

String theory, gravity and cosmology (SGC2020)

Based on 2006.11717

Unitarity of Entanglement and Islands in Two-sided Janus Black Holes

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Unitarity of Entanglement and Islands in Two-sided Janus Black Holes

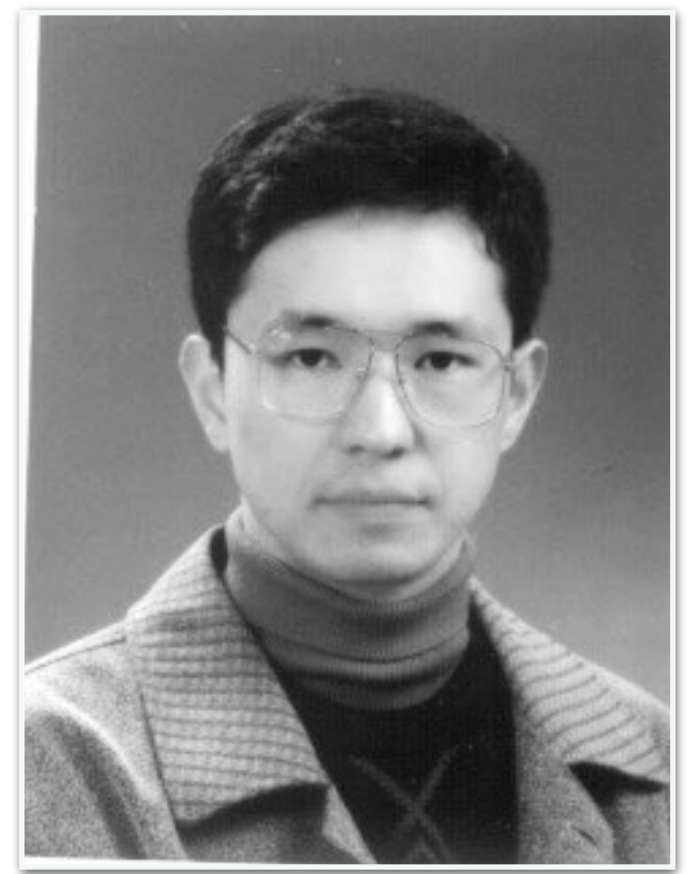
D. Bak, C. Kim, S. Yi and JY
arXiv: 2006.11717



Dongsu Bak
University of Seoul

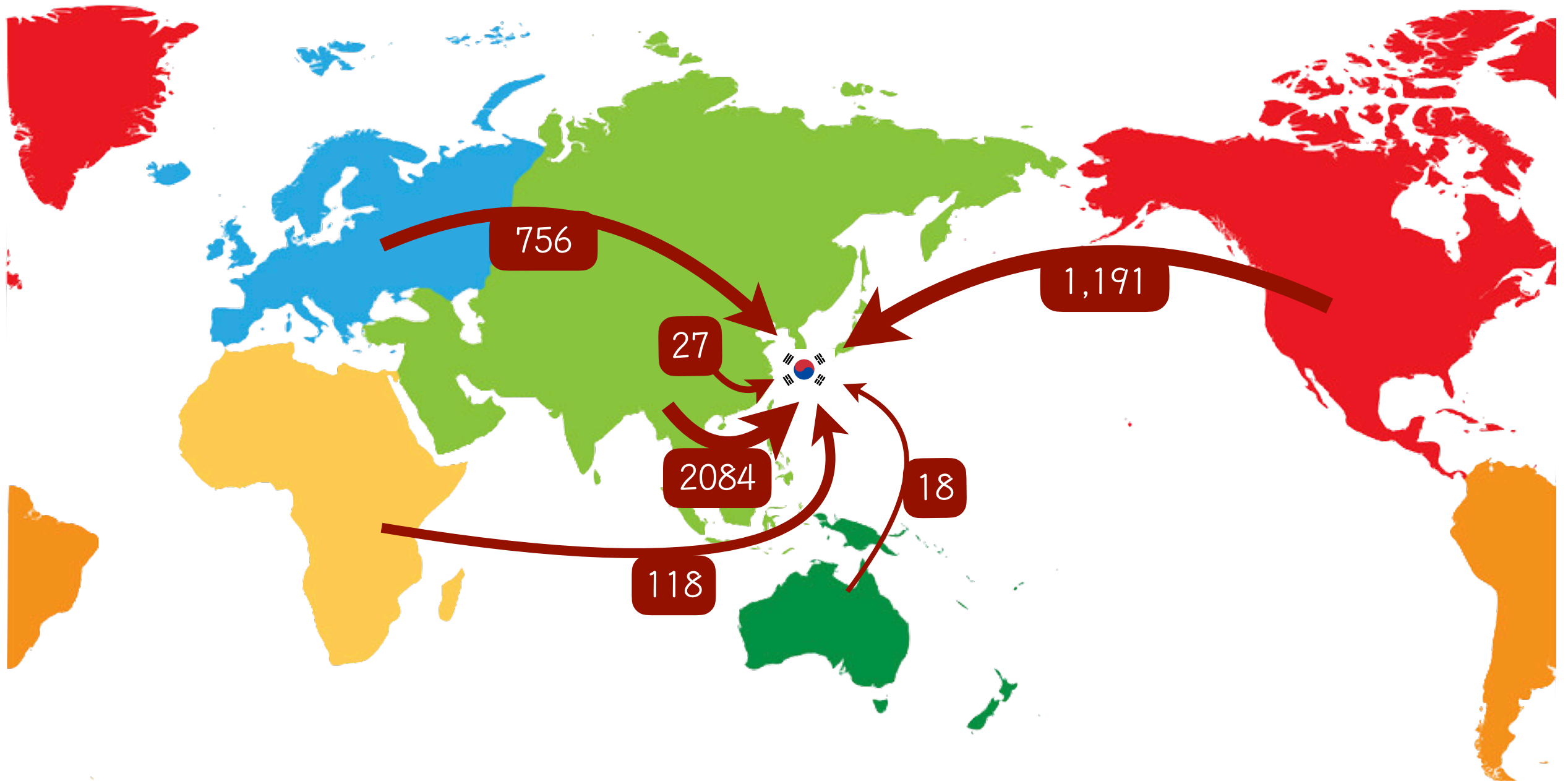


Chanju Kim
Ewha Women's University



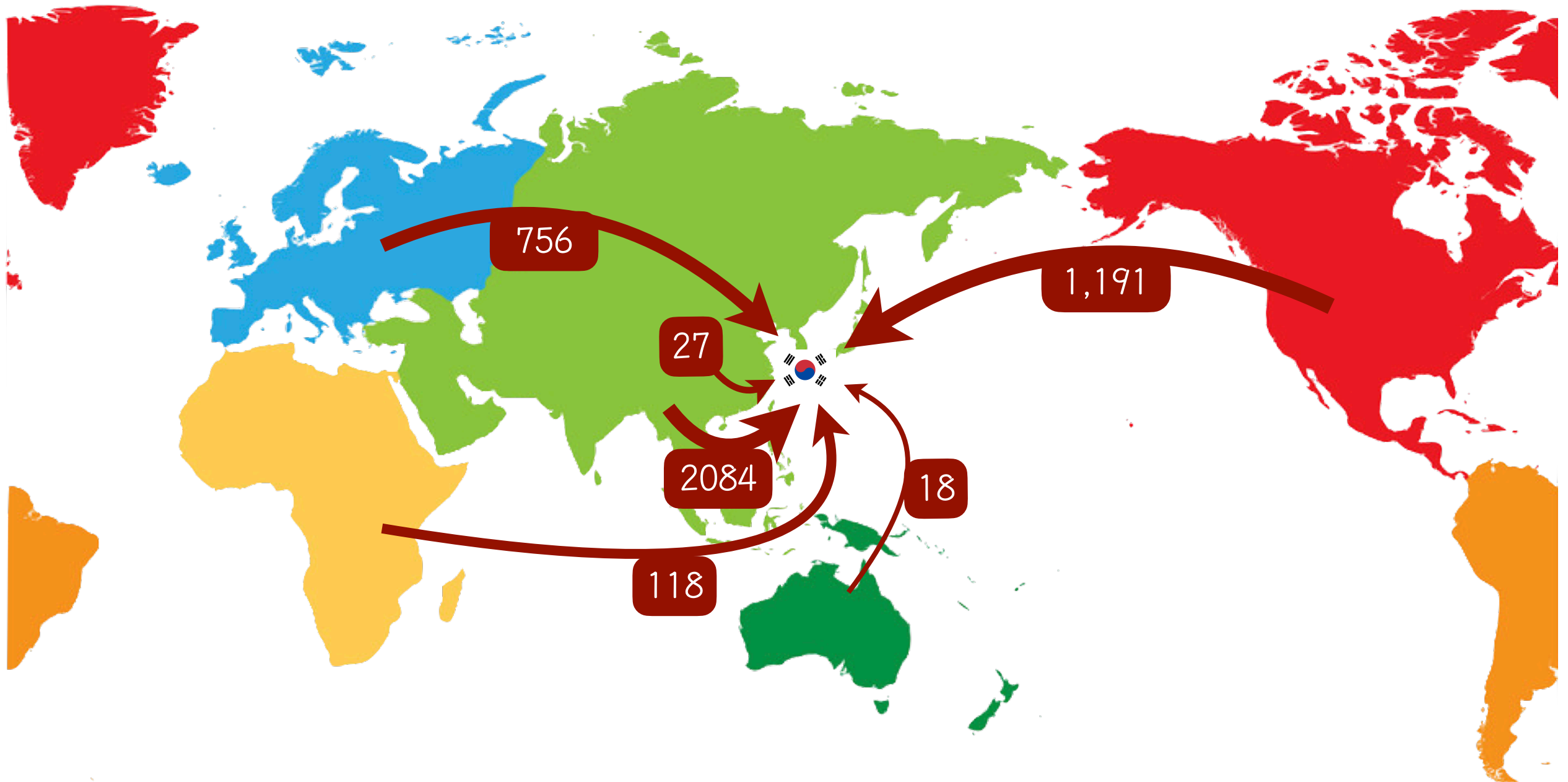
Sang-Heon Yi
CQUEST

COVID-19: Imported Cases from Outside of Korea

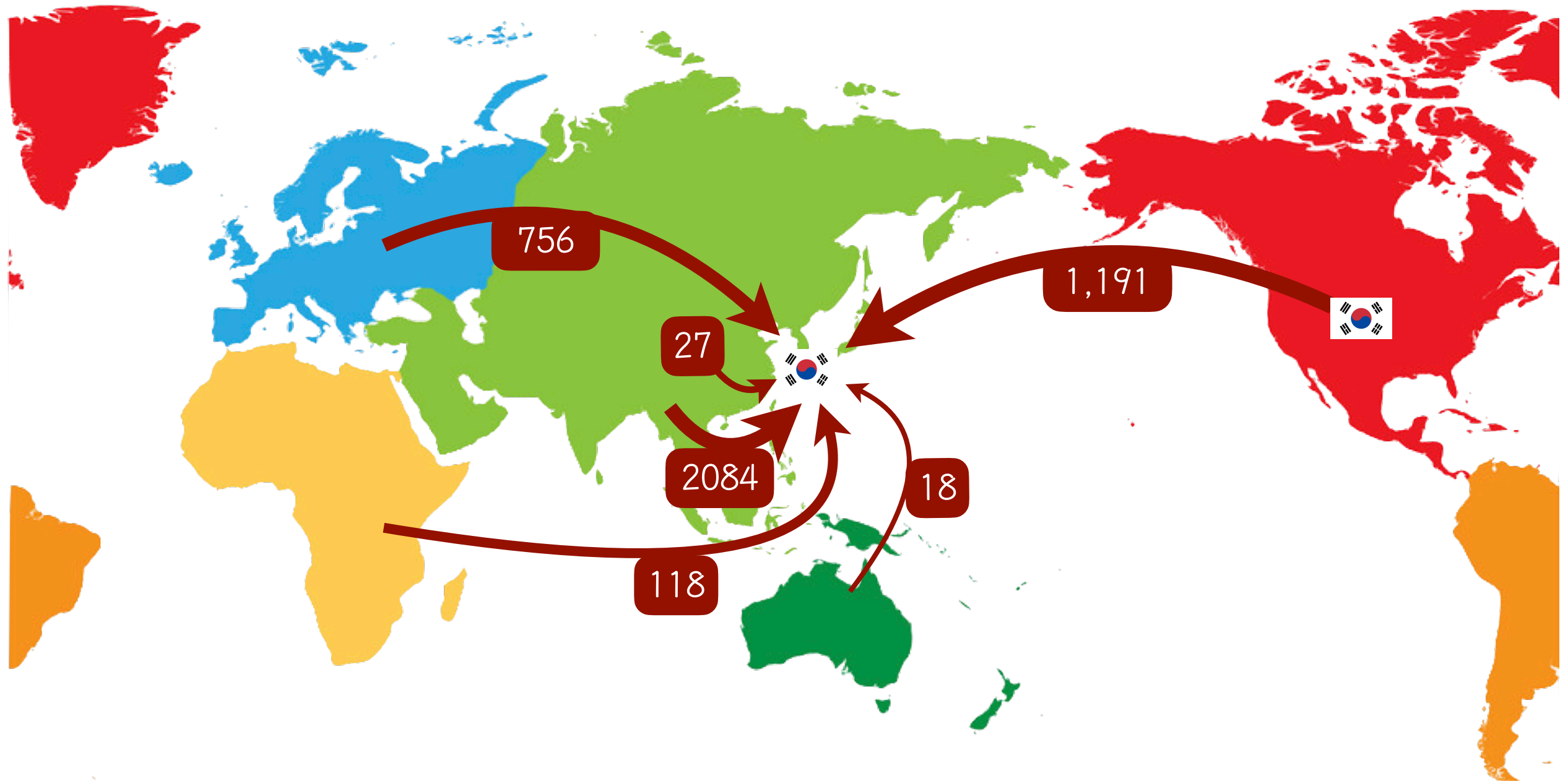


How can we reduce the number of
the imported cases?

