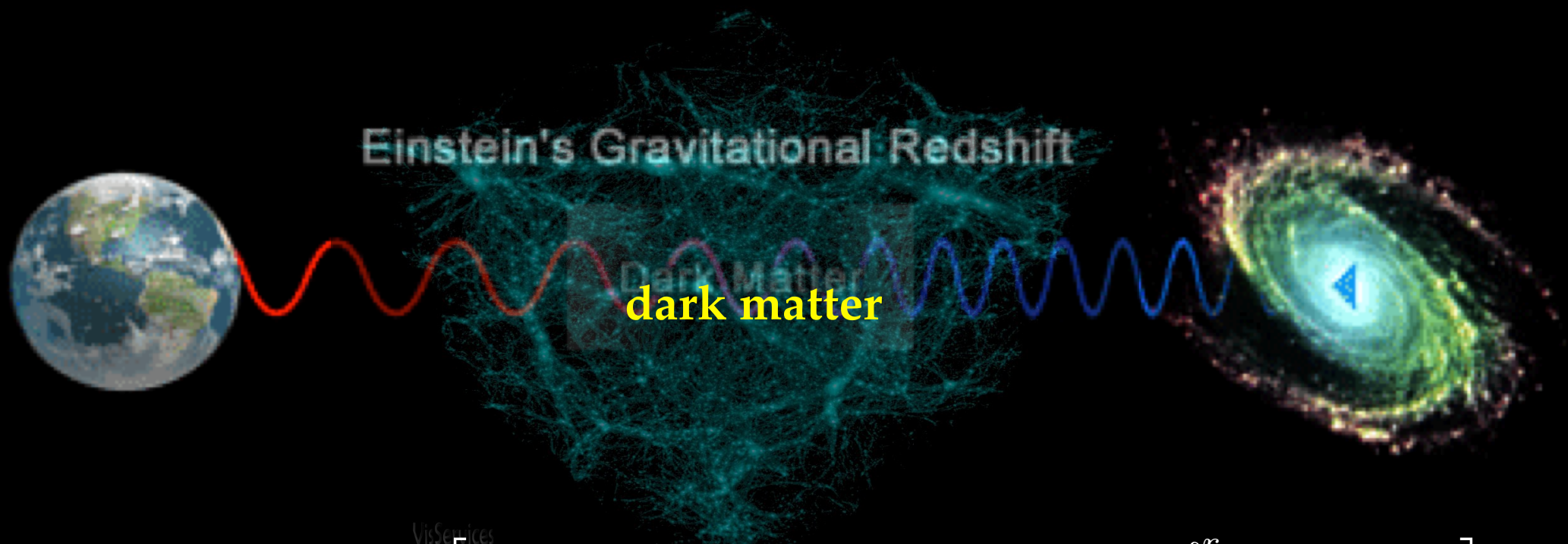
(a) Relativistic Effects

- all cosmological observations by measuring **photons**:
  - **well known**, but often *ignored!*
  - null **geodesic** for light path (vs instantaneous prop.)
  - **light cone** observation (vs same time volume)

- **missing** relativistic effects:
  - gravitational redshift, lensing, frame distortion, etc
  - photon propagation over cosmological scales
  - explicit accounts of all *needed*

Gravitational Redshift

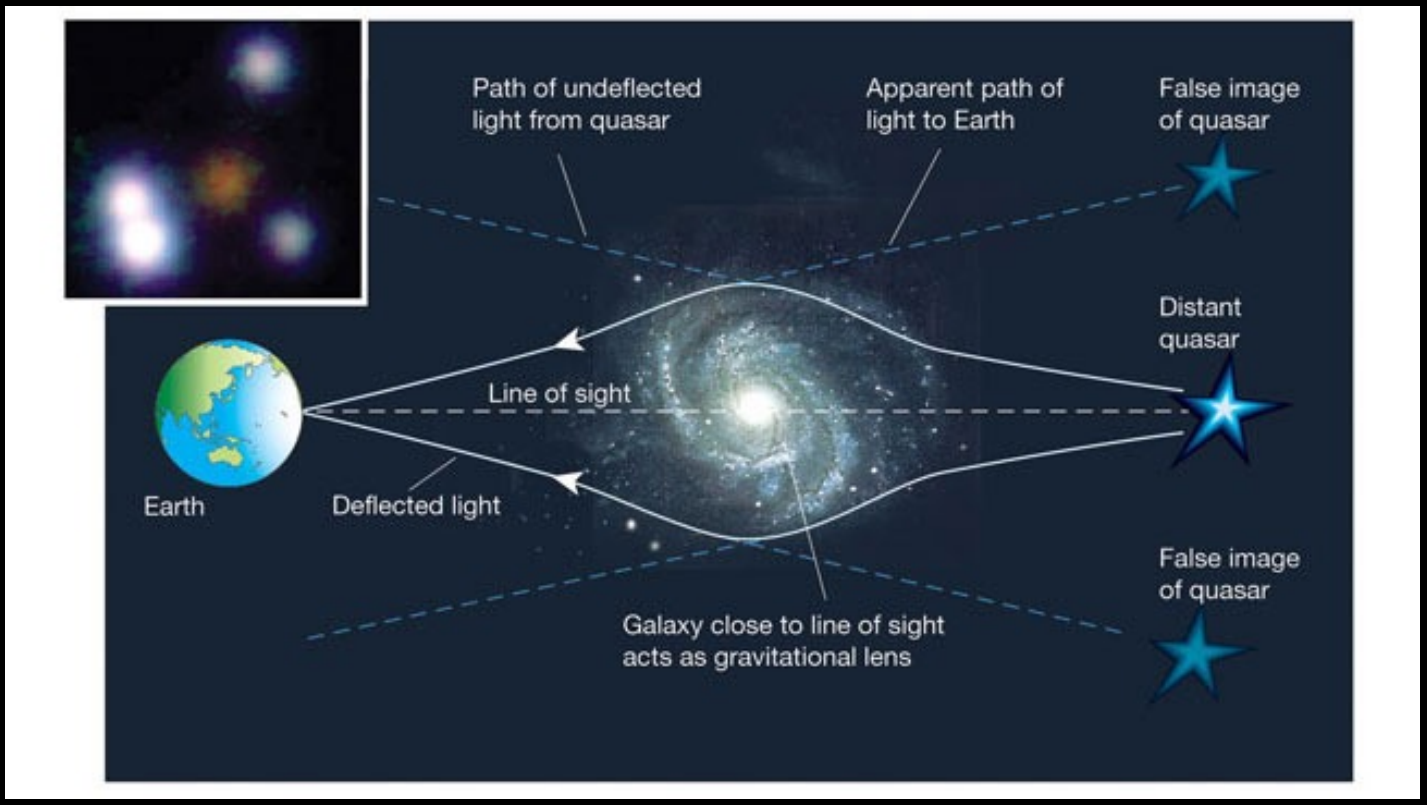
- **photon energy** is affected
- due to gravity at source and observer (*Sachs-Wolfe*)
- also change in gravity during propagation (*iSW*)