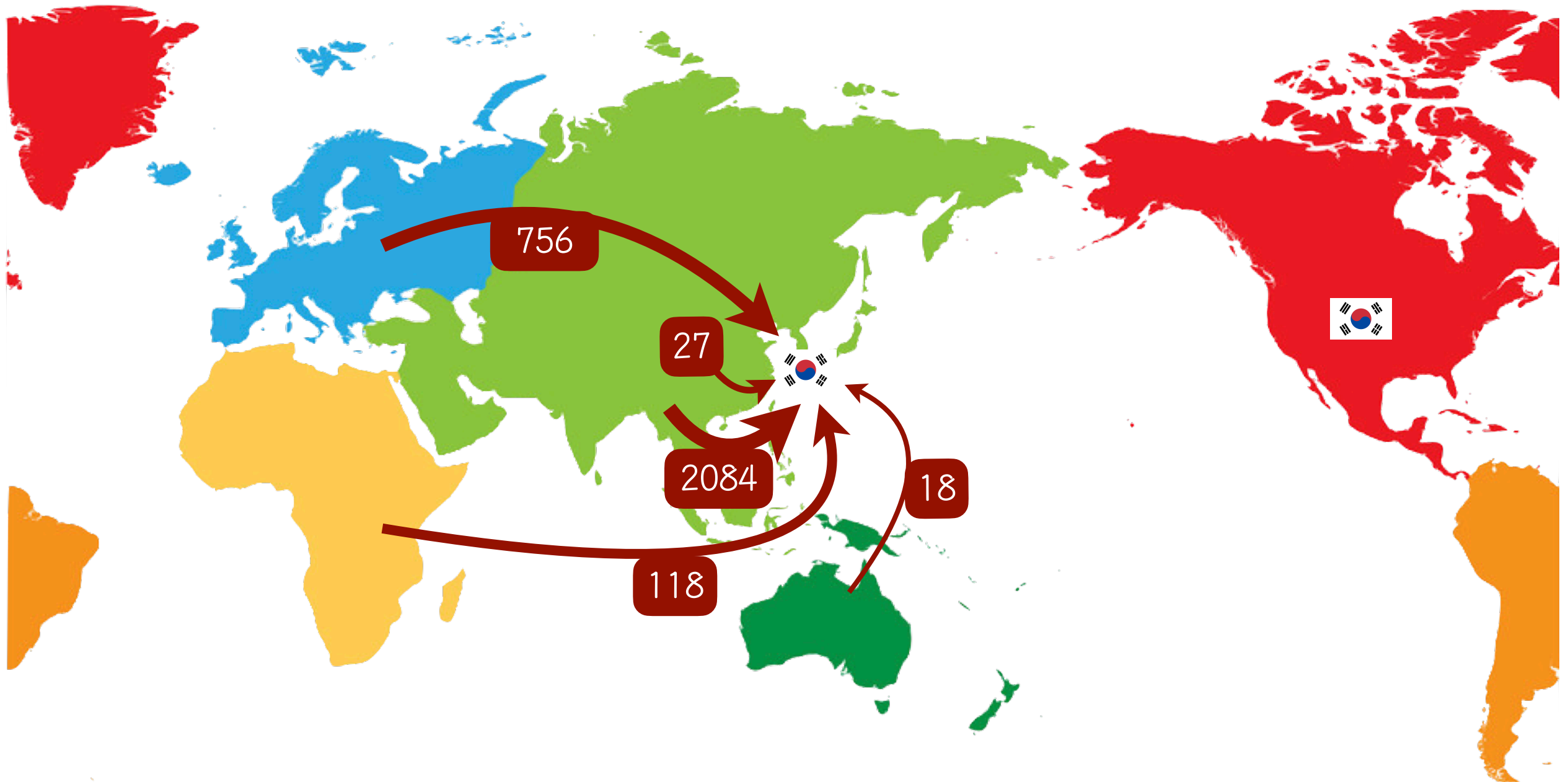
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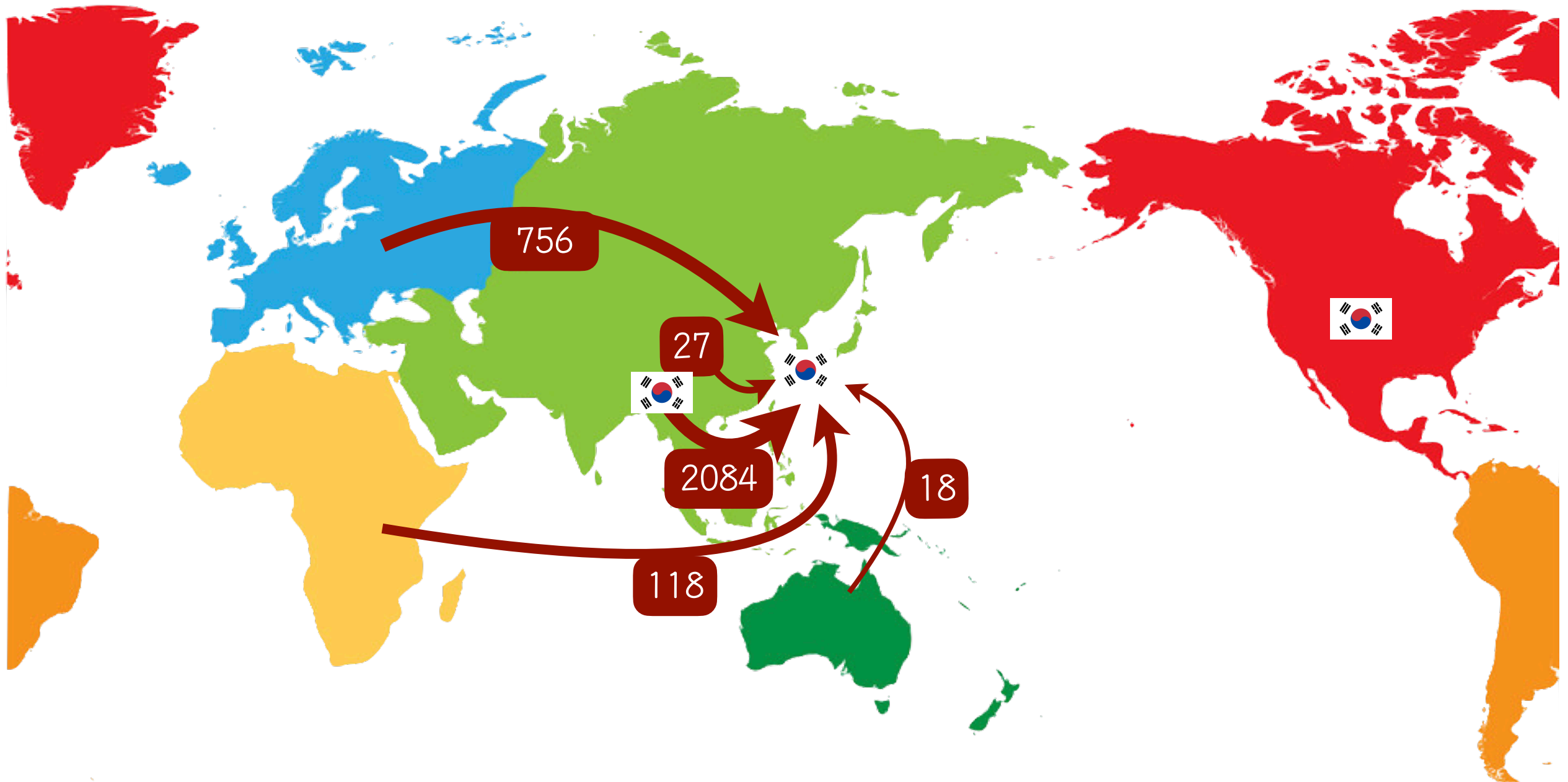
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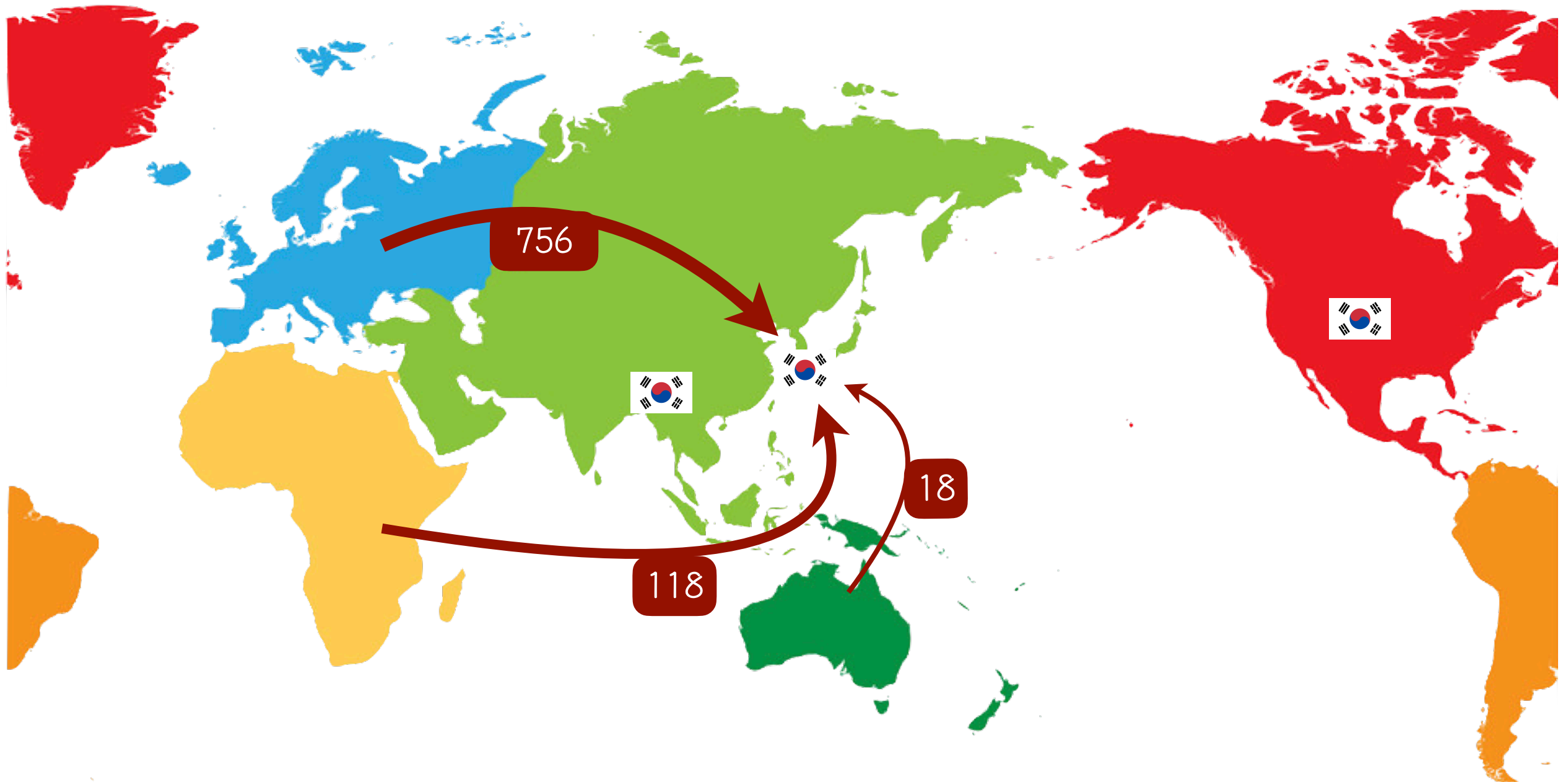
COVID-19: Imported Cases from Outside of Korea



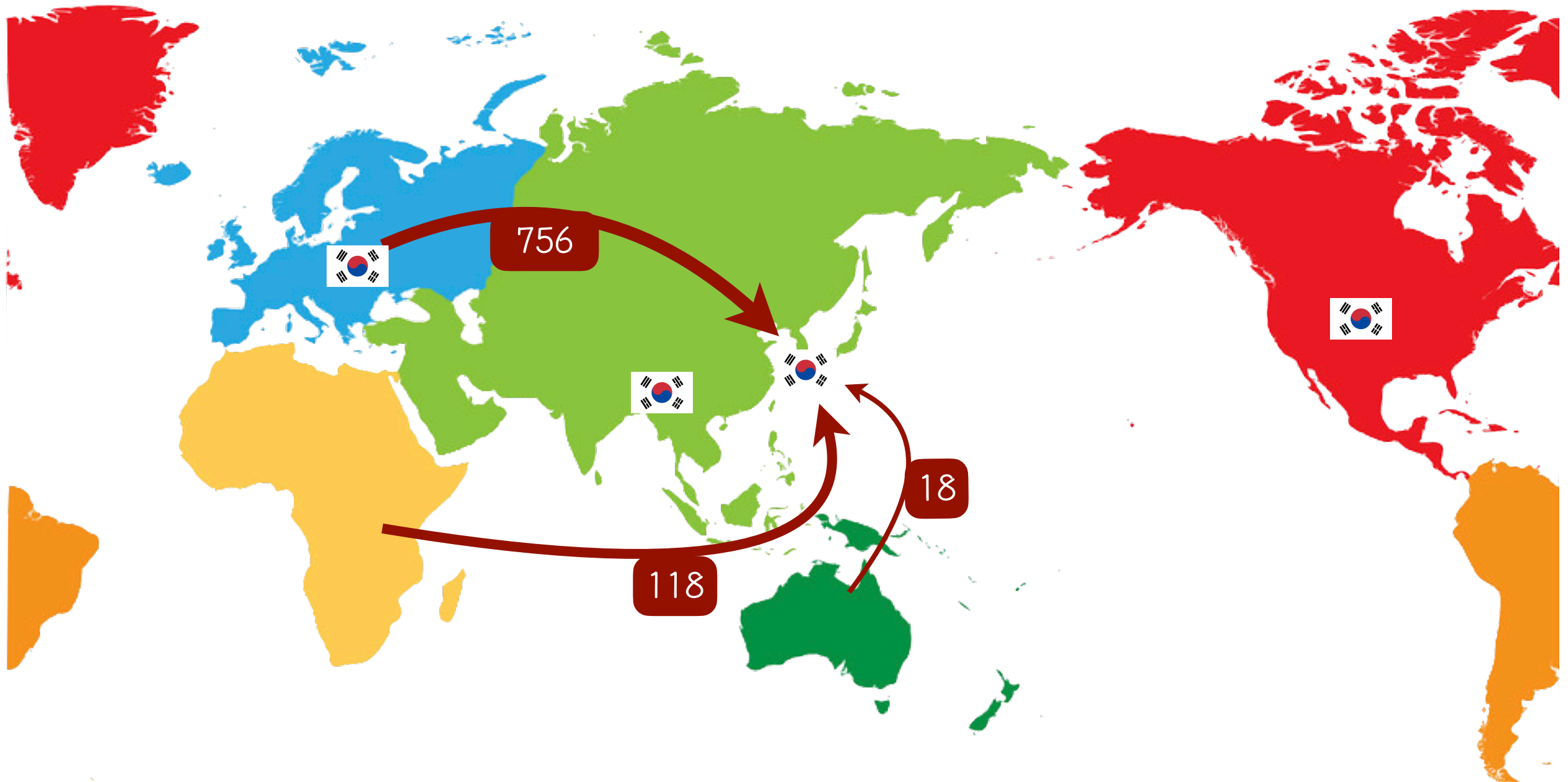
COVID-19: Imported Cases from Outside of Korea



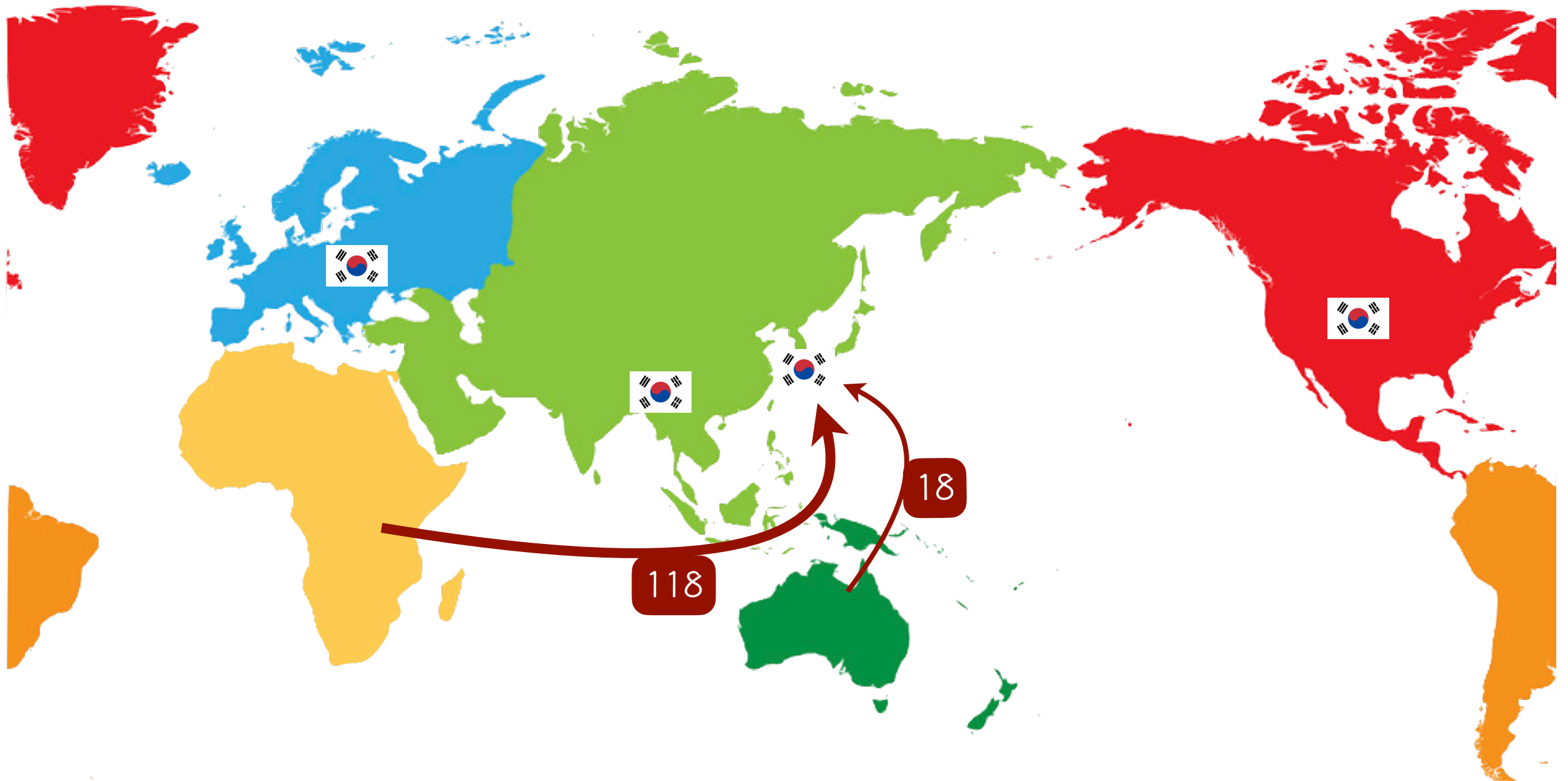
COVID-19: Imported Cases from Outside of Korea



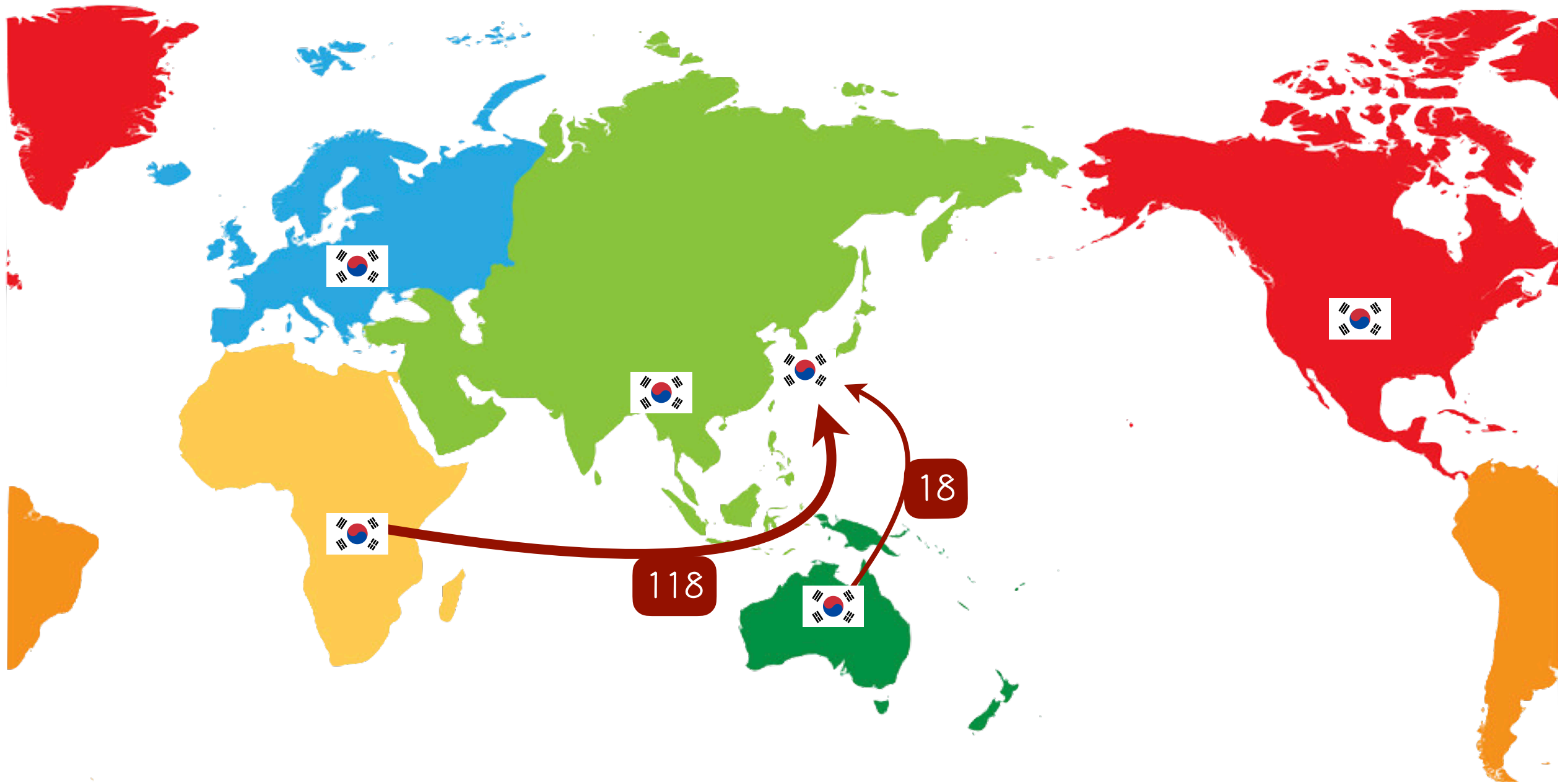
COVID-19: Imported Cases from Outside of Korea