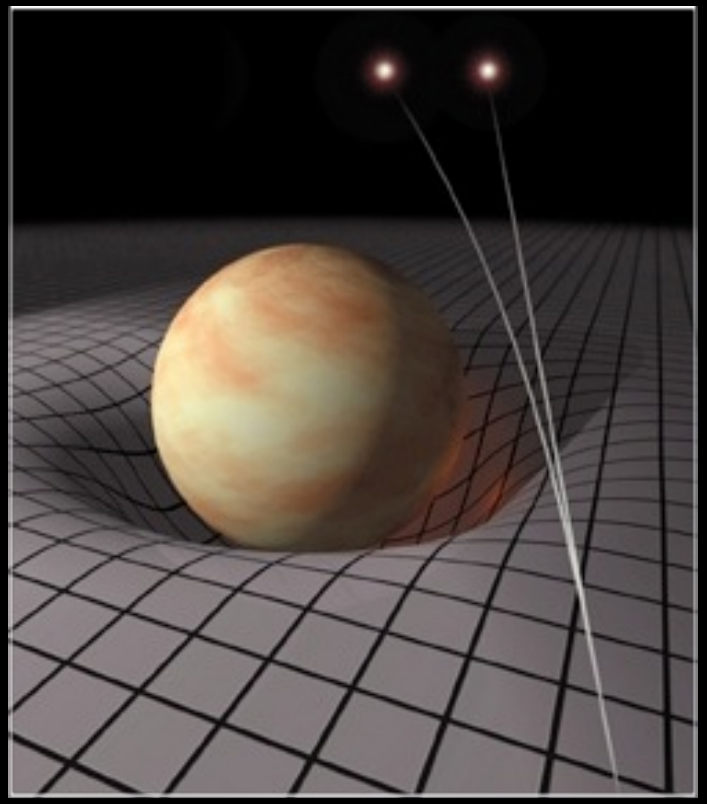


$$1 + z_{\rm obs} = (1+z)\left[1 + V(z) - V(0) - \psi(z) + \psi(0) - \int_0^r dr' \, (\dot{\psi} - \dot{\phi})\right] .$$

Gravitational Lensing

- ***observed*** **angular position is** ***not*** **real position**
- **matter distribution** ***deflect*** **light propagation**



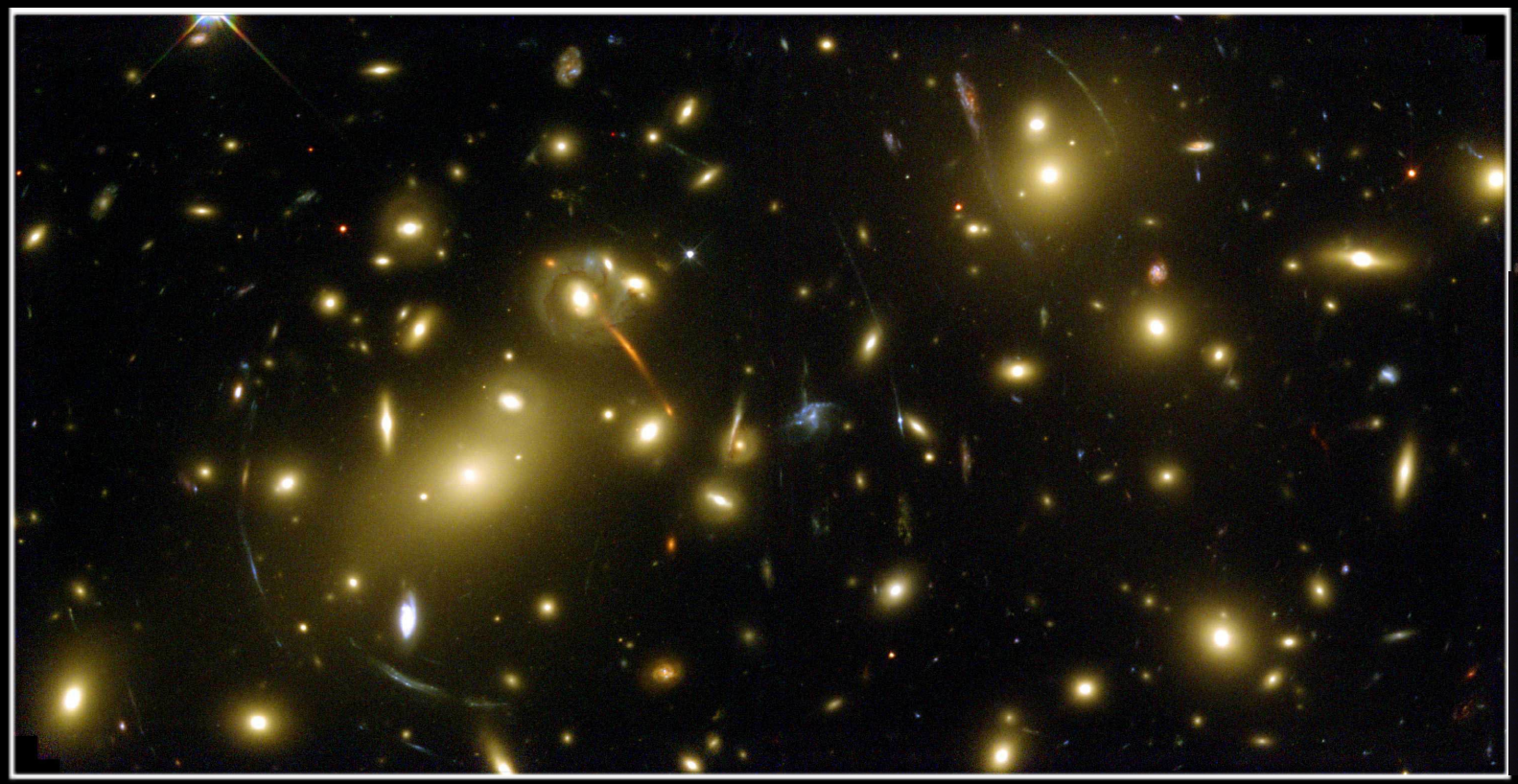


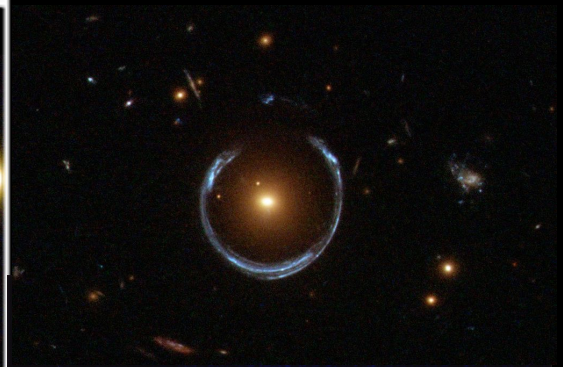
$$\hat{n}_{\rm obs} = (\theta_{\rm obs},\ \phi_{\rm obs}) = \hat{n}_{\rm true} + \delta n, \quad \delta n = (\delta\theta,\ \delta\phi)$$