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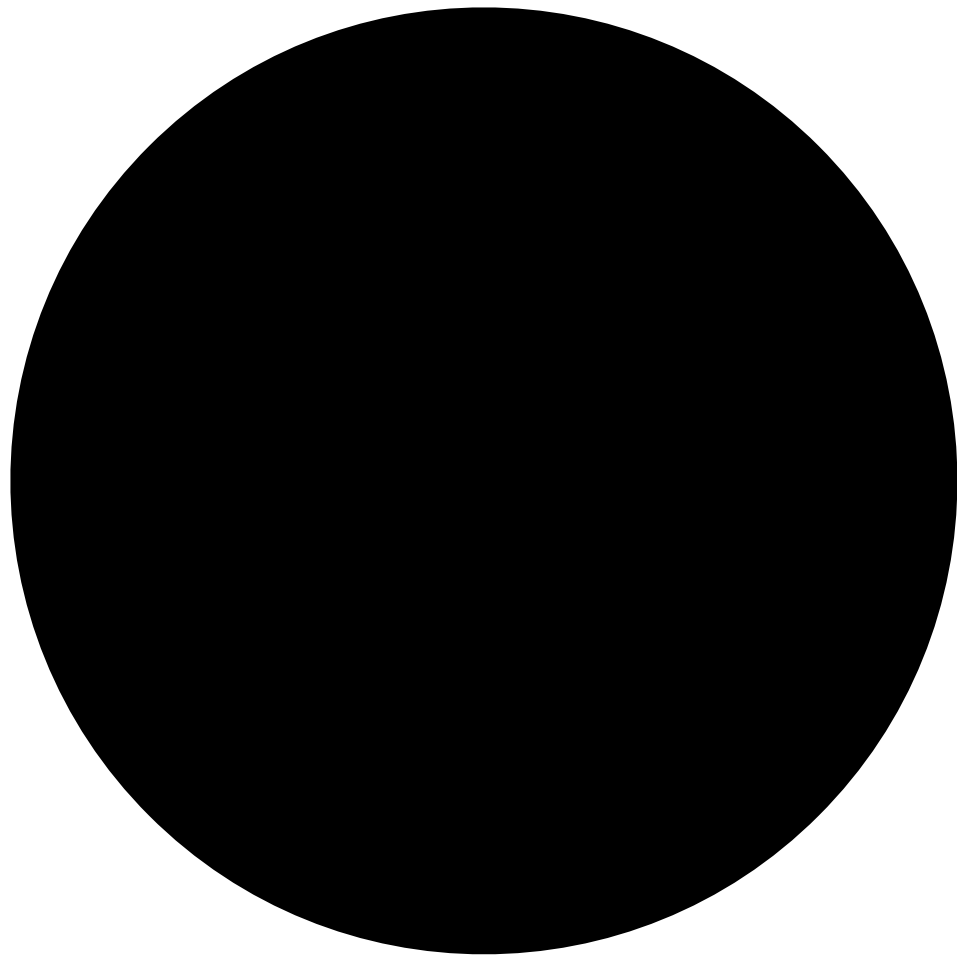


COVID-19: Imported Cases from Outside of Korea

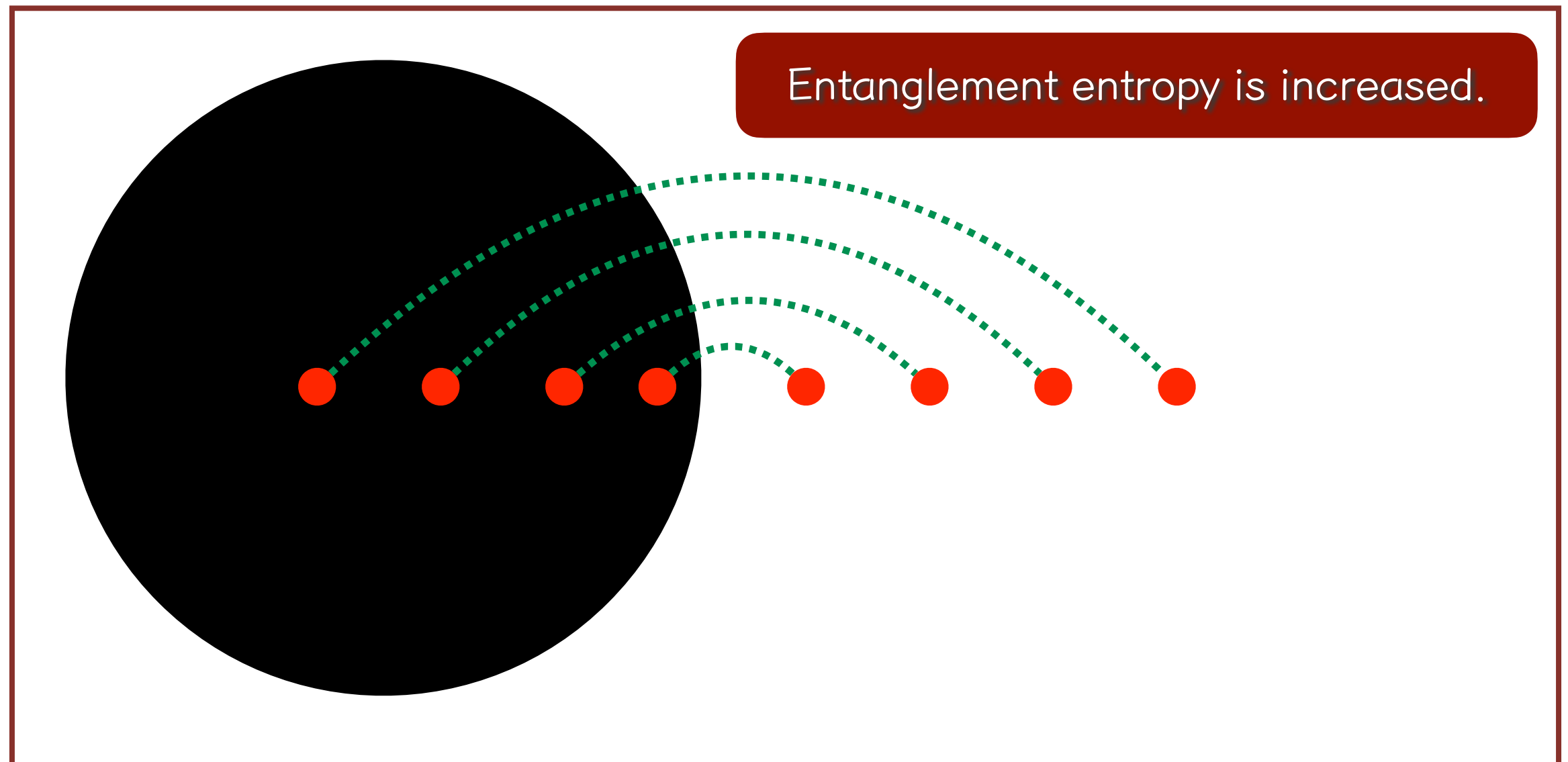


No Imported Case from Outside!

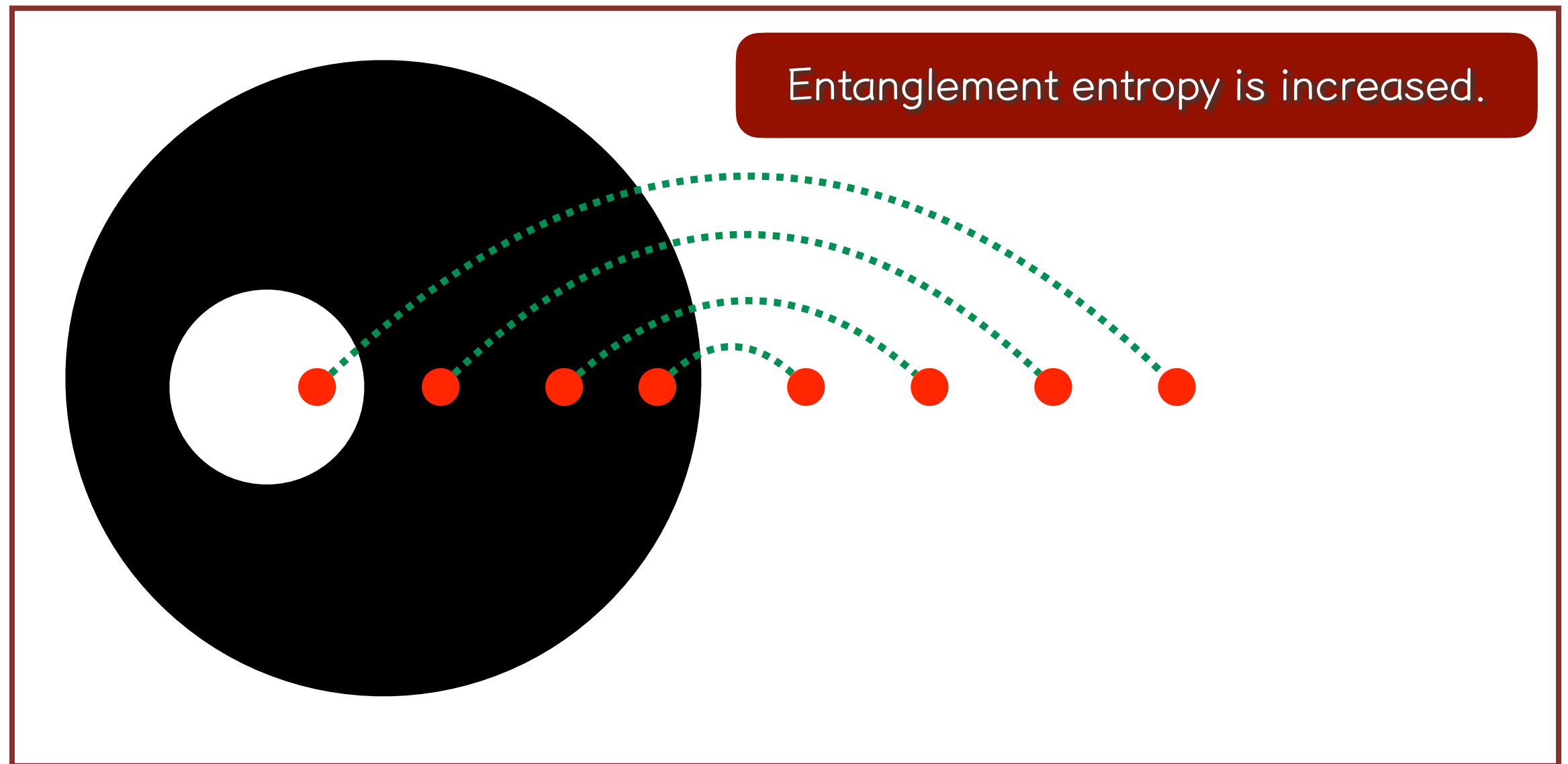
Island inside Black Hole



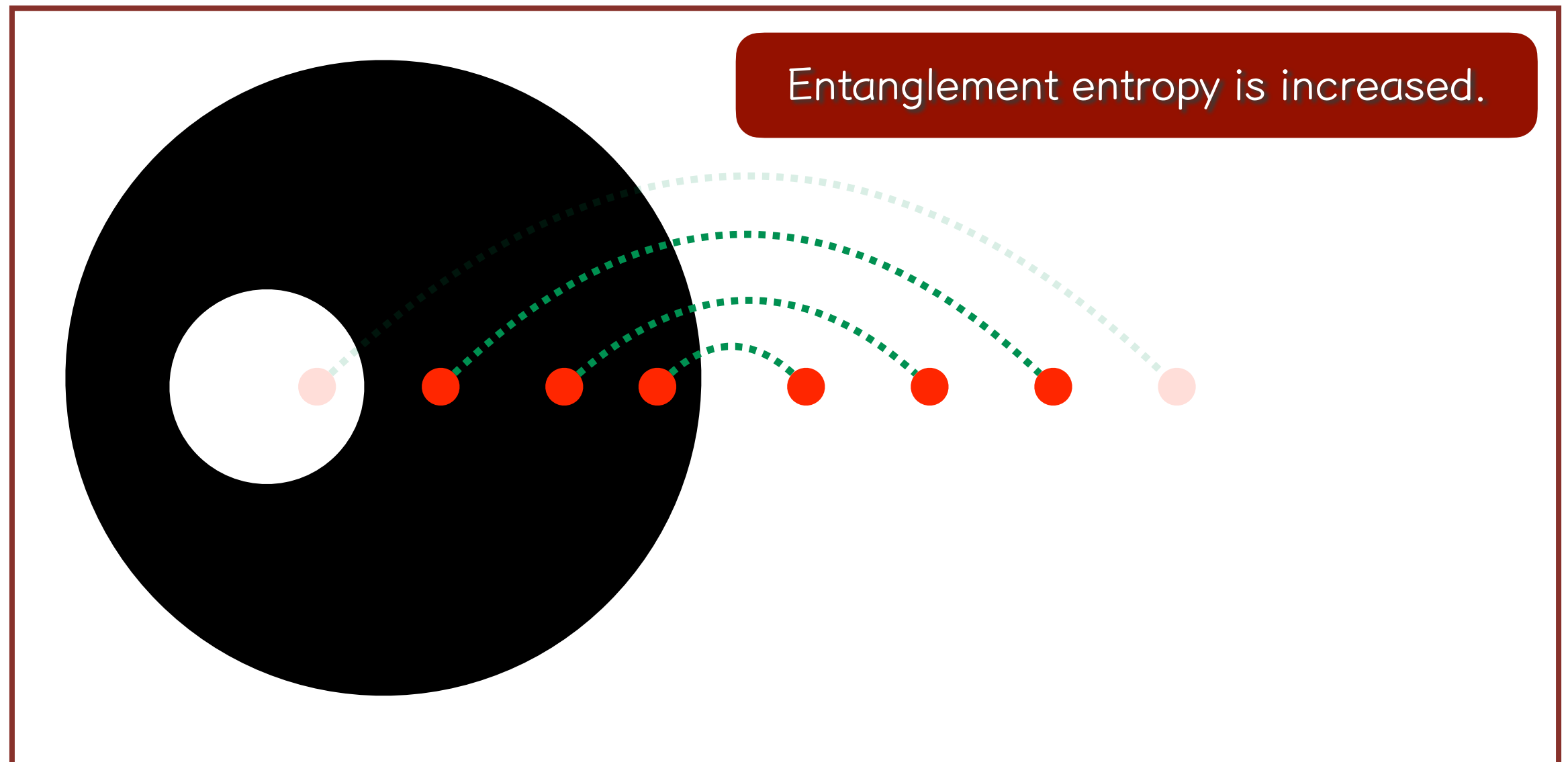
Island inside Black Hole



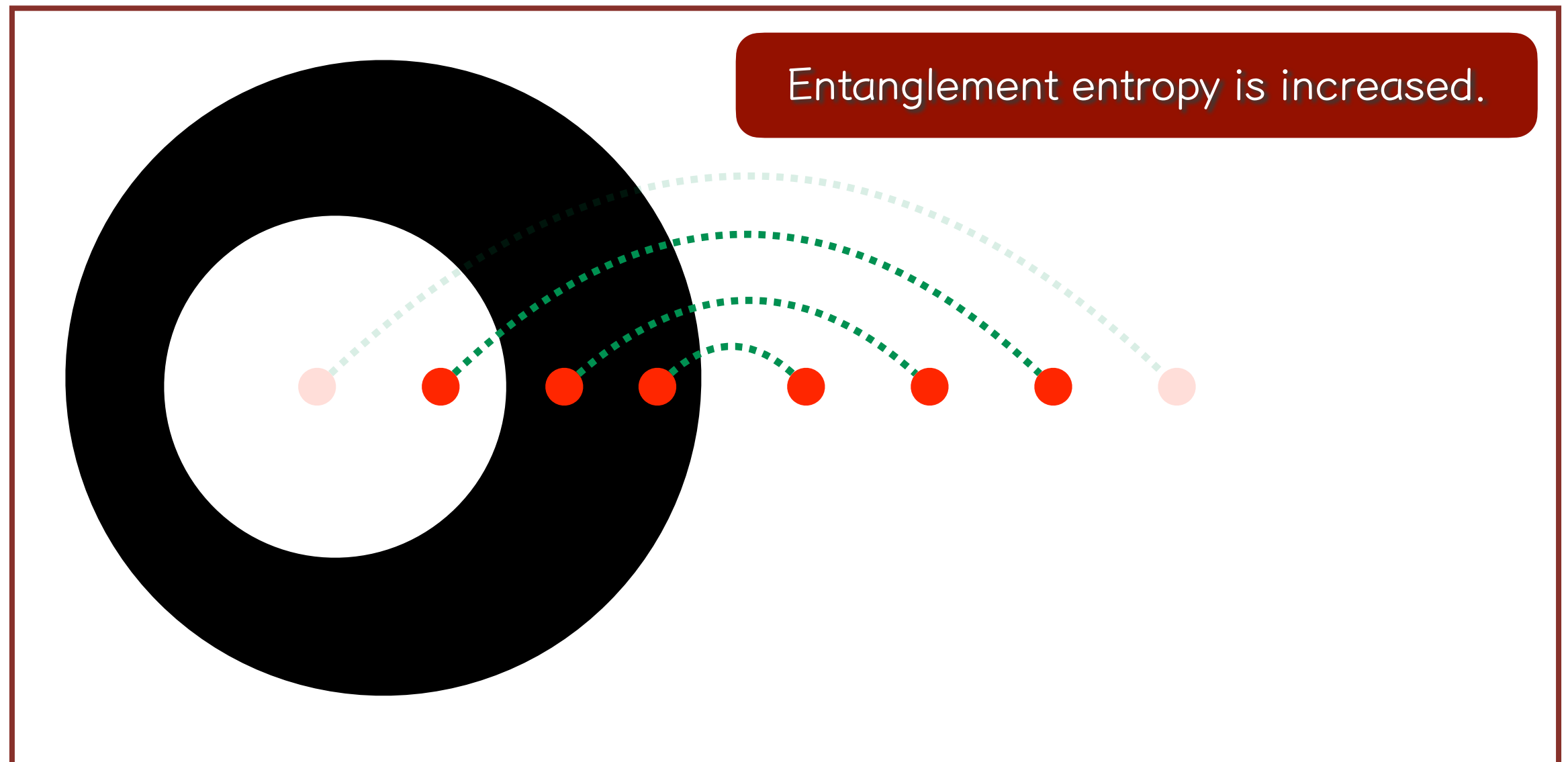
Island inside Black Hole



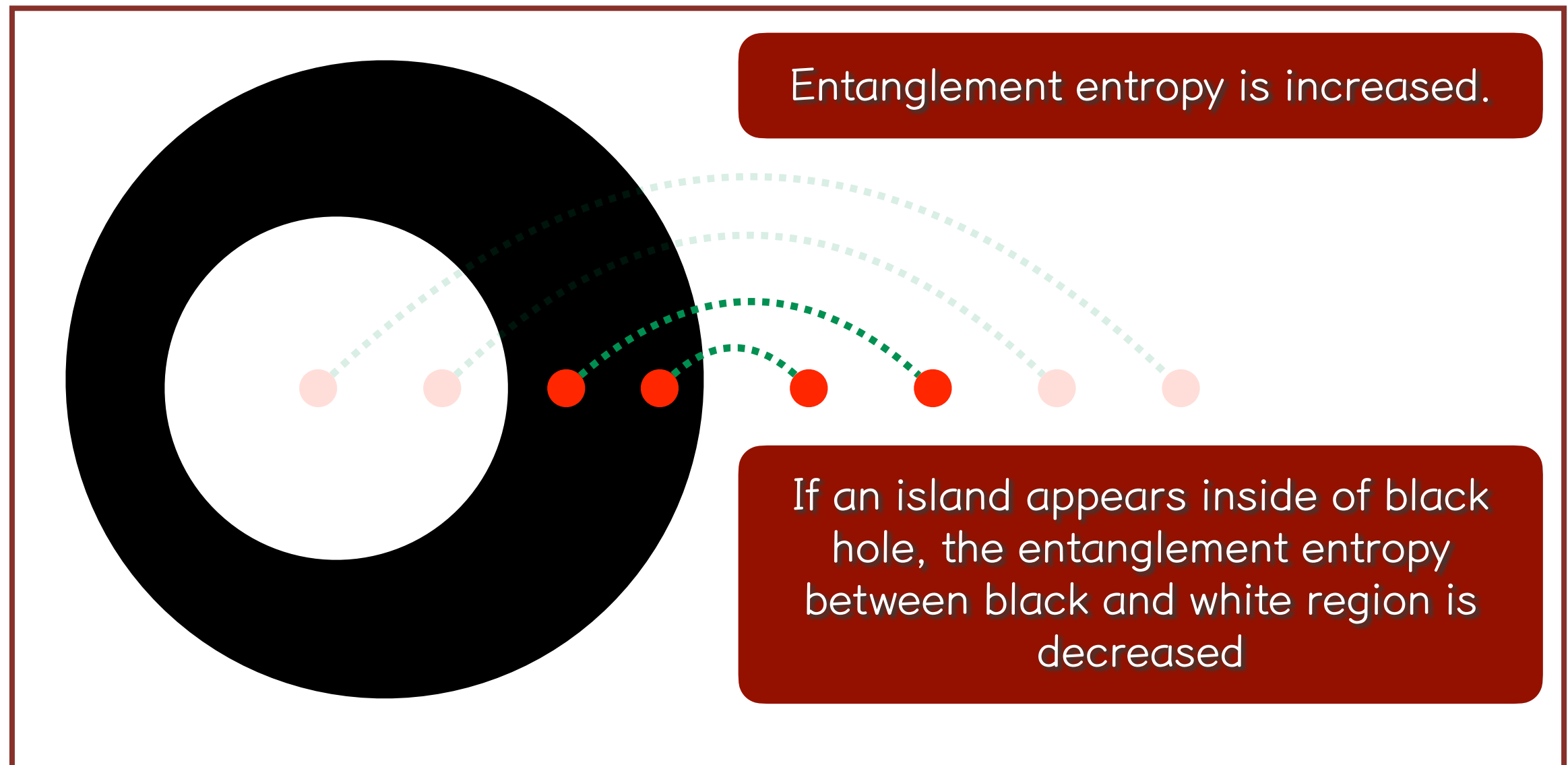
Island inside Black Hole



Island inside Black Hole



Island inside Black Hole



Nature of Island?