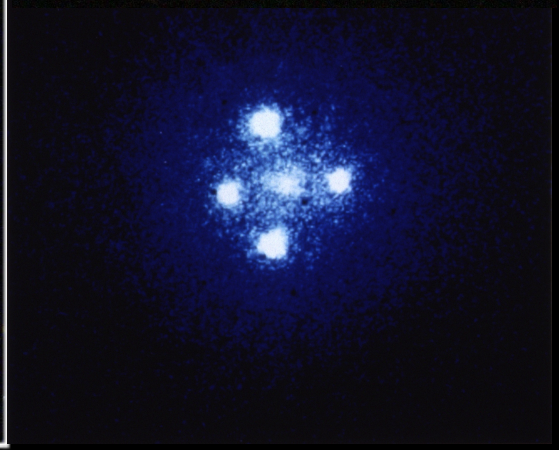
$$\delta_g \propto \kappa$$

Gravitational Lensing

- *observed* angular position is *not* real position
- matter distribution *deflect* light propagation







(b) Who Measures What?

- **cosmological observables:**
  - **photons: frequency, polarization, flux, position**
  - **derivables: redshift, shape, luminosity, number density, lensing shear, etc.**

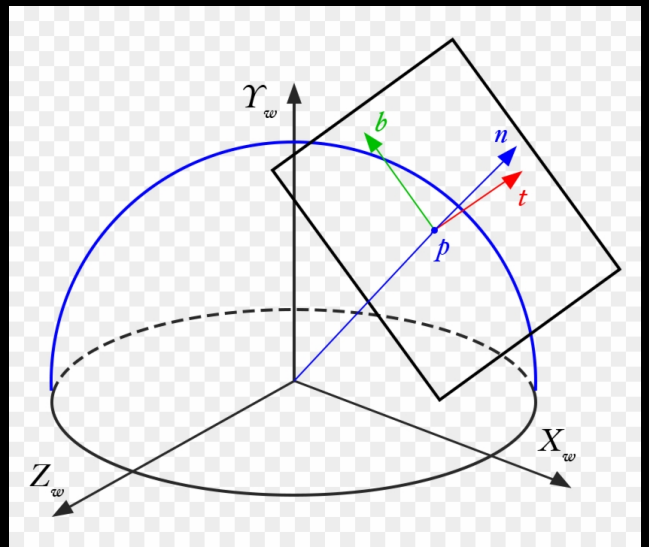
- ***observers* (*us*) in rest frame (Minkowski):**
  - **observer dependent, but so trivial, often ignored!**
  - **(FRW) coordinate independent! (diffeo. invariant)**
  - ***scalar* under diffeomorphism: FRW vectors, tensors, *all against our local basis***
  - **same for physical quantities in the *source* rest frame**

Symmetries

- general relativity (in cosmology):
  - diffeomorphism symmetry: any coordinates work
  - FRW metric with any gauge choice
  - cosmological observables: *gauge invariant*

Symmetries

- coordinates in observer rest frame (Minkowski):
  - (local) Lorentz symmetry (indep. of FRW coordinates) in the tangent space at observer position
  - boost is fixed, only rotational freedom
  - cosmological observables: *not invariant under Lorentz*
  - direct connection to *QFT* calculations



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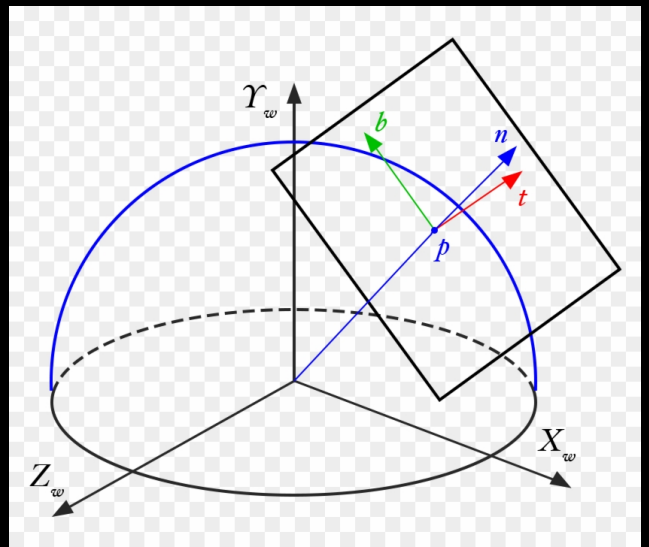
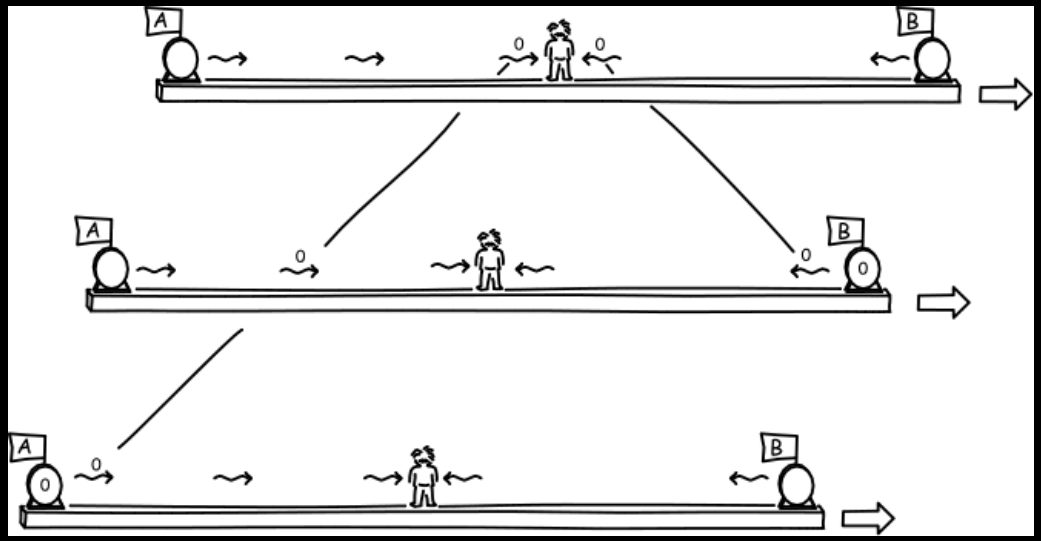
→ *tetrad* formalism (local internal gauge symmetry)



Simultaneity

- **no absolute simultaneity**
- **any choice of hypersurface is ok (*gauge freedom*)**
- **perturbations depend on *choice of hypersurface (gauge)***



simultaneity is relative!

Gauge Issues

- **theoretical predictions in cosmology**
  - **compute perturbations such as** $\delta_m,\ \psi,\ P_m(k),\ \cdots$
  - **compare to observations such as** $\delta_m^{\rm obs},\ P_m^{\rm obs}(k),\ \cdots$
  - **perturbations are *gauge-dependent***
  - ***cannot* be directly associated with *observables!***

- **observable quantities:**
  - **gauge-invariance is a *necessary* condition, but *not a sufficient* condition**
  - **explicit check is *needed***

(c) Gauge-Invariance

- **cosmological observables:**
  - **should be gauge-invariant, but std description: *no!***

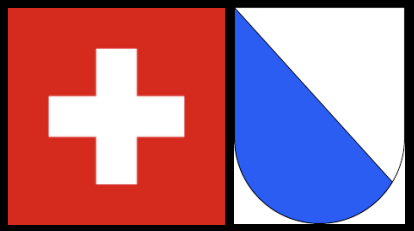
- **standard theoretical descriptions:**
  - **chose one gauge, e.g., conformal Newtonian gauge**
  - **complete gauge fixing: gauge-invariant, *not enough!***
  - ***gauge-dependent*: in general representations**

- **lessons learned:**
  - **gauge fixing: easier, but lose ability to verify**
  - **explicit check of gauge transformations**

Take-Home Message

- standard descriptions: **incomplete**
  - gauge dependent: *different values* in different gauges
  - *no* specification of observer & source
  - *nor* frames in which physical events take place

- **limited** to linear order:
  - rely on background FRW metric
  - similar to Minkowski metric in observer rest frame
  - *only* at 1st order, not generally valid



need to *re-write* cosmology: what we do in Zürich!

III. IMPROVING STANDARD COSMOLOGY

Gauge-Invariant Formalism

Observables

- **use <u>observables</u>, not <u>unobservables</u>!**
- **unobservables: (*gauge-dependent*)**
  - **most quantities in theoretical descriptions**

$$\bar{z},\ \hat{s},\ \delta_m,\ \psi,\ P_m(k)\ ,\cdots$$

- **observables: (*physical*)**
  - **photon wave vector in observer rest frame**

$$k^a = \omega\left(1, -n^i\right)\ , \qquad n^i = (\theta, \phi)\ , \qquad \omega = 2\pi\nu\ ,$$

  - ***ind. FRW*, gauge-invariant, subject to *Lorentz trans.***
  - **same for other observables & derivables**
  - **e.g., redshift, angular size,** $1 + z = \dfrac{\nu_{\rm src}}{\nu}\ , \qquad (d\theta, d\phi)\ ,\cdots$

FRW Coordinates

- going to coordinates: tetrad vectors at observer $e^\mu_a(x^\nu_o)$
- photon wave vector in FRW:
  - *different* from what we measure in rest frame!

$$k^\mu = e^\mu_a k^a = \frac{\omega}{a}\left[1 - \mathcal{A} - n^i \delta^\beta_i \left(\mathcal{U}_\beta - \mathcal{B}_\beta\right), -n^i \delta^\alpha_i + \mathcal{U}^\alpha + n^i \delta^\beta_i \left(\varphi\, \delta^\alpha_\beta + \mathcal{G}^\alpha{}_{,\beta} + C^\alpha_\beta\right) + \epsilon^\alpha{}_{ij} n^i \Omega^j\right]$$

  - Doppler effect, gravitational redshift, distortion, etc.
  - same for other observables & derivables

Yoo, Grimm, Mitsou, Amara, Refregier 2018 JCAP 04, 029

- transport them to source position:
  - geodesic equation but with *boundary condition*!
  - geodesic deviation equation for shapes
  - check gauge dependence (yes, specific way)

Physical Quantities at SRC

- going to *source rest frame*: tetrad vectors at src $e^{\mu}_{a}(x^{\nu}_{s})$
  - same effects at src: Doppler effect, gravitational redshift, frame distortion, misalignment, etc.
  - src has different velocity, potential, etc

- relation bw observables & physical quantities:
  - should be gauge invariant, explicit check needed
  - e.g. src physical length vs observed angular size
  - emitted frequency vs measured frequency (redshift)
  - intrinsic luminosity vs apparent flux, etc.
  - also subject to LLT at observer & source

Yoo, Grimm, Mitsou, Amara, Refregier 2018 JCAP 04, 029

Boltzmann EQ & QFT

- evolution equations along trajectory:
  - LHS: propagation of any fluids in spacetime manifold
  - RHS: particle interactions in rest frame
  - S-matrix invariant under LLT, not under diffeo.

- problems in standard approach:
  - parametrize geodesic & momentum $x^\mu(\lambda)\ ,\quad p^\mu = \dfrac{dx^\mu}{d\lambda}$
  - coordinate transformation

$$d\tilde{x}^\mu = \frac{\partial \tilde{x}^\mu}{\partial x^\nu}\, dx^\nu\ ,\quad \tilde{p}^\mu = \frac{\partial \tilde{x}^\mu}{\partial x^\nu}\, p^\nu \qquad |\mathcal{M}(p_i^\mu)|^2 \neq |\tilde{\mathcal{M}}(\tilde{p}_i^\mu)|^2$$

  - microscopic physics in rest frame

$$p^a = e^a_\mu p^\mu\ ,\quad \tilde{p}^a = \Lambda^a{}_b p^b \qquad |\mathcal{M}(p_i^a)|^2 = |\tilde{\mathcal{M}}(\tilde{p}_i^a)|^2$$

Mitsou & Yoo, Springer Briefs 2020

Tetrad Formalism

- **tetrad fields:** e^μ_a
  - **metric is fully contained in tetrad** $g_{\mu\nu} = \eta_{ab} e^a_\mu e^b_\nu$
  - **transparency: diffeo. & LLT (internal gauge symm.)**
  - **spinors: a representation of SL(2,C), not in diffeo.**
  - **already well developed in general relativity**

- **application to cosmology: natural generalization**
  - **not only at observer or src, but fields (*everywhere!*)**
  - **observer family: all possible observers everywhere**
  - **when projected, "*observables*"& *gauge invariant***
  - **natural connection to QFT in Minkowski spacetime**

Mitsou & Yoo, Springer Briefs 2020

IV. DO WE HAVE TO CARE?

Impacts of the Relativistic Effects

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Impacts of the Relativistic Effects

(a) Luminosity Distance

(b) Galaxy Clustering

(c) Weak Gravitational Lensing

(d) Cosmic Microwave Background Anisotropies

(e) Cosmological Information: Cosmic Variance

(b) Galaxy Clustering

- **measure of how galaxies are distributed:**

- **construct fluctuation in galaxy counts:**
  - **total number of observed galaxies** $dN_{\rm tot}$
  - **observed volume** $dV_{\rm obs}$ **given** $(z_{\rm obs}, \theta_{\rm obs}, \phi_{\rm obs})$
  - **fluctuation field** $\delta_g^{\rm obs} = \frac{n_g^{\rm obs}}{\langle n_g^{\rm obs} \rangle} - 1$