- * What is the nature of island?
 - ✓ Does it exist physically?
 - ✓ Is it nothing but ad hoc to explain the Page curve?
- * Unfortunately, I can't answer those questions.
 - ✓ In this talk, I will try to explain a possible scenario to have Island. But, **this does not prove anything**.
 - ✓ Then, **assuming** the existence of Island, I will present some calculations to reproduce the Page curve.

Contents

- I. Short Review: black hole information paradox
- II. Black hole Complementarity
- III. Quantum Extremal Surface
- IV. Island Conjecture
- V. Doubly-Holography: Bottom-up
- VI. Doubly-holography in 3D Janus Black Hole

Short Review: Black Hole Information Paradox

What is the state of black hole?

Is it pure or thermal (mixed)?

Black Hole Information Paradox

- * Black hole seems to be thermal object
- * Bekenstein-Hawking entropy of black hole:

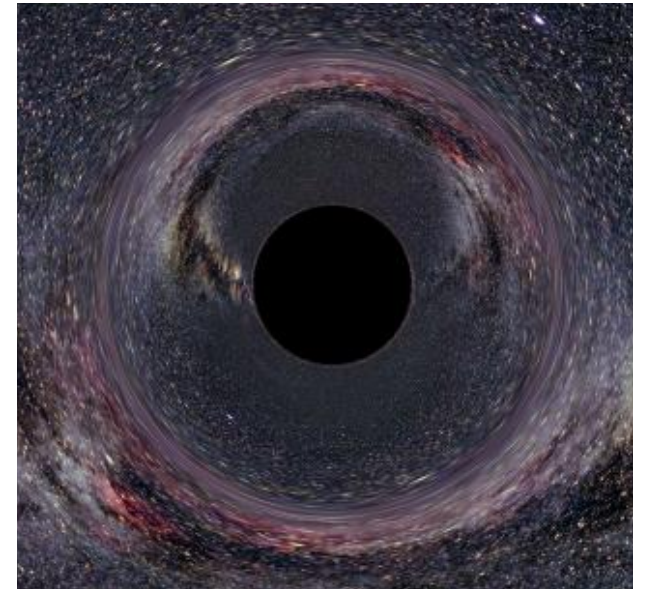
$$S = \frac{A}{4G} \quad A: \text{Area of horizon}$$

✓ 1st law of thermodynamics: $dM = \frac{\kappa}{2\pi} d\left(\frac{A}{4G}\right)$



$$dE = TdS$$

- * Is Black hole in mixed state?



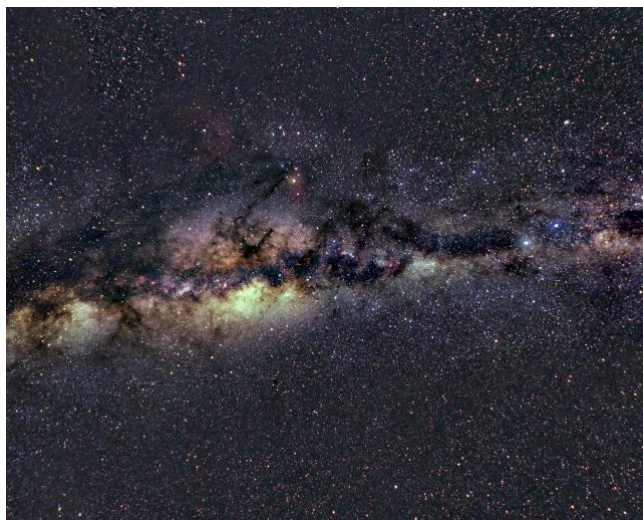
Black Hole Information Paradox

* Try Gedanken experiment for black hole formation and evaporation

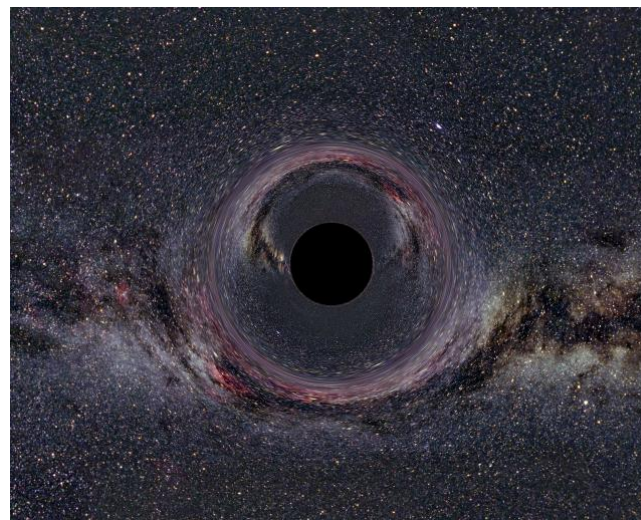
- Assumption

- ✓ Initial state (before black hole is formed): pure state

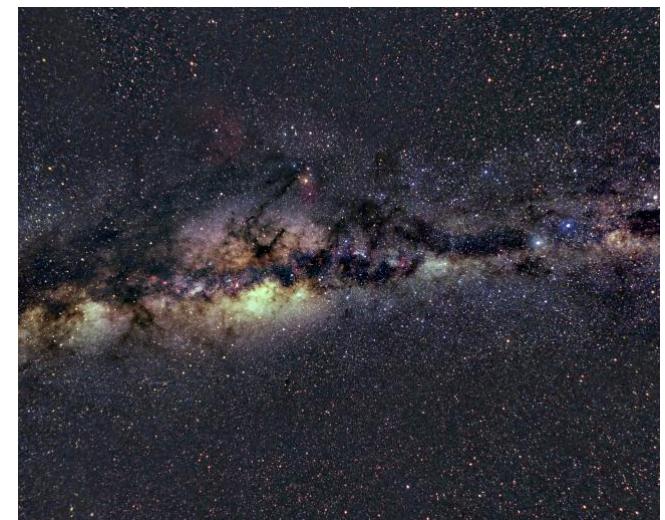
- ✓ Unitary evolution



Before formation
pure state



Black hole



evaporated

* The final state should be pure if the time evolution is unitary

Black Hole Information Paradox

- * When BH is in **pure** state, how does pure state look like thermal state?
- * The pure density matrix is very close to the thermal density matrix
 - ✓ Difficult to distinguish density matrix of pure state and thermal state

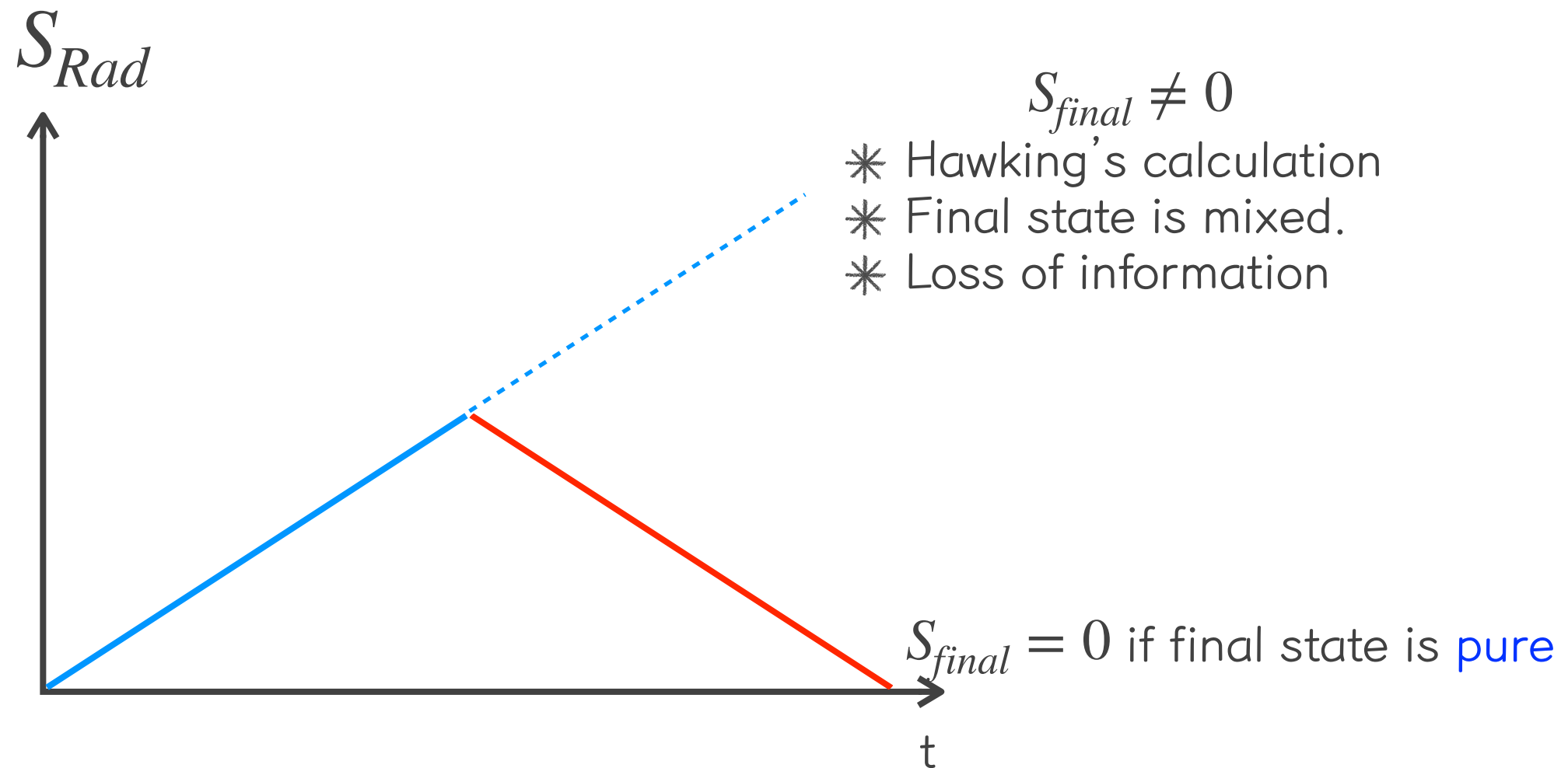
$$\rho_{\text{pure}} = \frac{1}{Z} e^{-\beta H} + e^{-S} \delta$$

exponentially suppressed
non-perturbative

- * At early time, semi-classical perturbative calculation is valid
 - ✓ e.g. Hawking's calculation
- * In a long time later, **one cannot ignore the difference** $e^{-S} \delta$
 - ✓ non-perturbative

Page Curve

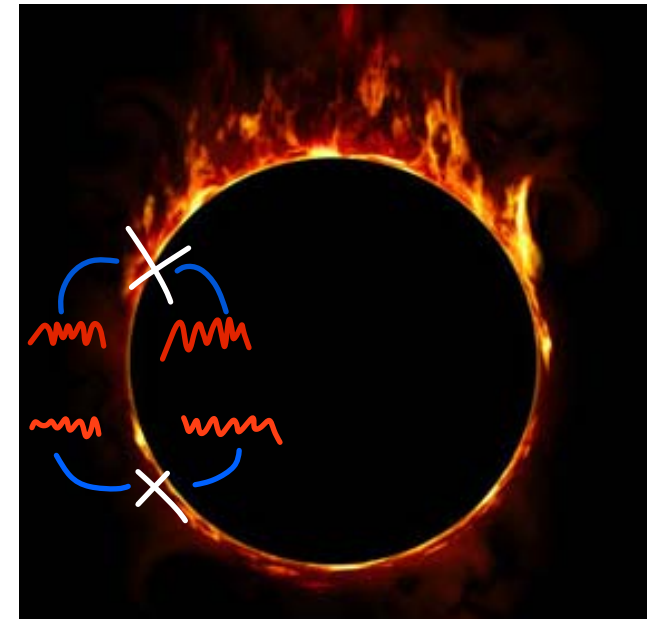
* Assumption for Page curve: **unitary**, closed quantum system



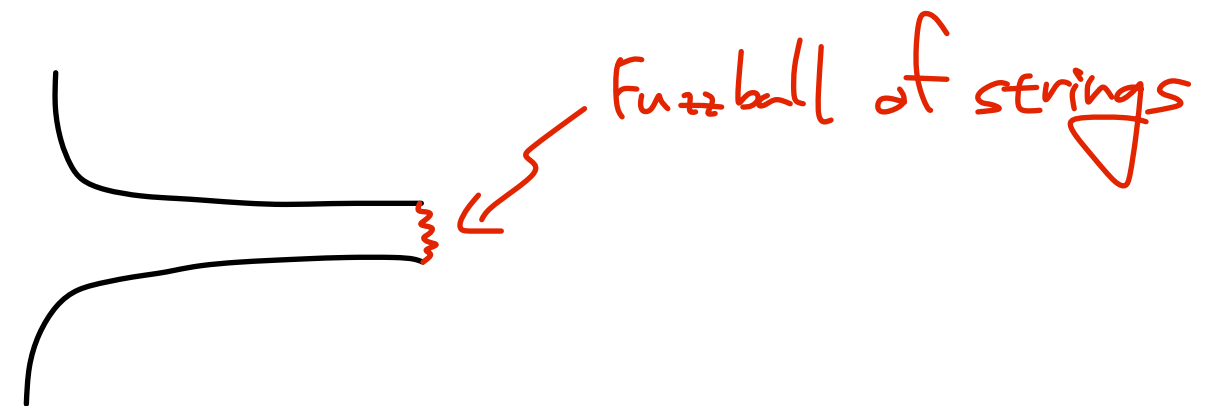
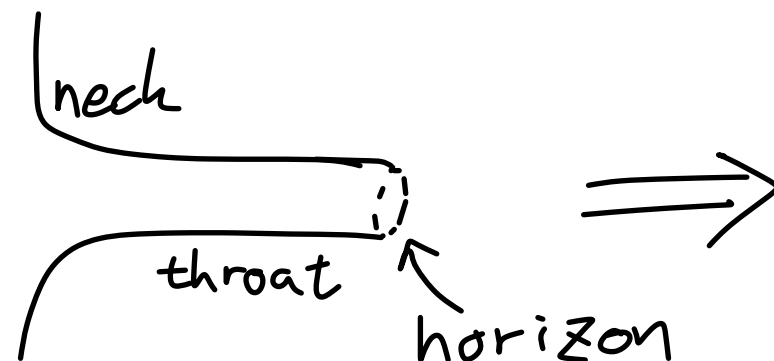
Proposed Resolutions

* Firewall at horizon [AMPS, 2012]

- ✓ proposed to resolve **strong subadditivity (SSA) paradox** of black hole
- ✓ **non-unitarity** (by firewall)
- ✓ Recall Wontae Kim's talk.