- **relation to physical number density:**
  - **number conservation** $dN_{\rm tot} = n_g^{\rm phy} dV_{\rm phy} = n_g^{\rm obs} dV_{\rm obs}$
  - **observed number density** $n_g^{\rm obs} = n_g^{\rm phy} \frac{dV_{\rm phy}}{dV_{\rm obs}}$

$$z_{\rm obs} \neq z \ , \quad f_{\rm obs} \neq f_{\rm phy} \ , \quad dV_{\rm obs} \neq dV_{\rm phy}$$

Unified Treatment

- **observable:** $dN_{\rm tot} = n_g^{\rm phy} dV_{\rm phy} = n_g^{\rm obs} dV_{\rm obs}$

dV : volume
V : velocity

- **volume effects:** $\dfrac{dV_{\rm phy}}{dV_{\rm obs}}$
  - **redshift-space distortion:** $\dfrac{\partial z}{\partial z_{\rm obs}} \simeq \dfrac{1+z}{H}\dfrac{\partial V}{\partial r}$
  - **lensing magnification:** $\dfrac{\partial \Omega}{\partial \Omega_{\rm obs}}\dfrac{\partial f}{\partial f_{\rm obs}} \simeq \dfrac{1}{\mu^2} = 1 - 4\,\kappa$
- **source effects:** $n_g^{\rm obs} \simeq n_g^{\rm phy}$
  - **observed redshift:** $n_g^{\rm obs}(z_{\rm obs}) \simeq n_g^{\rm phy}(z)$
  - **magnification bias:** $n_g^{\rm obs}(f_{\rm obs}) \simeq n_g^{\rm phy}(f_{\rm phy})$

$z_{\rm obs} \neq z$
$f_{\rm obs} \neq f_{\rm phy}$

- ***complete description*** **of** ***all*** **the effects**
  - **holds in Newtonian & GR descriptions**
- ***subtlety*****: relation to physical quantities** $dN_g,\ n_g^{\rm phy},\ dV_{\rm phy}$

Yoo 2009 PRD

Observed Physical Volume in 4D

- **unified treatment:** $N_{\rm tot} = n_g^{\rm phy}\, dV_{\rm phy} = n_g^{\rm obs}\, dV_{\rm obs}$
- **integral of 3-form in 4D spacetime manifold:**
  - **observables** $z_{\rm obs},\ \theta_{\rm obs},\ \phi_{\rm obs}$
  - **photon geodesic path** $x^a(\lambda) = \bar{x}^a(\lambda) + \delta x^a(\lambda)$
  - **redshift-space distortion, lensing effects, and etc.**
  - **distortion in local Lorentz frame**
  - ***manifestly gauge-invariant***

$$dV_{\rm phy} = \int \sqrt{-g}\, \varepsilon_{abcd}\, u^d\, \frac{\partial x^a}{\partial z_{\rm obs}} \frac{\partial x^b}{\partial \theta_{\rm obs}} \frac{\partial x^c}{\partial \phi_{\rm obs}}\, dz_{\rm obs}\, d\theta_{\rm obs}\, d\phi_{\rm obs}$$

Yoo, Fitzpatrick, Zaldarriaga, PRD, 2009

Levi-Civita symbol ε_{abcd}, **comoving velocity** u^a

Relativistic Formula

- **standard Kaiser formula:** $\delta_g = b\,\delta_m - \frac{1+z}{H}\frac{\partial V}{\partial r} = b\,\delta_m + f\,\mu_k^2\delta_m$
- **general relativistic formula:**

$$\delta_g = b\,\delta_m^{(v)} - e\,\delta z^{(v)} + \alpha_\chi + 2\,\varphi_\chi + V - C_{\alpha\beta}\,e^\alpha e^\beta$$
$$+3\,\delta z_\chi + 2\,\frac{\delta\mathcal{R}}{r} - H\frac{\partial}{\partial z}\left(\frac{\delta z_\chi}{\mathcal{H}}\right) - 5p\,\delta\mathcal{D}_L - 2\,\mathcal{K}\,,$$

(all the individual terms are gauge-invariant variables)

Yoo, Fitzpatrick, Zaldarriaga 2009, Yoo 2010, Bonvin & Durrer 2011
Challinor & Lewis 2011, Bruni et al. 2012, Jeong, Schmidt, Hirata 2012
Yoo, Hamaus, Seljak, Zaldarriaga 2012, and many more

- **2nd-order relativistic description:**

Yoo & Zaldarriaga 2014, Di Dio, Durrer, et al. 2014, Bertacca et al. 2014

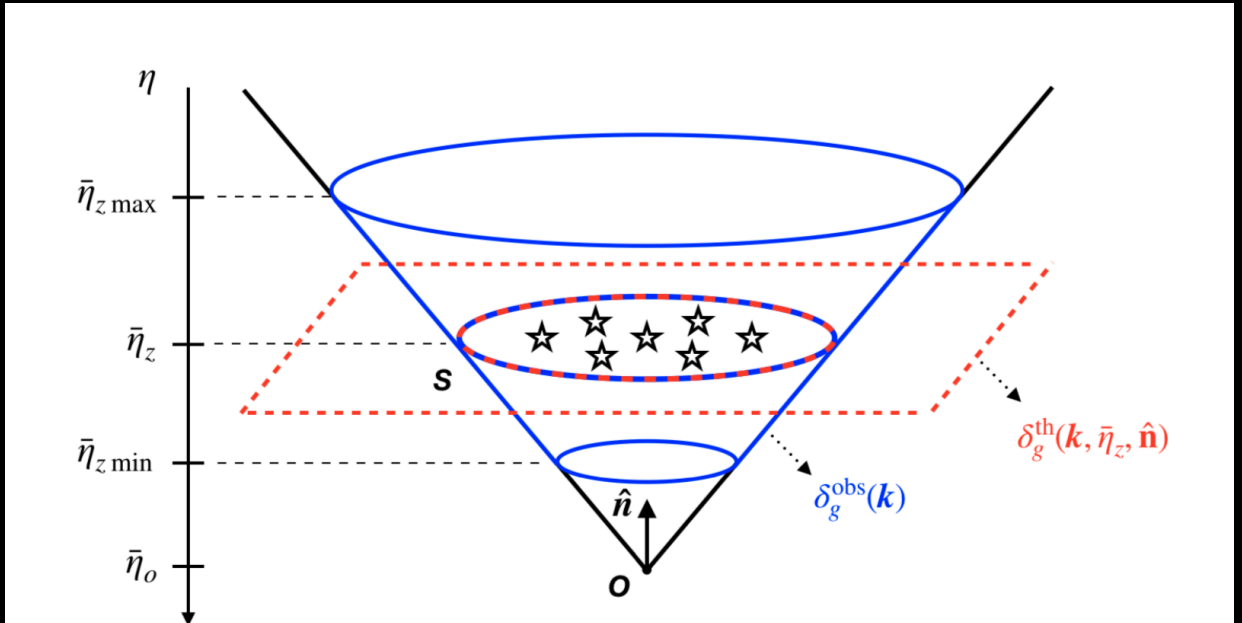
Power Spectrum Analysis

- *full relativistic description* of galaxy clustering:
  - split into *three types* of contributions at source position, line-of-sight, observer position at *1st order*
  - *gauge-invariant*, respect *equivalence principle*
  - compute the galaxy power spectrum!