* Fuzzball [Mathur, Lunin, 2002]



- ✓ **No inside of black hole, no singularity**

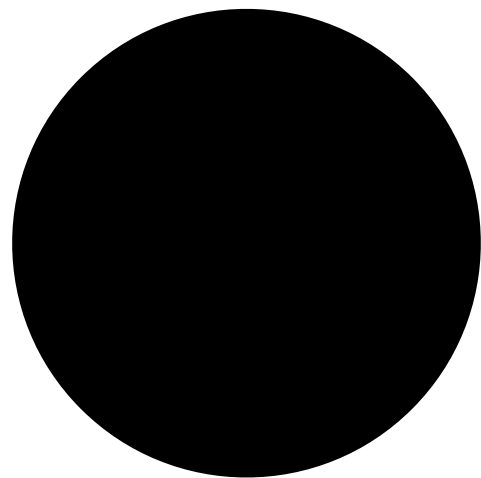
* Black hole complementarity (next slide)

Black Hole Complementary

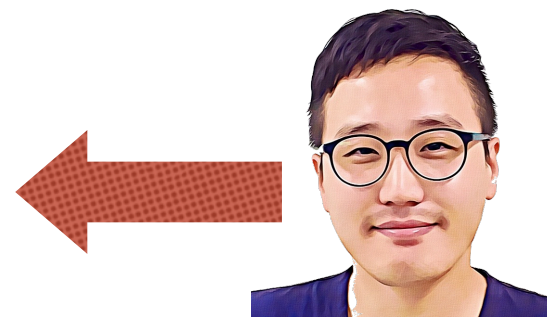
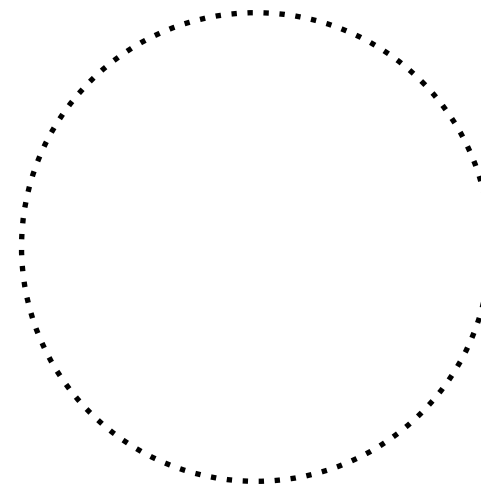
Original Black Hole Complementarity

* Black Hole Complementary [Susskind, Thorlacius, 1993]

- ▶ Two alternative descriptions of black hole
 - ✓ observer outside of horizon vs infalling observer



vs



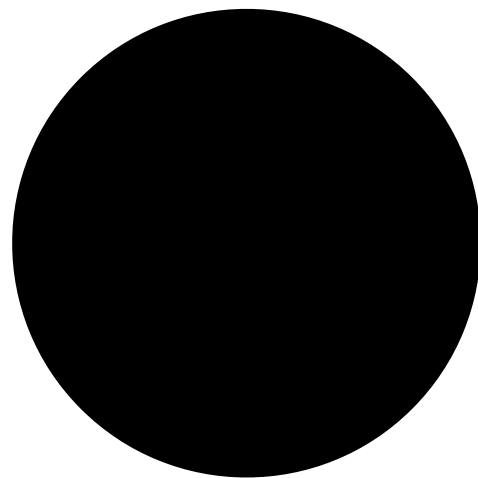
Stationary observer sees
“**membrane**” at stretched horizon

Infalling observer sees **nothing**,
and ends up with singularity

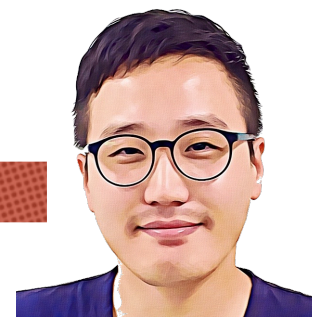
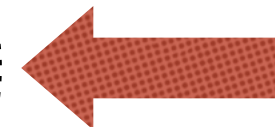
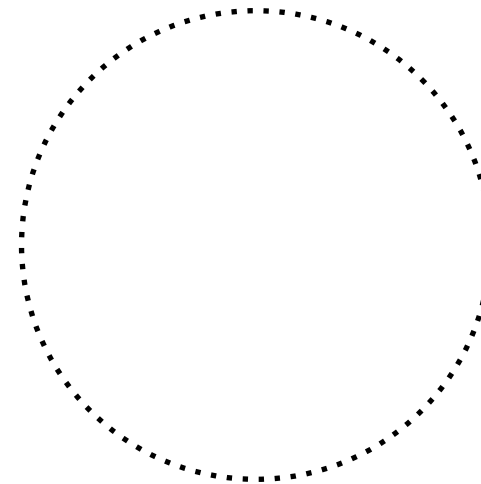
Original Black Hole Complementarity

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vs

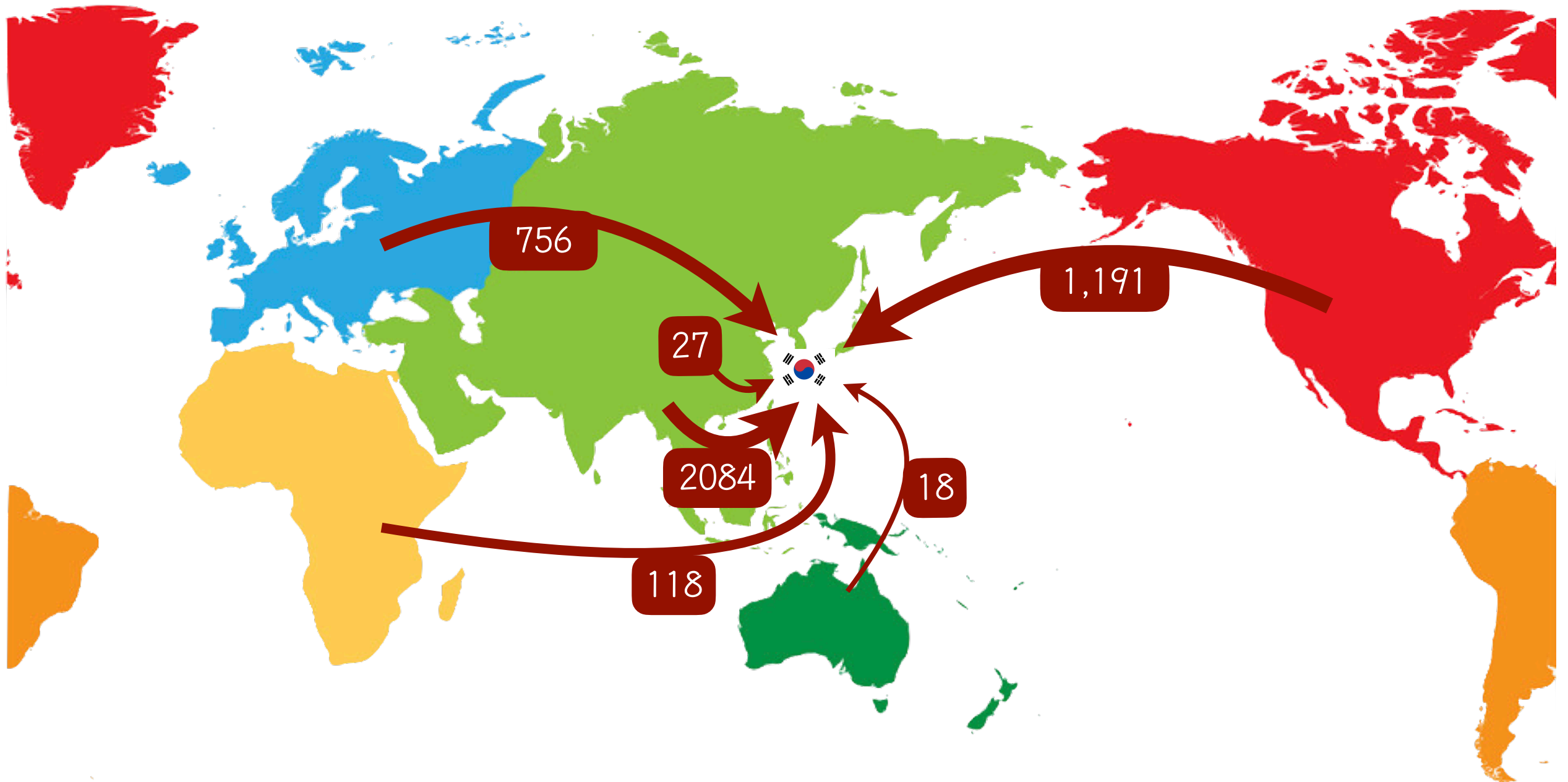


Stationary observer sees
“**membrane**” at stretched horizon

Infalling observer sees **nothing**,
and ends up with singularity

Two descriptions need not be equal.

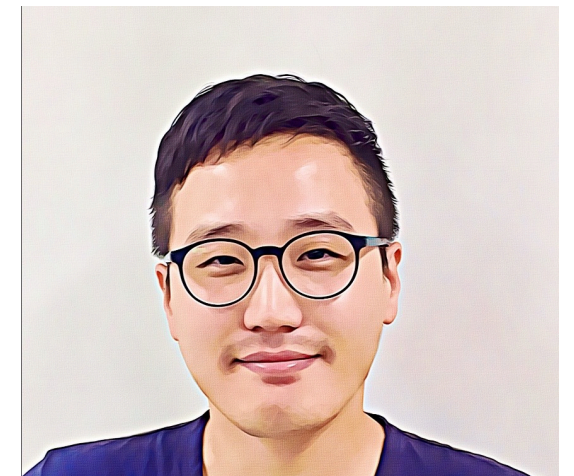
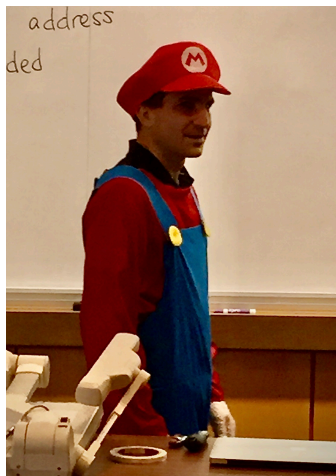
Recap: Imported Cases from Outside of Korea



We are the World



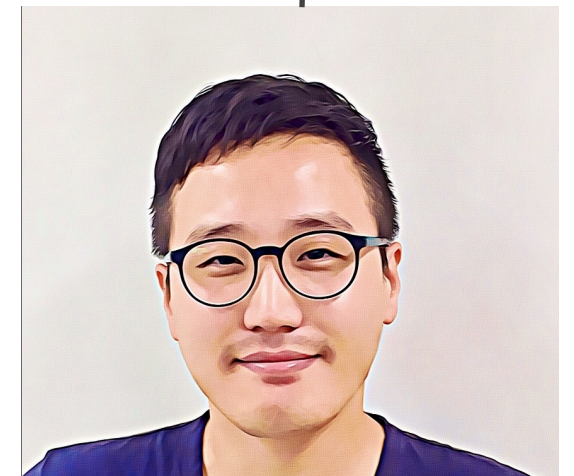
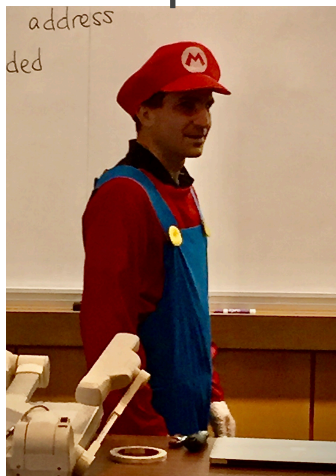
Australopithecus africanus



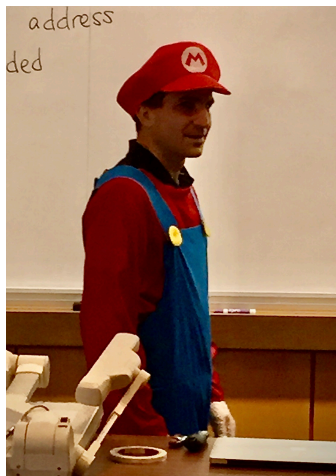
We are the World



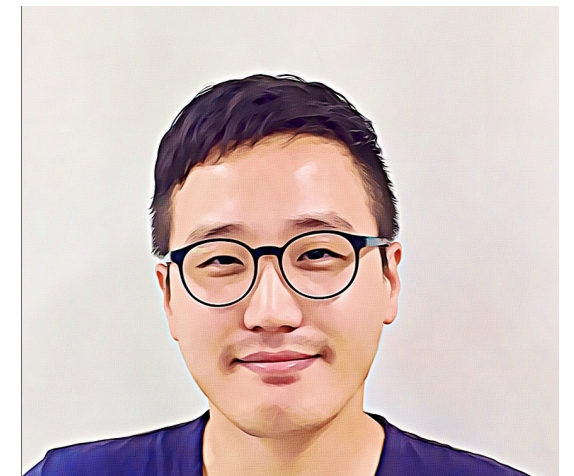
Australopithecus africanus



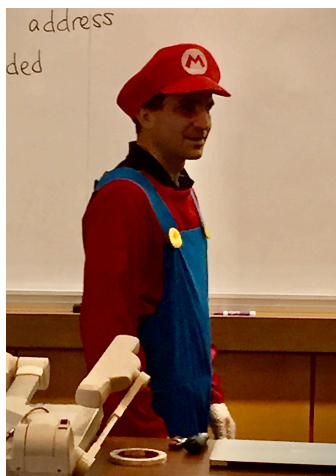
We are the World



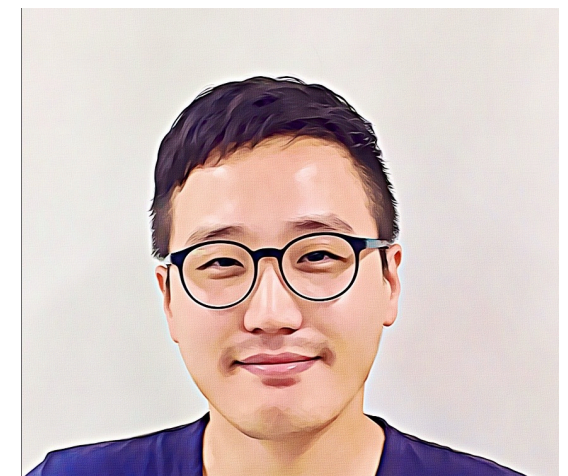
* It is difficult to track the family tree practically so that we are divided into many countries.



We are the World

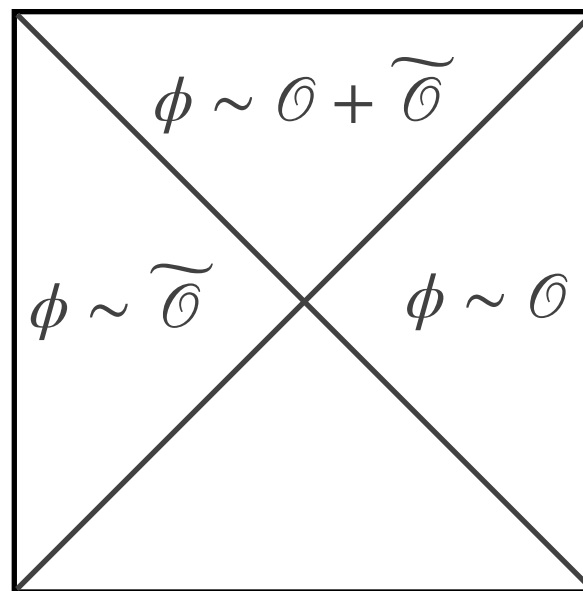


- * It is difficult to track the family tree practically so that we are divided into many countries.
- * This could be an analogy of the black hole complementary. i.e. Hilbert space of inside and outside of horizon



Factorization of Hilbert Space

- * Quantization of scalar field in the **eternal black hole** background.

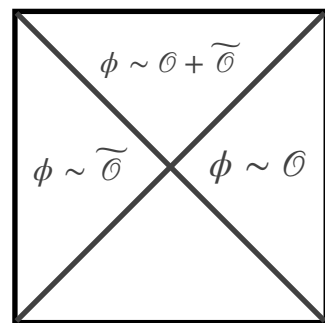


- ✓ B.C. on the horizon identifies d.o.f. across the horizon
- ✓ In AdS/CFT, $\mathcal{O}$ lives CFT_R and $\widetilde{\mathcal{O}}$ lives CFT_L .
- ✓ We usually think the Hilbert space is **factorized** into $H_L \times H_R$.
e.g. Thermofield dynamics (TFD) of CFT

Black Hole Complementarity

* d.o.f inside of horizon is **not independent** of d.o.f outside of horizon

✓ $\widetilde{\mathcal{O}}$ is a (complicated) function of $\mathcal{O}$'s.



$$\widetilde{\mathcal{O}} = f[\mathcal{O}]$$

✓ The degree of the function is very large. e.g. $\deg[f] = e^S$

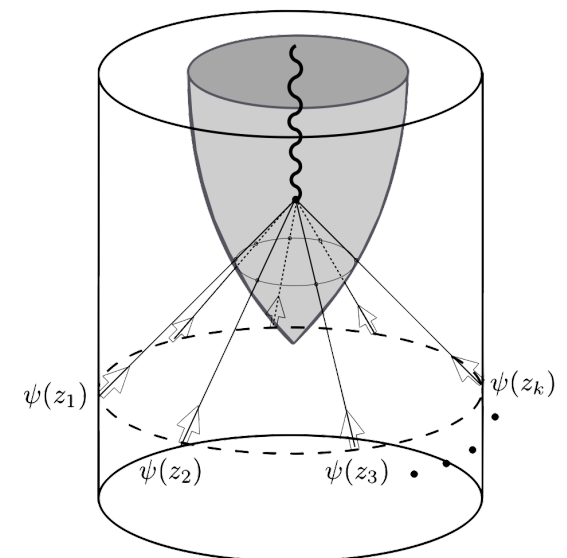
✓ This is **failure of the factorization of the Hilbert space**.

* **Tomita-Takesaki theory** in math provides underlying mechanism.

✓ For given $\{\mathcal{O}\}$, there exists an operator J such that $\widetilde{\mathcal{O}} = J\mathcal{O}J$

“Small Hilbert Space” for Effective Field Theory

- * For Physics, we need additional ingredient to Tomita–Takesatki theorem.
- * For effective field theory (EFT) on the curved background, we take a **small Hilbert space**.
 - ✓ For example, we don't consider 100 black holes moving in our spacetime.
 - ✓ We consider a small d.o.f which does not background too much for EFT.
 - ✓ e.g. **EFT will be broken down** when we calculate **a correlation functions of n operators ($n > e^S$)**
[Anous, Hartman, Rovai, Sonner, 1603.04856]
[Ghosh, Raju, 1706.07424]
 - ✓ cf) code space



Black Hole Complementary

[Raju, Papadodimas, 2012]

Full Hilbert Space

$\mathcal{O}, \mathcal{O}^2, \dots$

Simple operators describes
the outside of black hole

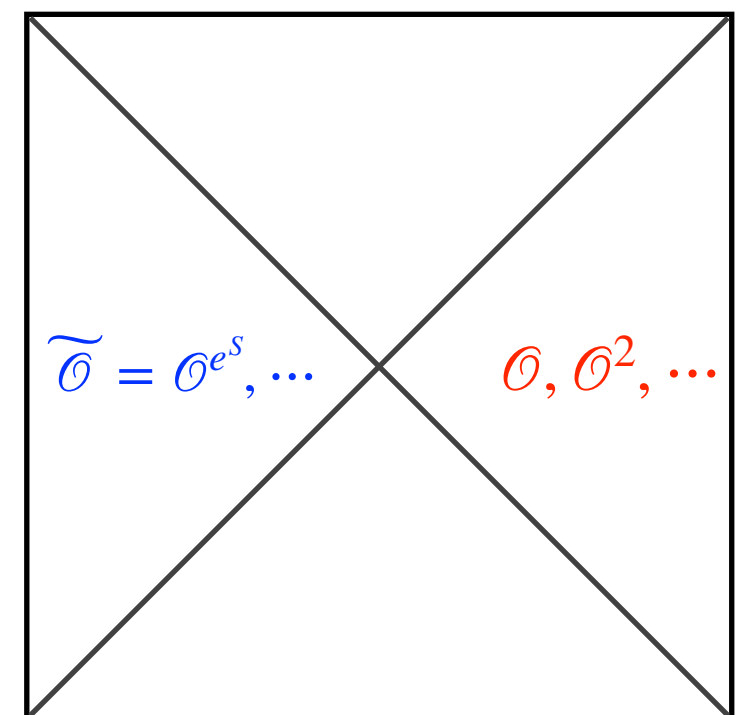
➡ EFT for outside of black hole

$\mathcal{O}^{e^s}, \dots$

Complicated operators describes
the inside of black hole

➡ corresponds to mirror operator $\tilde{\mathcal{O}}$

- * The full Hilbert space is **effectively factorized** into $\{\mathcal{O}\}$ and $\{\tilde{\mathcal{O}}\}$ within EFT.
- * This will hold until you measure large d.o.f.
- * The construction of the mirror operator $\tilde{\mathcal{O}}$ depends on the state: **State-dependence**
- * The bulk causality and locality is not exact.

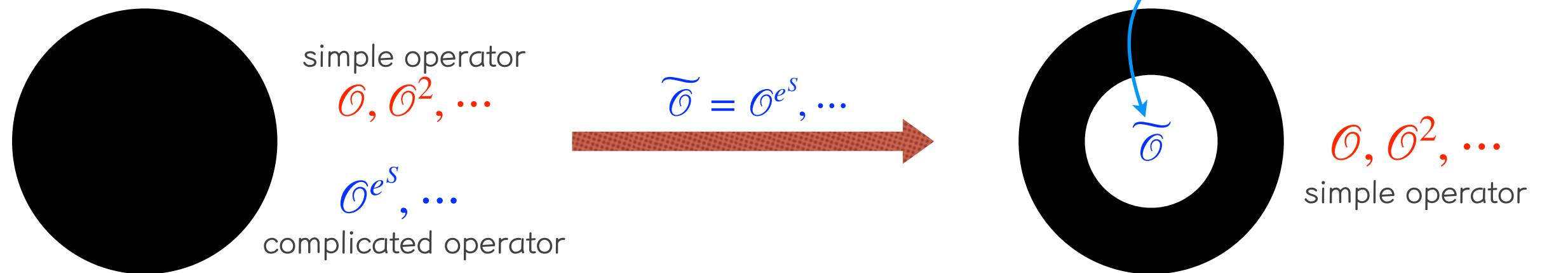


Island and Black Hole Complementary

- * After Page time, half of black hole d.o.f. comes out. For entanglement entropy between black hole and outside of Hawking radiations, it is inevitable to consider Hilbert space including complicated operators.

✓ EFT cannot usually describe this large Hilbert space.

- * We might be able to think complicated operators as “simple operators” living inside of horizon.

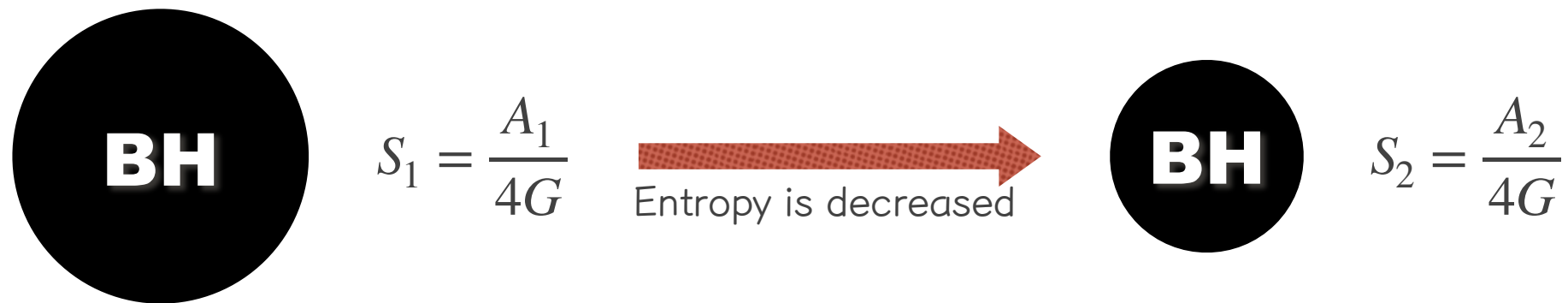


- * As I mentioned in the beginning, this is nothing but speculation.

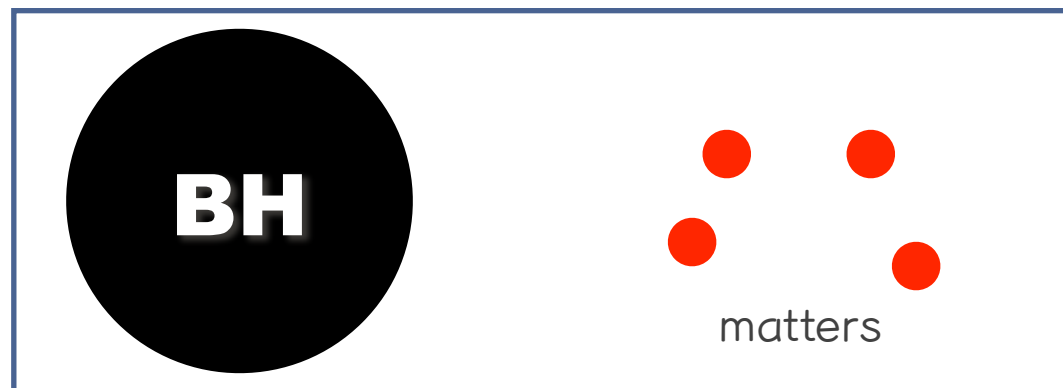
Quantum Extremal Surface

Generalized Entropy

- * Bekenstein-Hawking entropy: $s = \frac{A}{4G}$
- * Entropy of black hole is decreased as black hole evaporates.



- * Need to include **matter (radiation) contribution** for 2nd law of thermodynamics.



- ✓ Define Generalized Entropy: $s_{\text{gen}} = \frac{A}{4G} + S_{\text{outside}}$

Quantum Extremal Surface (QES)

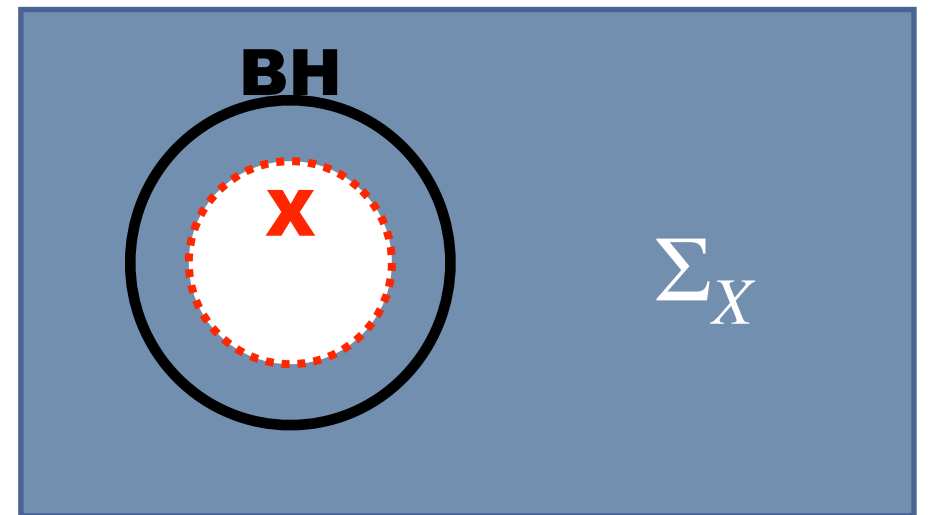
* Consider generalized entropy with (codim-2) surface X instead of horizon.

- ✓ Include matters in Σ_X

$$S_{\text{gen}}(X) = \frac{A(X)}{4G} + S_{\text{matter}}(\Sigma_X)$$

- ✓ Semi-classical calculation

- ❖ $A(X)$: Area from the fixed background
- ❖ $S_{\text{matter}}(\Sigma_X)$: Quantum calculation from ETF on the fixed background



* Quantum Extremal Surface: Extremize $S_{\text{gen}}(X)$

$$S_{\text{QES}} = \min_X \left[\text{Ext}_X S_{\text{gen}}(X) \right]$$

- ✓ In extremization, there could be several local extrema.

- ✓ Among local extrema, choose global minima

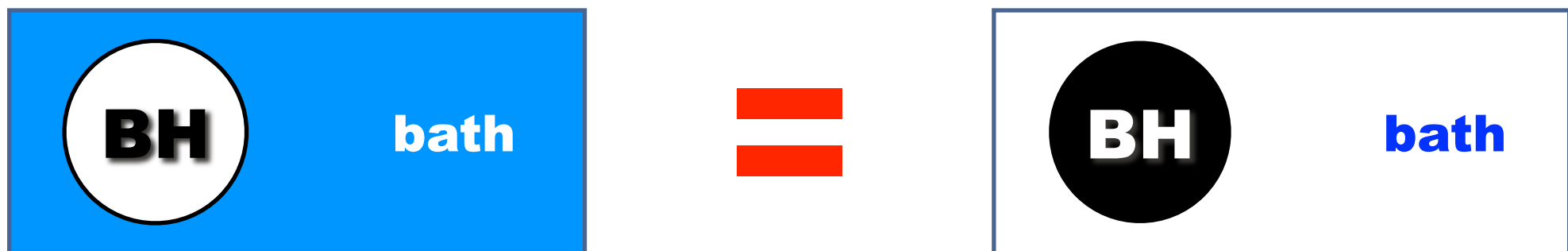
➡ Transition between minima leads to Page curve!

Calculation of QES

- * In higher dimension, the calculation of QES is difficult.

$$S_{\text{gen}}(X) = \frac{A(X)}{4G} + S_{\text{matter}}(\Sigma_X)$$

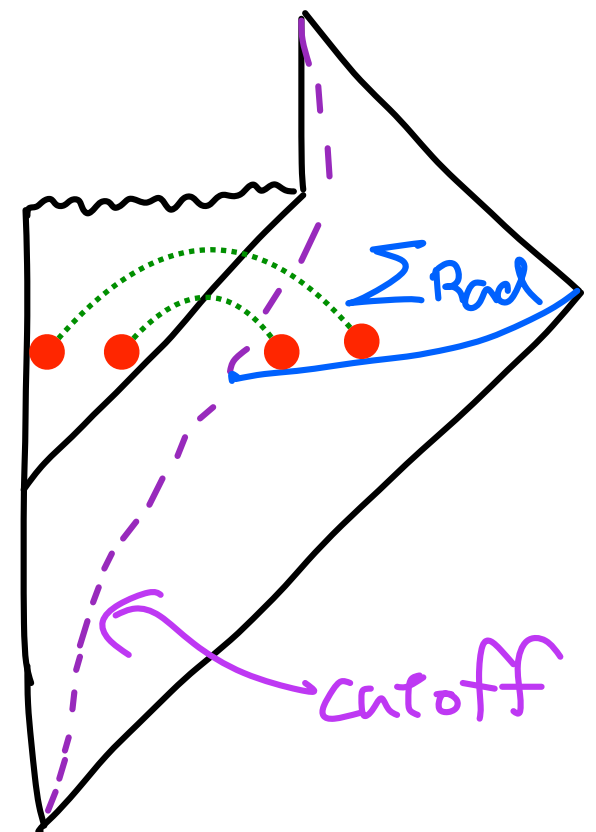
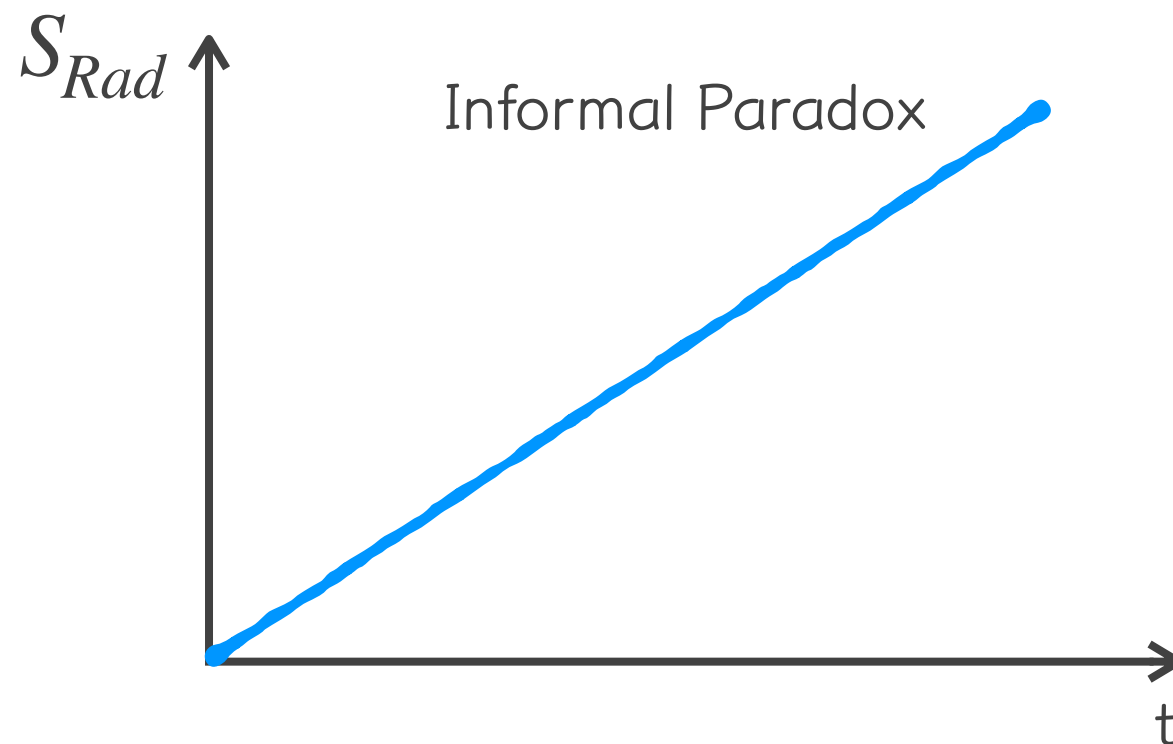
- ✓ The calculation of entanglement entropy $S_{\text{matter}}(\Sigma_X)$ is very difficult in general except for 2D.
 - ✓ The 2D conformal symmetry allows us to evaluate $S_{\text{matter}}(\Sigma_X)$.
 - ✓ You can learn the detailed calculations (for 2D case) in [APCTP winter school](#) from Jan 7 ~ Jan 16 2021 by Raghu Mahajan.
- * One may calculate the entanglement entropy between black hole and environments (bath) from the point of view of black hole or from the point of view of bath.



- ✓ e.g. $S_{EE}(A) = S_{EE}(B)$ if $A \cup B$ is pure state

Old Version: Information Loss

- * Let us consider time slice Σ_{Rad} outside of horizon.
- * We will count EPR pairs between black hole and Σ_{Rad} .
 - ✓ The number of EPR pairs is increased as time passes.
 - ✓ This is equivalent to Hawking's calculation.



Island Conjecture

based on [Almheiri, Hartman, Maldacena, Shaghoulian, Tajdini, 2006.06872]

- * New version: introduce an **island** inside of horizon

$$S_{\text{radiation}} = \min_X \left[\text{Ext}_X \left(\frac{A(X)}{4G} + S_{\text{semi-cl}}(\Sigma_X \cup \Sigma_{\text{rad}}) \right) \right]$$

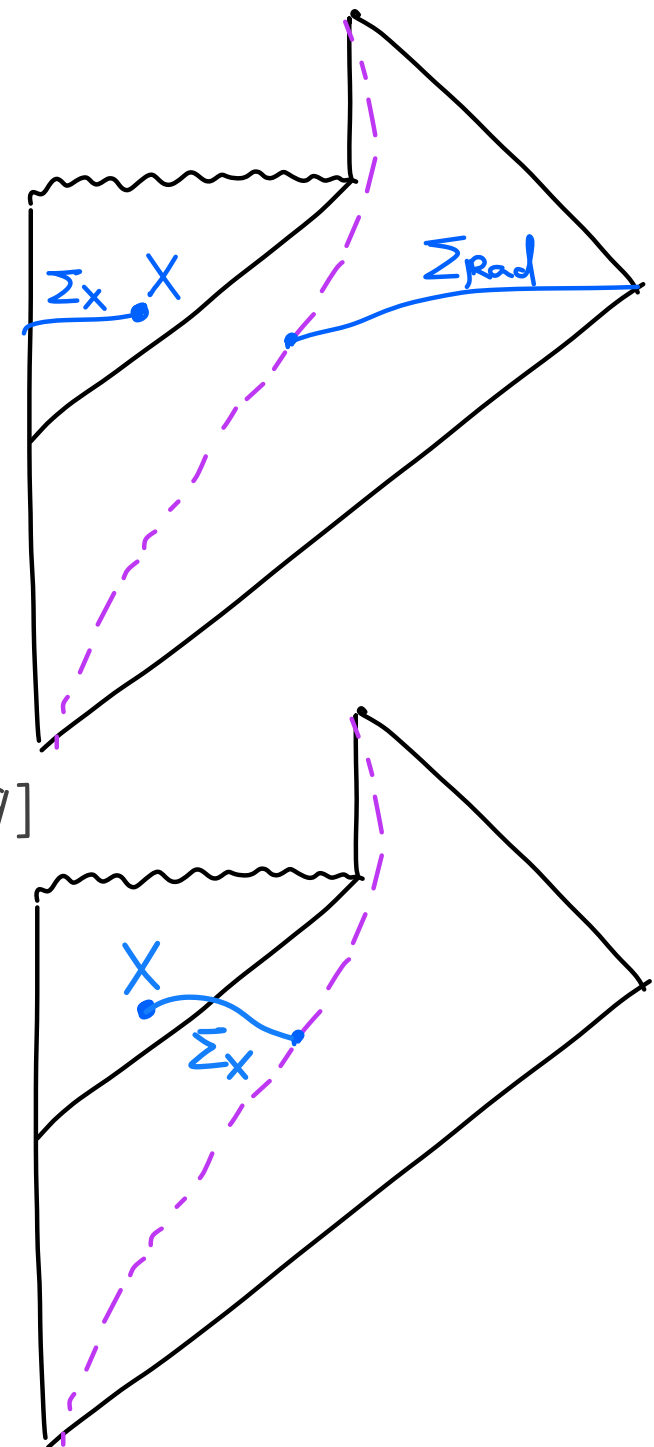
- ✓ Including Σ_X seems counter-intuitive.
cf) Recall imported cases of COVID-19.

- * The first paper [Almheiri, Engelhardt, Marolf, Maxfield, 2019] considered the complementary region

- ✓ EE from the point of view of black hole)

$$S_{BH} = \min_X \left[\text{Ext}_X \frac{A(X)}{4G} + S_{\text{semiclass}}(\Sigma_X) \right]$$

- ✓ Gives equivalent result.