- issues associated with power spectrum:
  - infinite hyper surface vs observed light-cone volume
  - *boundaries*, *time-evolution*, *wide angle*, …
  - theory power spec: suited for theoretical investigation
  - obs power spectrum: what we measure in surveys

Theory vs Observed?

- **relation between theory and obs power spectra:**
  - ***convolution*** **of theory power spectrum over survey volume in multiple time slices**
  - $\delta_g^{\rm obs}(k) = \int \frac{d^3k'}{(2\pi)^3} \int_V d^3x_{\rm obs}\; e^{i(k'-k)x_{\rm obs}} \delta_g^{\rm th}(k';\eta_z)$ $\qquad \delta_g^{\rm th}(k;\eta_z) =: T(k,\eta_z)\zeta(k)$
  - $P_{\rm obs}(k) = \frac{1}{V}\int \frac{d^3k'}{(2\pi)^3}\; P_\zeta(k')\,|F(k',k)|^2$ $\qquad F(k,k') := \int_V d^3x_{\rm obs}\; e^{i(k'-k)x_{\rm obs}} T(k,\eta_z)$
- **focus on theory power spectrum for further** ***theoretical investigations***



Relativistic Description

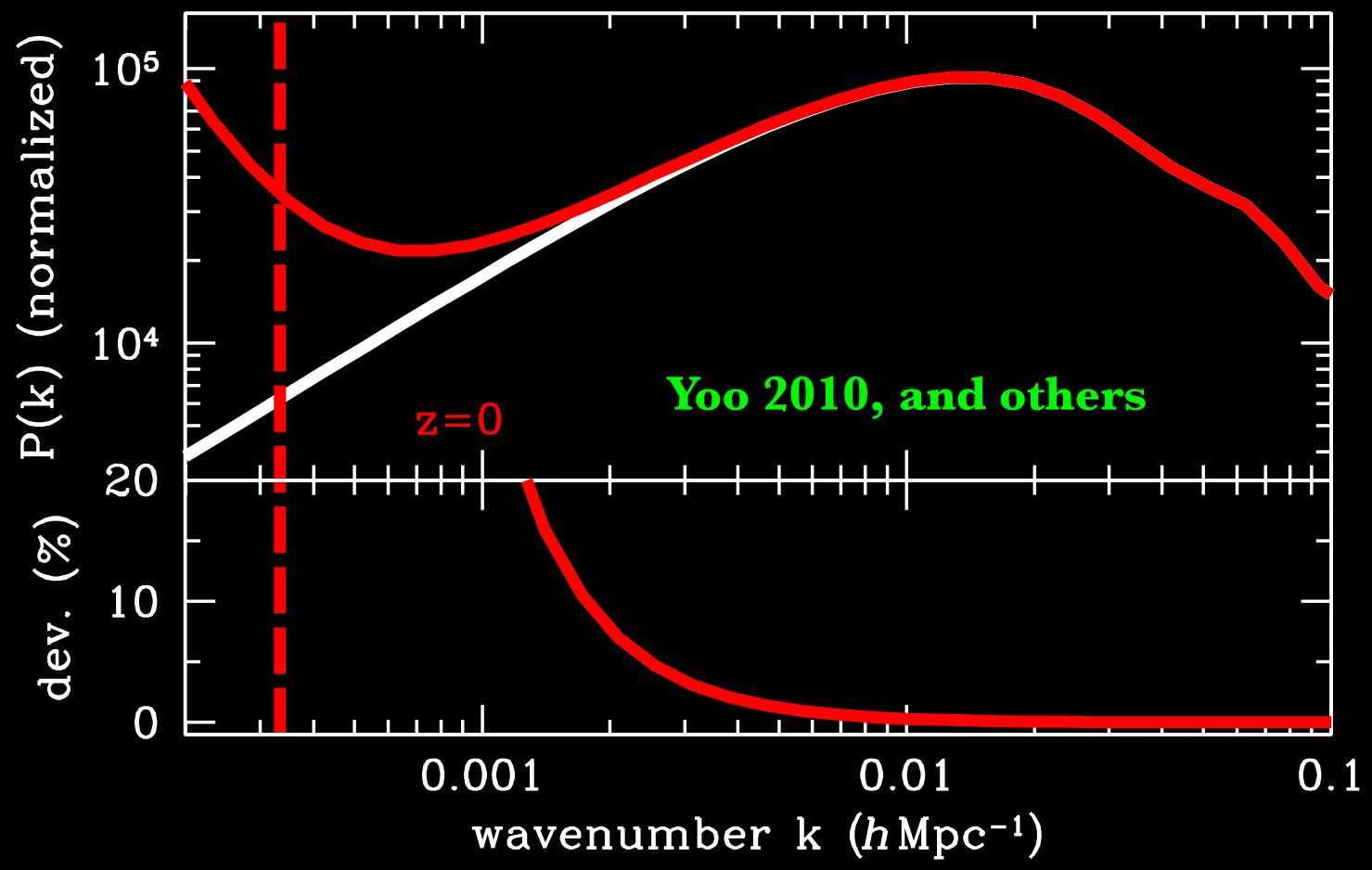
- **three types** of contributions
  - **source**: standard RSD, gravitational potential
  - **line-of-sight**: standard lensing, iSW
  - **observer**: velocity, potential
  - separation into three types: *gauge dependent*
  - *only the whole combination* is gauge invariant
  - three types *mix* beyond linear order

- previous work on power spectrum: **incomplete!**
  - **only source**: RSD, relativistic contribution, fnl cont.
  - contributions at **observer** & along **los**: *ignored*

Galaxy Power Spectrum

- relativistic effect at source *only*:
  - gravitational potential *dominant!*
  - relativistic effect ~ primordial non-Gaussianity!

- relativistic effects in *early* Universe vs *late* Universe
- power spectrum *diverge in the infrared*
- *sensitivity* to super-horizon scale fluct*?*