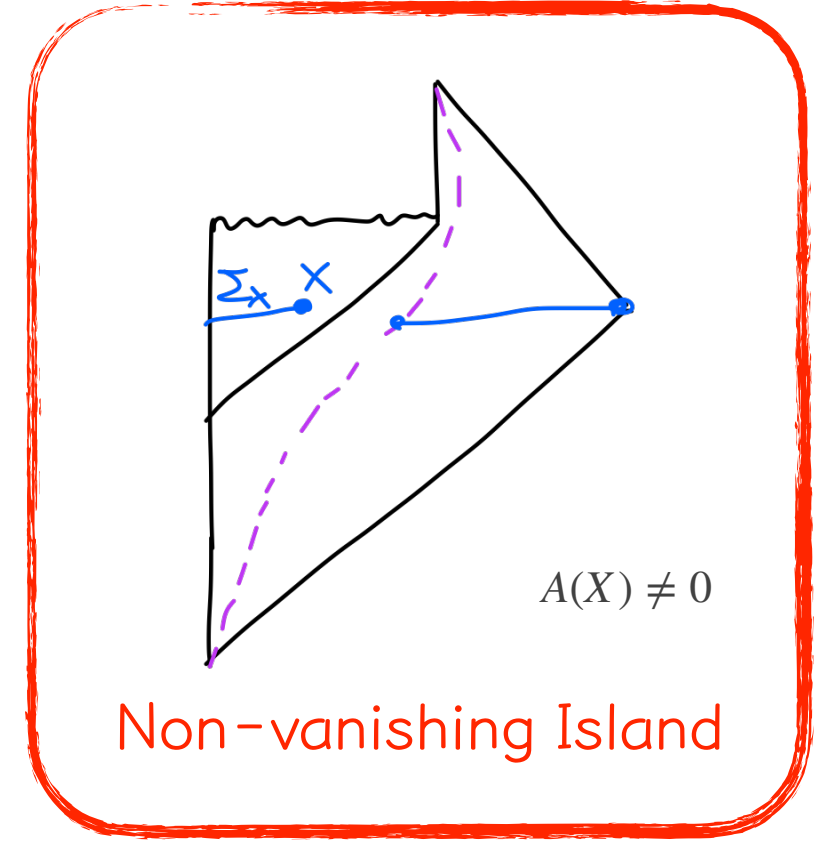
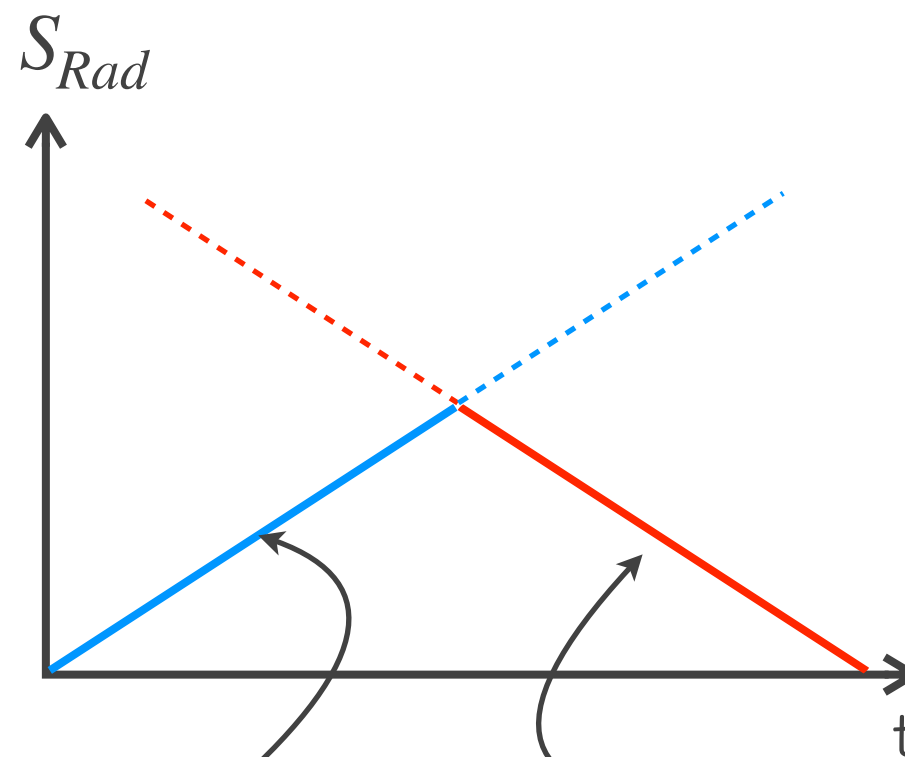
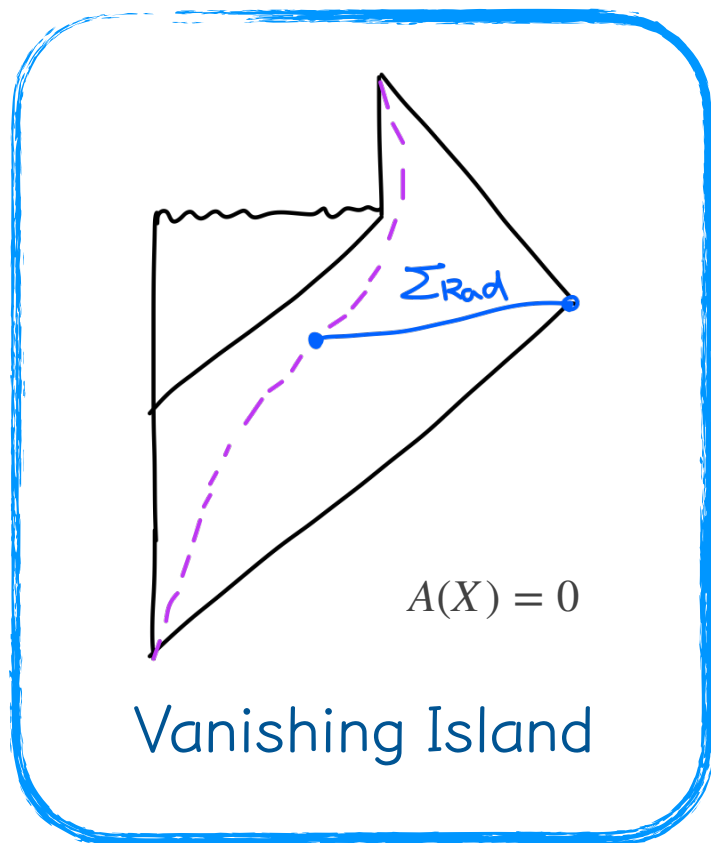


Two Local Extrema

based on [Almheiri, Hartman, Maldacena, Shaghoulian, Tajdini, 2006.06872]

✱ When extremizing the generalized entropy, one can find

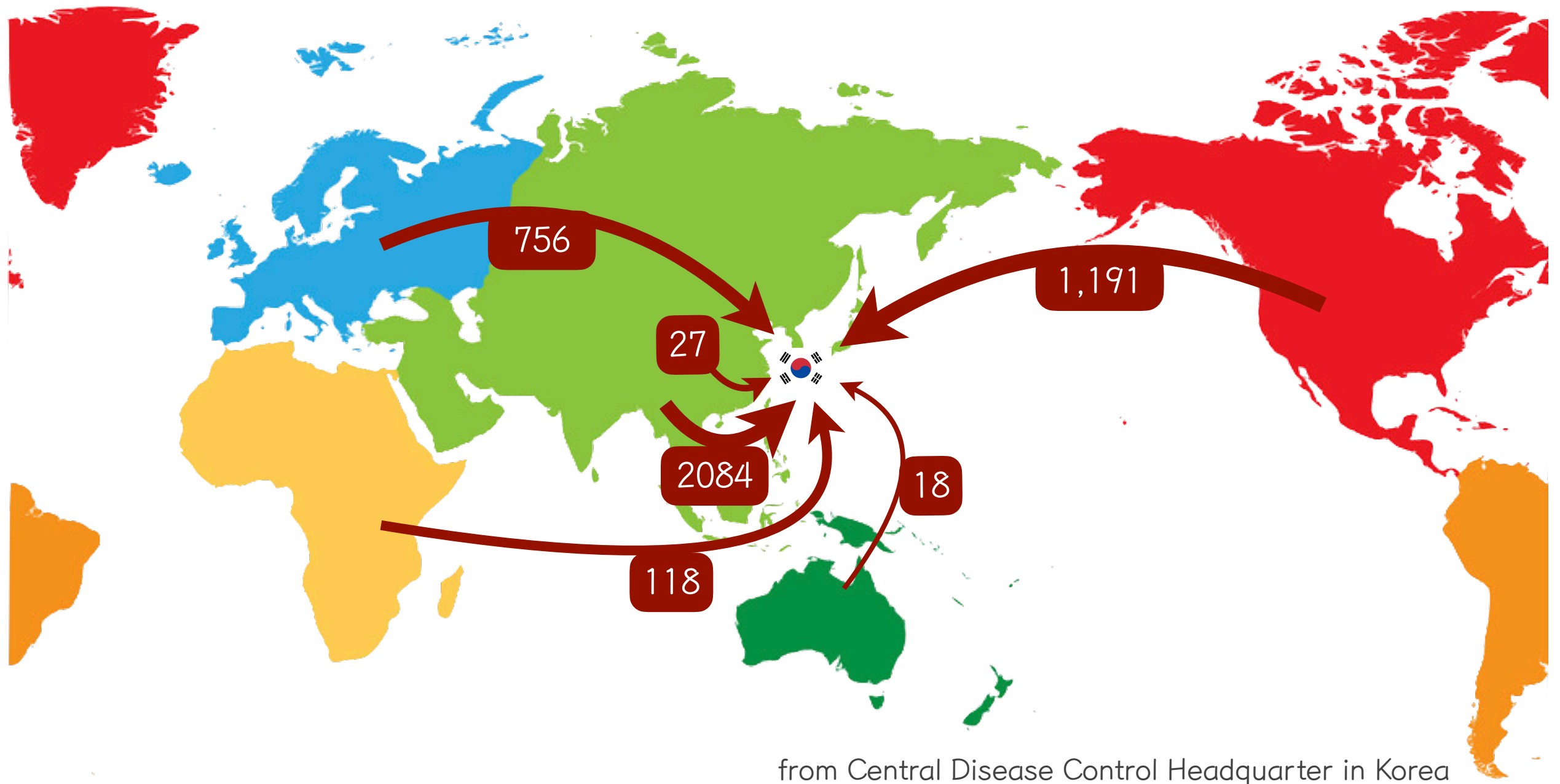
$$S_{\text{radiation}} = \min_X \left[\text{Ext}_X \left(\frac{A(X)}{4G} + S_{\text{semi-cl}}(\Sigma_X \cup \Sigma_{\text{rad}}) \right) \right]$$