Equivalence Principle

- **equivalence principle: already at *one point level***

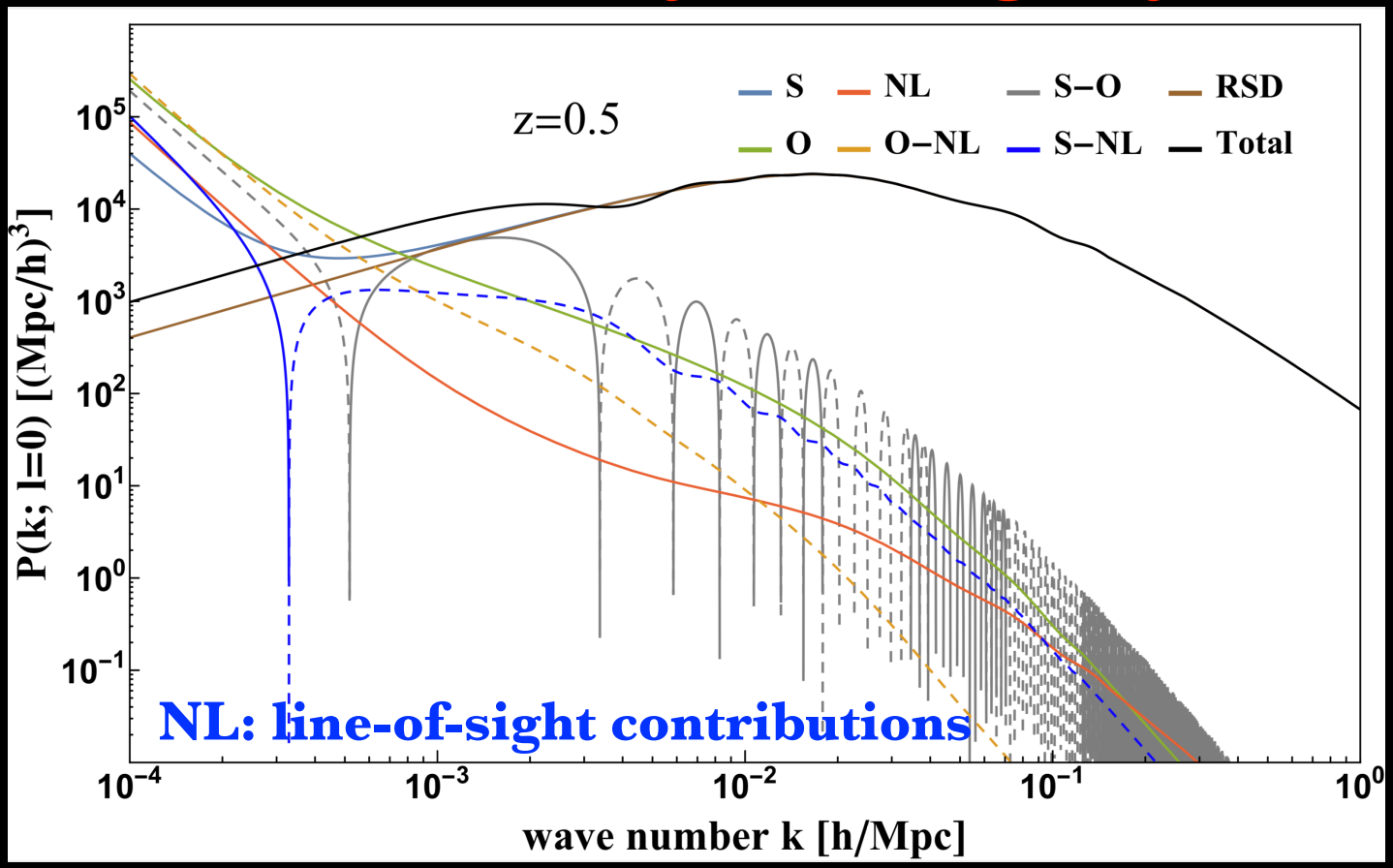
  Jeong, Schmidt, Hirata 2012, Schmidt & Jeong 2012, Biern & Yoo 2017, Scaccabarozzi, Yoo, Biern 2018

  - ***independent* of uniform gravitational potential and acceleration** $\phi(x) \to \phi(x) + \phi_0$, $\delta_g^{\rm obs}(x) \to \delta_g^{\rm obs}(x)$
  - **cancellation of ϕ_0 is made possible with *all* three-types of contributions via *Einstein equation***
  - ***no cancellation* <u>without Einstein equation</u>**

- **luminosity distance:** Biern & Yoo 2018
  - **standard expression *ignored* δD_L at <u>observer position</u>**
  - $\delta D_L^{\rm std}(x)$ **respond to uniform ϕ_0, variance *diverges* in IR**
  - **gauge-invariant expression: *no divergence***

Galaxy Power Spectrum

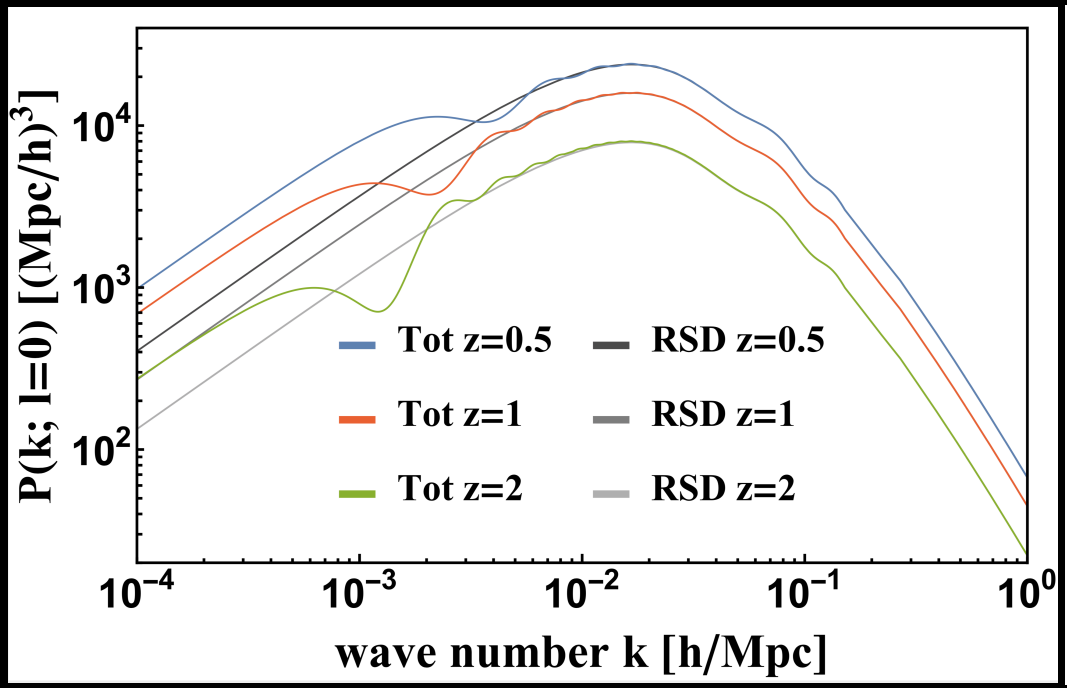
- **full** relativistic effects on **monopole** power spectrum:
  - **many** contributions are ***diverging!***
  - ***all cancel at low k! no fnl-like signal from GR!***

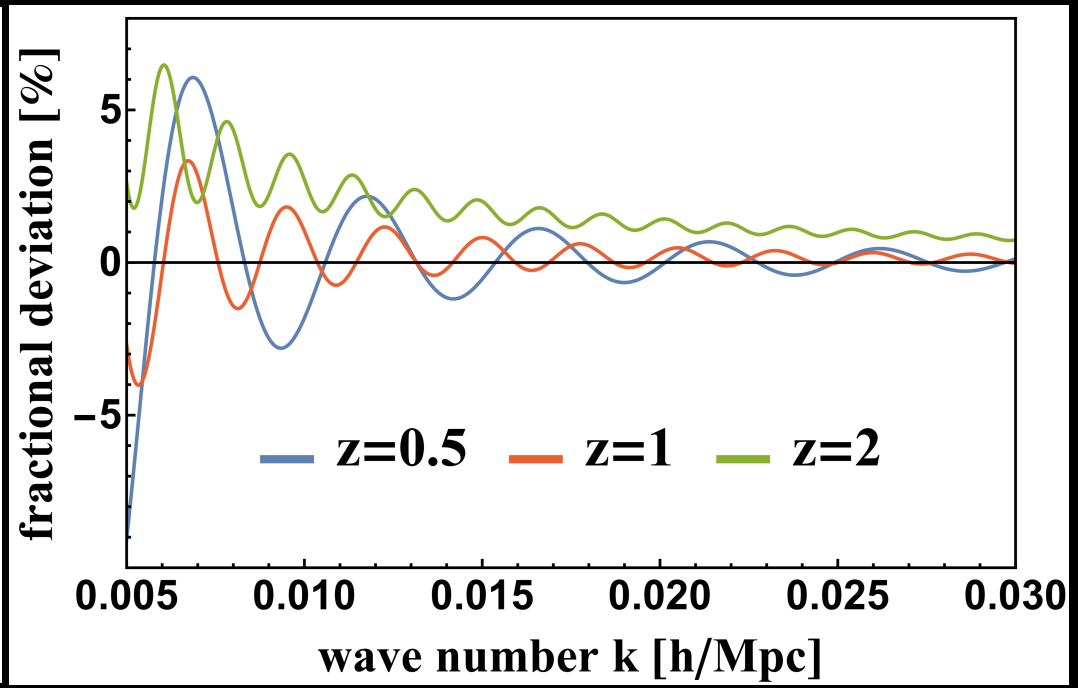


Grimm, Scaccabarozzi, Yoo, Biern, Gong, JCAP, 2020

Galaxy Power Spectrum

- relativistic effects on "small" scales:
  - deviations compared to standard RSD: *few percents!*
  - stronger at higher redshift
  - *observed pok*: convolution with survey window





Grimm, Scaccabarozzi, Yoo, Biern, Gong, JCAP, 2020

Implications on local-type fnl

- **standard theoretical description:**

  Dalal, Dore, Huterer, Shirokov 2008, Matarrese & Verde 2008, and more

  - $\delta_g^{\rm std}(x) = b_L \delta_m(x) + 2 f_{\rm NL} b_L \delta_c \; \phi(x)$ **respond to uniform** ϕ_0
  - ***not compatible*** **with equivalence principle*!***
  - **source of rising power at low** k

- **simple remedy to standard Newtonian method:**
  - **consider relativistic effects with light-propagation**
  - $\delta_g^{\rm std}(x) \to \delta_g^{\rm GR}(x) + 2 f_{\rm NL} b_L \delta_c \; \phi(x)$ ***not working!***
  - **those relativistic effects:** ***not contribute*** **to** $f_{\rm NL}$**,** ***nor cure*** **the situation in** $\delta_g^{\rm std}(x)$

(b) Galaxy Clustering

- **IR divergences & Equivalence principle:**
  - **IR-∞ common in relativistic calculations**
  - ***observables should be devoid of such pathology***
  - **cancellation is very *subtle***

- **relativistic effects in the power spectrum:**
  - **relativistic effects: *not contribute* to $f_{\rm NL}$ or $\Delta f_{\rm NL} = 0$**
  - ***non-negligible* at $k \simeq 0.01\ h/{\rm Mpc}$ or larger scales**
  - **standard formula for $f_{\rm NL}$: need improvements?**

V. MORE WORK & FUTURE DIRECTION:

In the Next Five Years and the Coming Decade

What can We Learn?

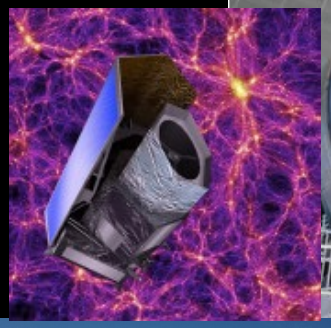
- **relativistic effects**: ***small***, but ***detectable!***
  - ***extra*** and ***critical*** information
  - difficult, but high gain (***new*** opportunities)

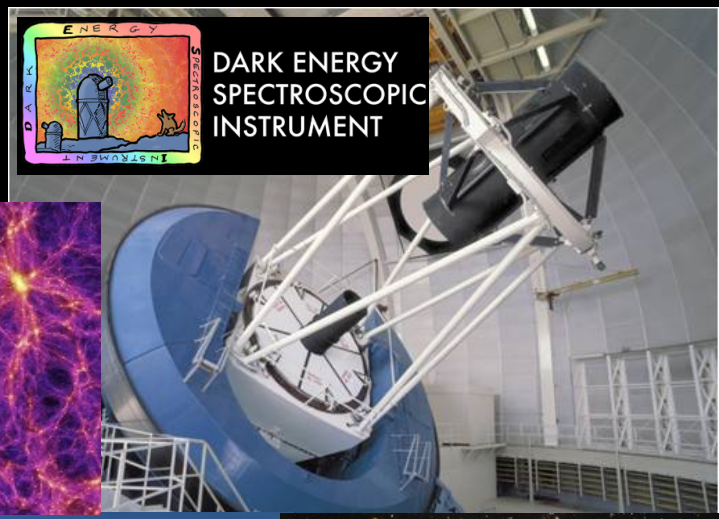
- **key role**: ***deviations*** from standard cosmology
  - ***higher-order signatures*** (fossil fields, DE fluct.)
  - ***not*** present in Newtonian description

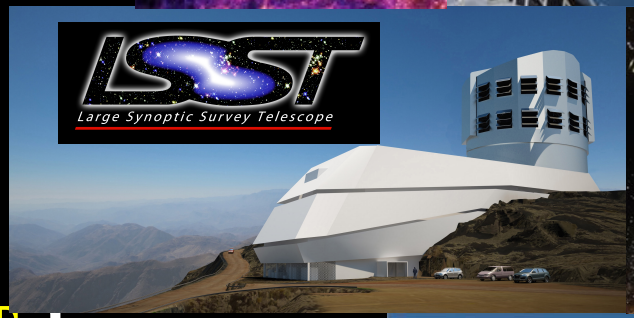
- **complementary role**: ***enigmatic*** standard cosmology
  - (better) ***complementary*** to CMB constraints
  - ***convincing*** constraints on dark energy

Future Surveys

- future ground-based surveys:
  - Dark Energy Spectroscopic Instrument
  - Large Synoptic Survey Telescope
  - Square Kilometer Array
  - CMB Stage-IV







- future space missions:
  - Euclid
  - Wide-Field Infrared Survey Telescope

Are We Ready for Precision Cosmology?

General Relativistic Effects and Gauge-Invariant Formalism

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19 November 2020

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