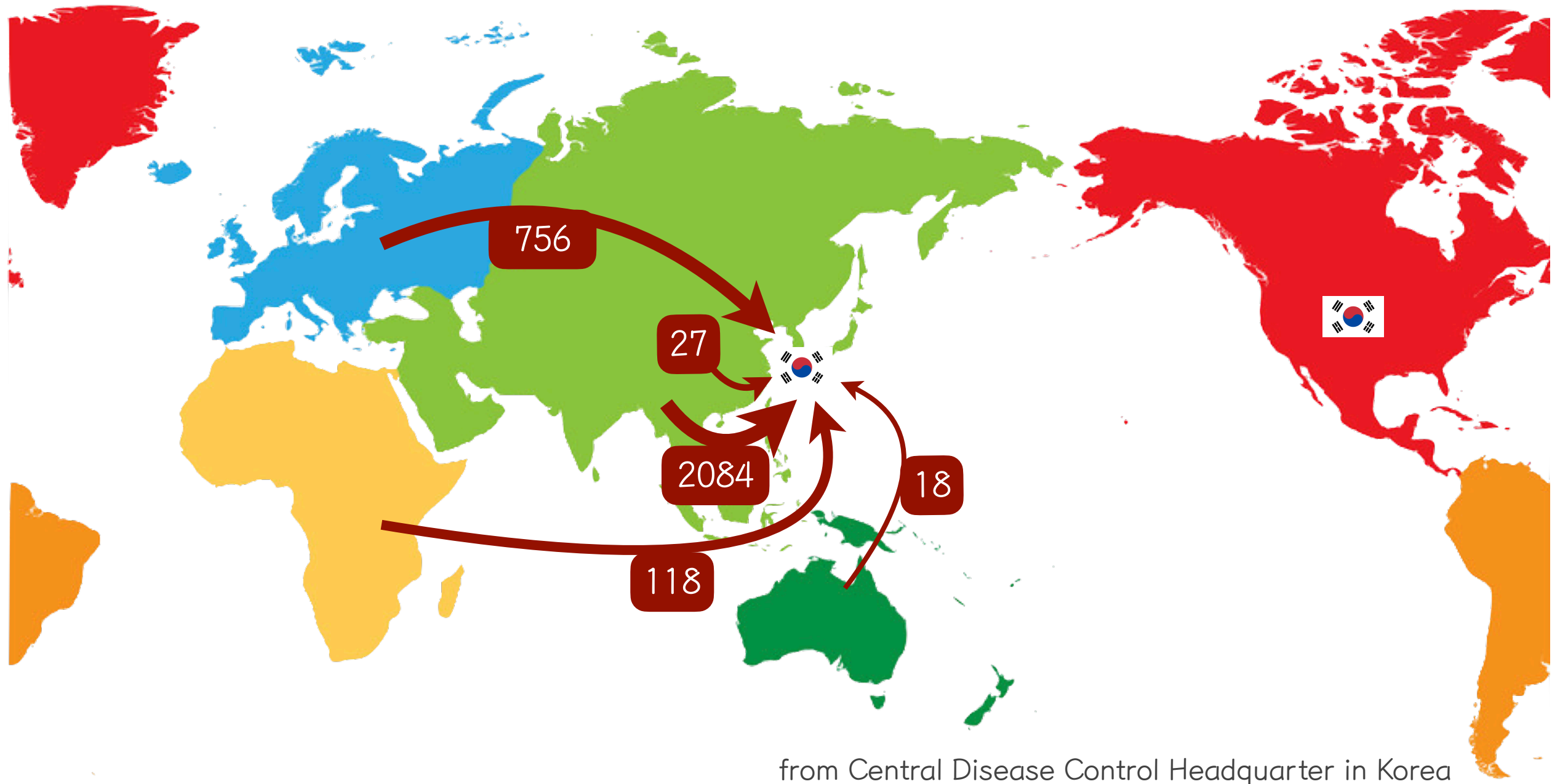
Extremum with vanishing Island

Extremum with Non-vanishing Island

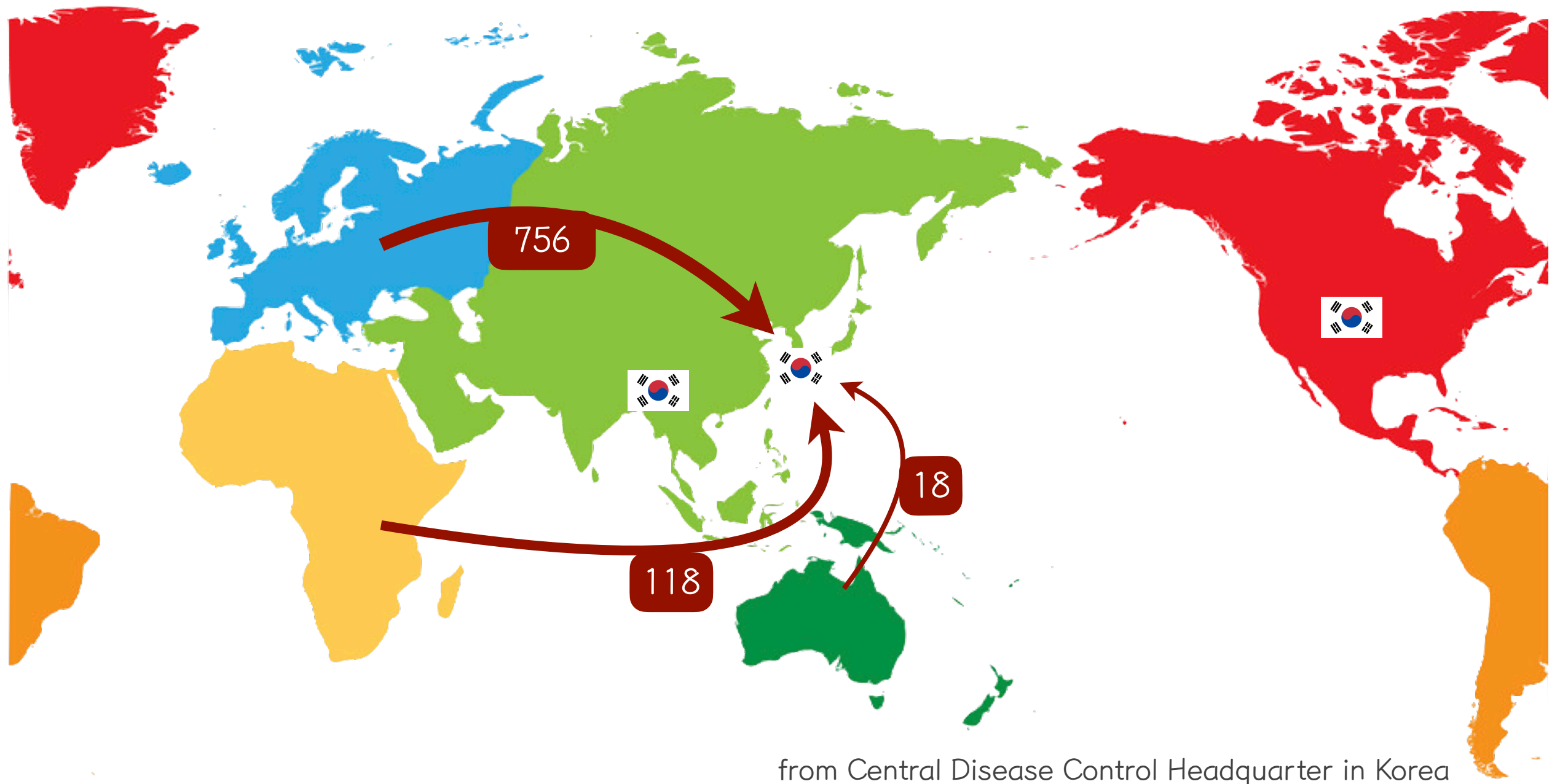
Mechanism for Decreasing EE



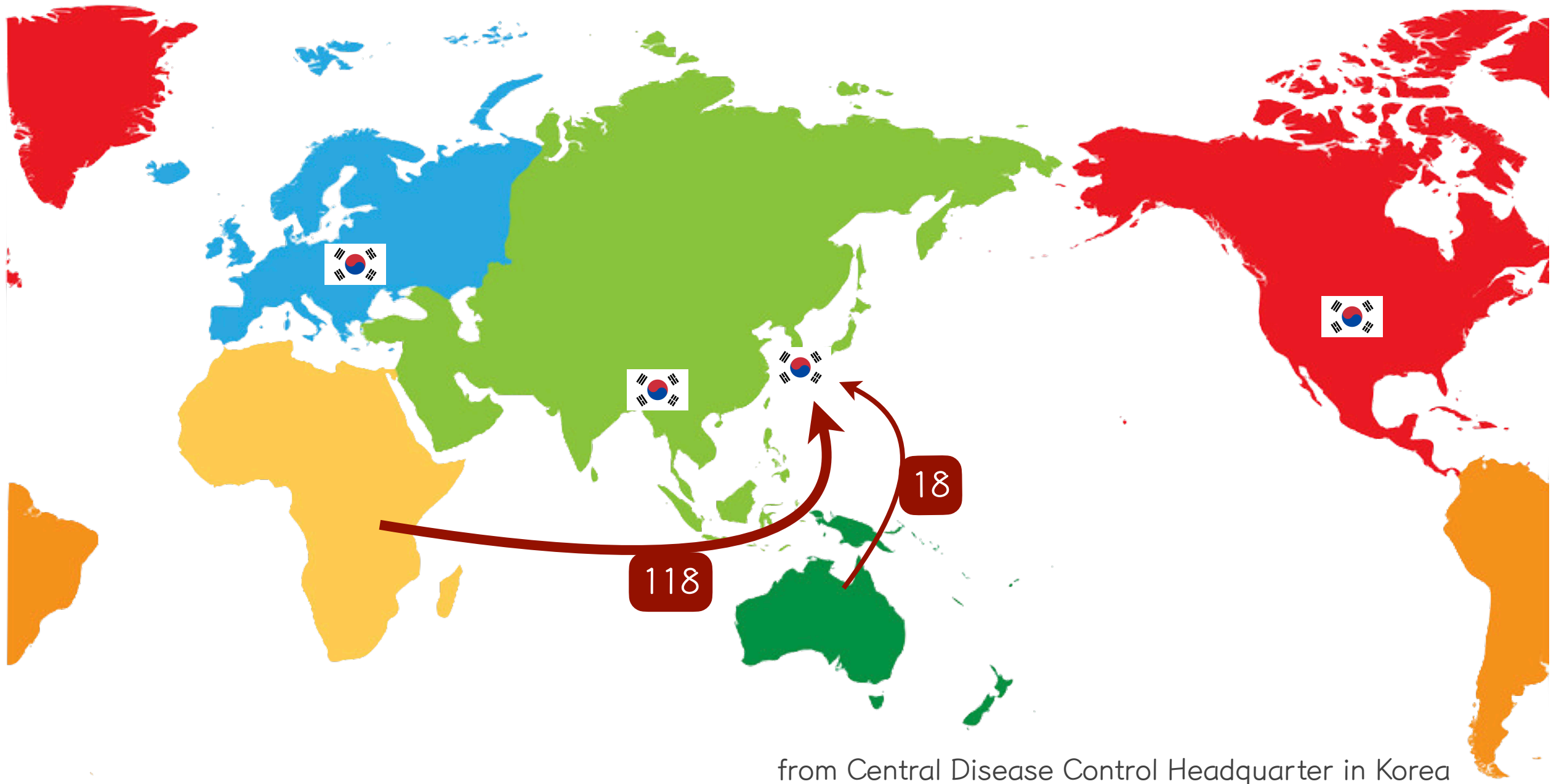
Mechanism for Decreasing EE



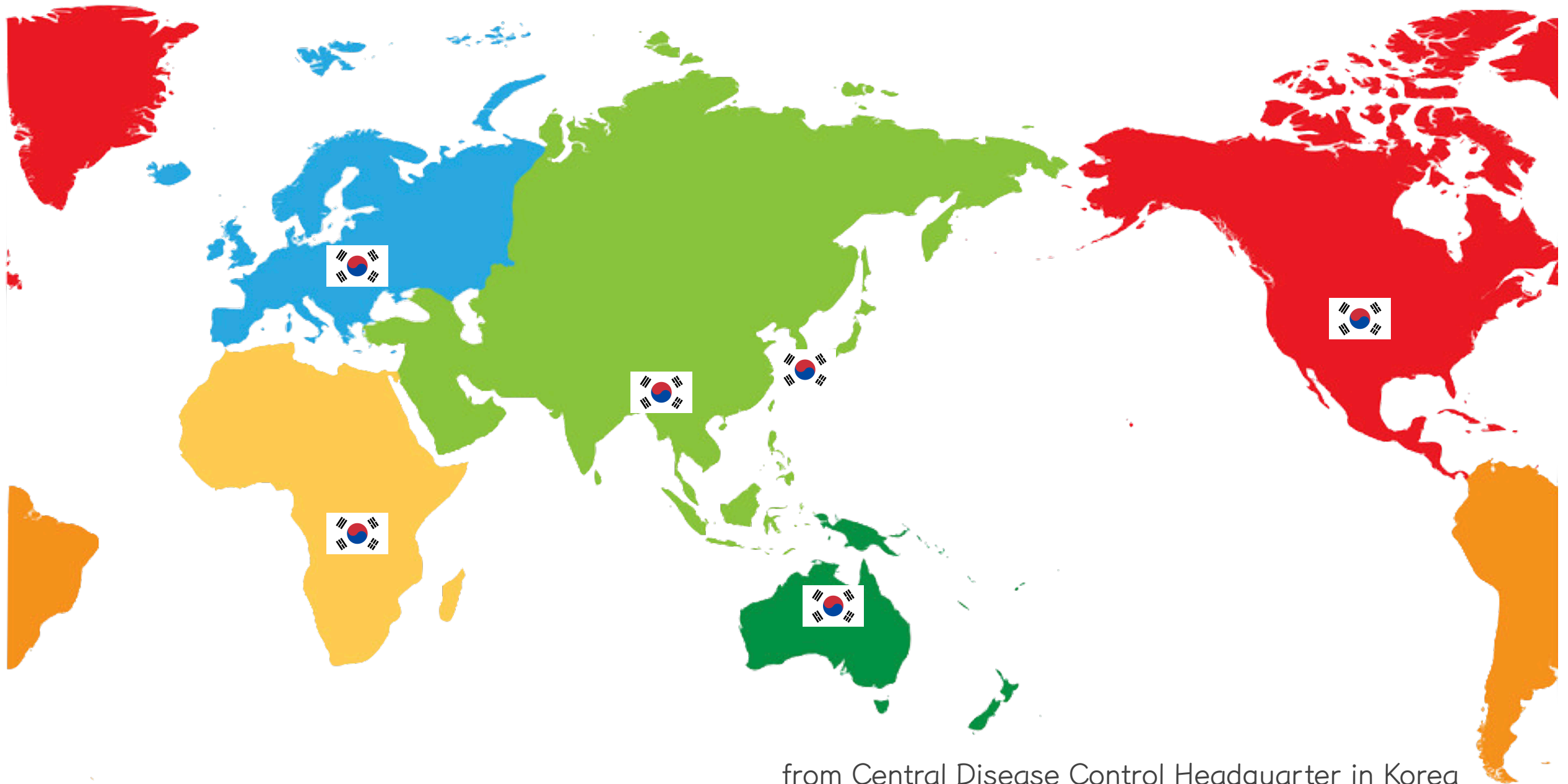
Mechanism for Decreasing EE



Mechanism for Decreasing EE

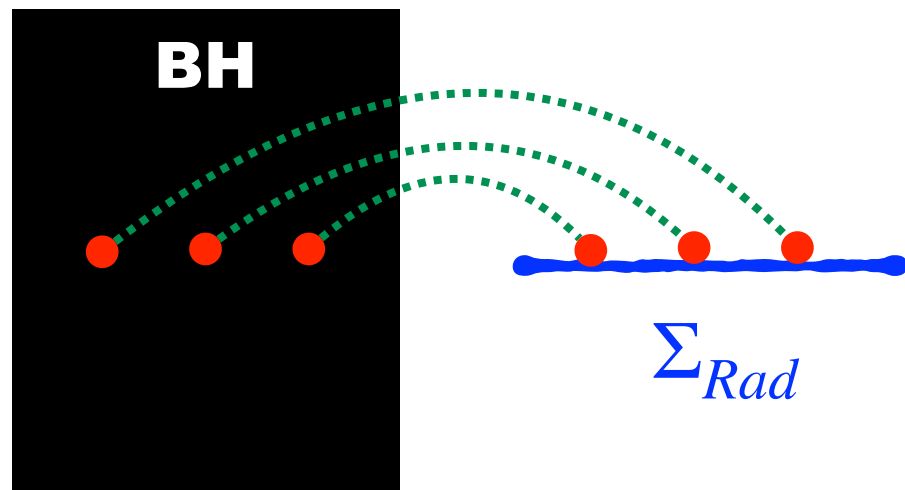


Mechanism for Decreasing EE



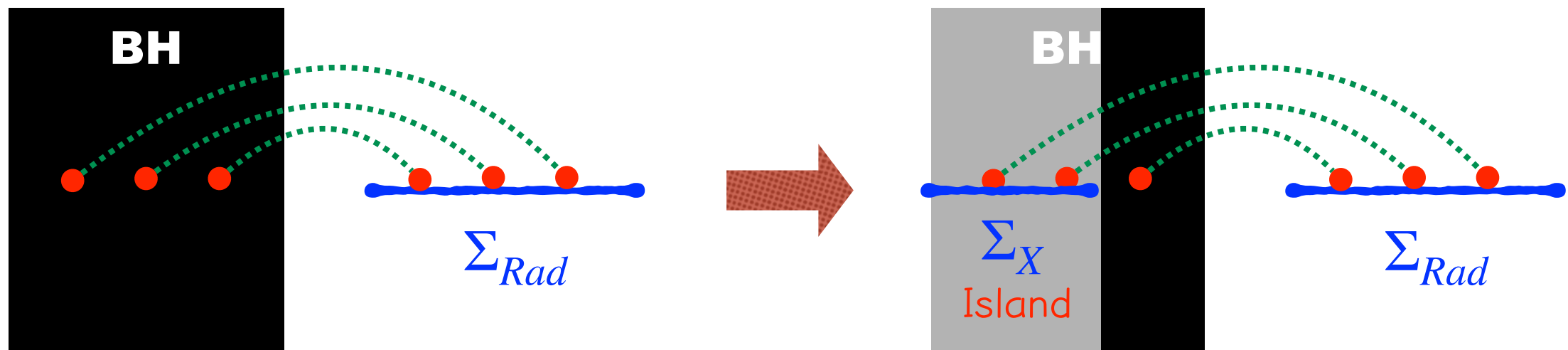
Mechanism for Decreasing EE

based on [Almheiri, Hartman, Maldacena, Shaghoulian, Tajdini, 2006.06872]



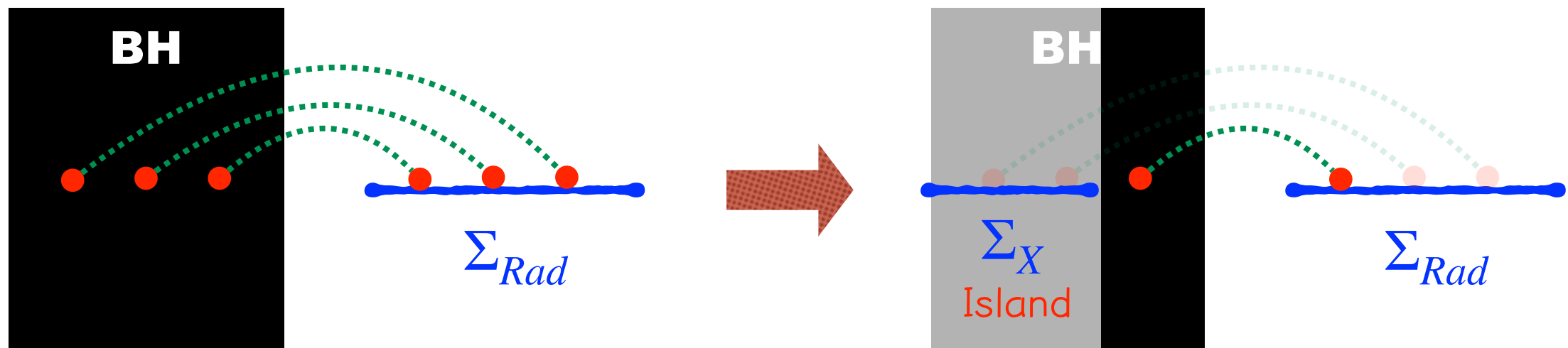
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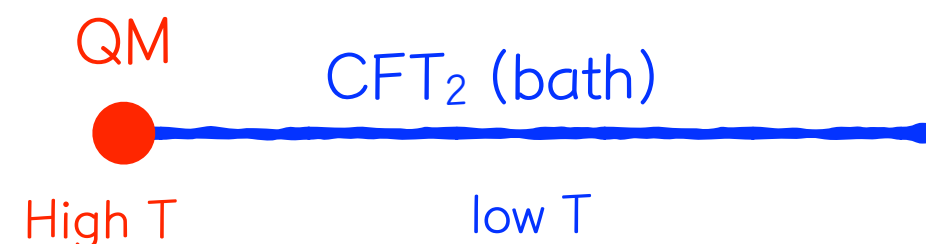


Island “purifies” the EPR pair of Hawking radiation.
This decreases the entanglement entropy.

Doubly-Holography

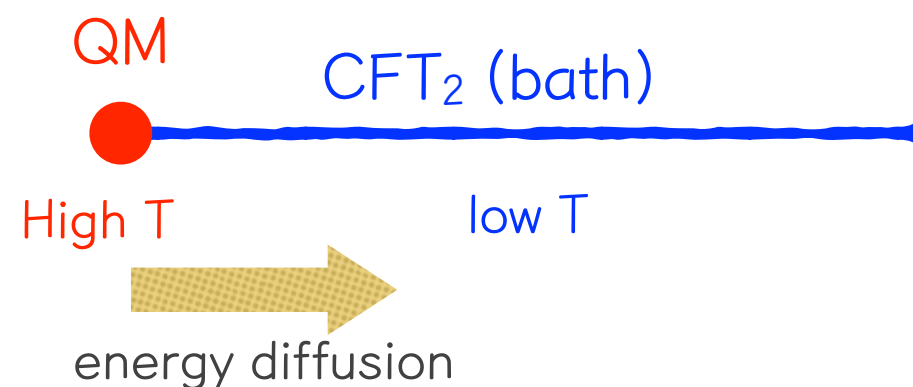
Low-Dimensional Model (Bottom-up)

* Quantum Mechanics (1+0)dim
+ CFT₂ (1+1)dim



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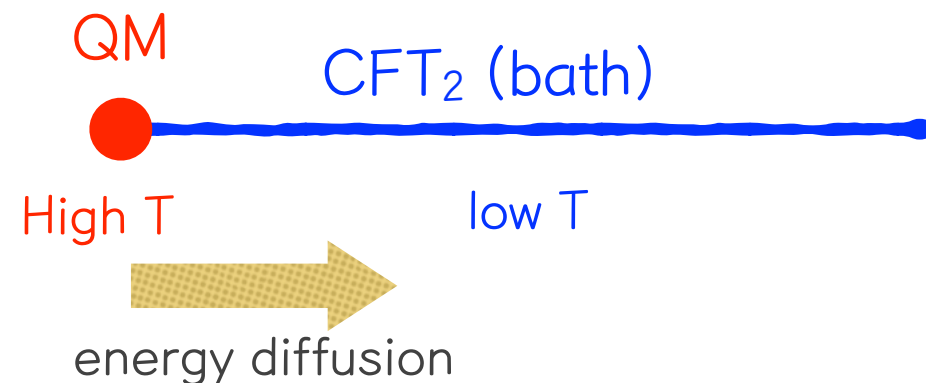


Low-Dimensional Model (Bottom-up)

* Holography:
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[Almheiri, Engelhardt, Marolf, Maxfield, 2019]

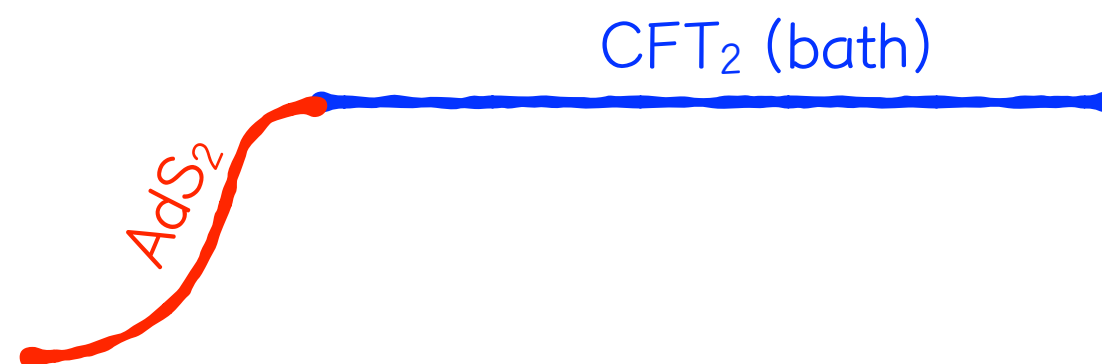


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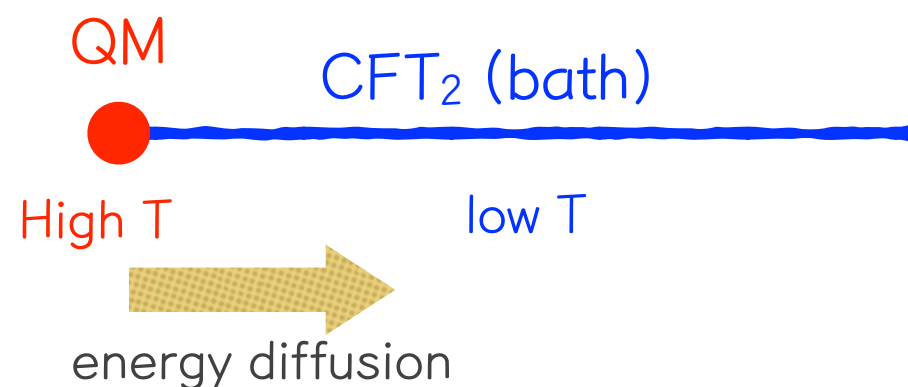


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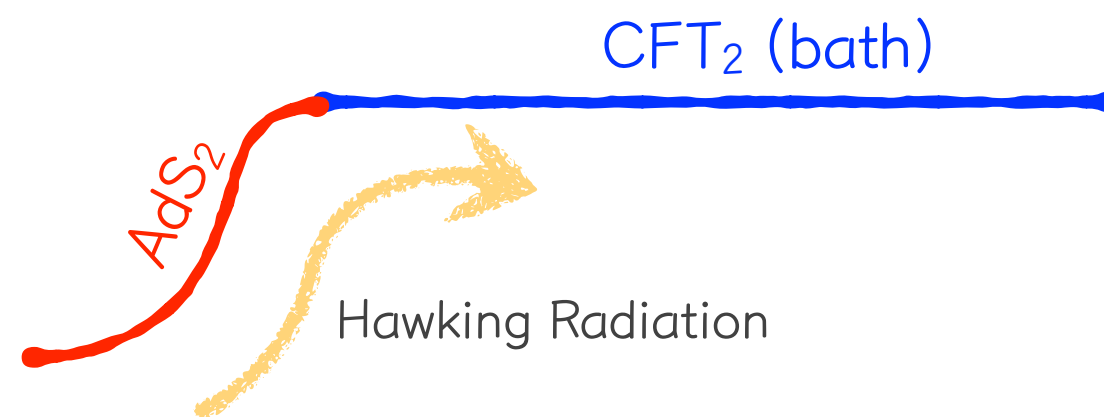


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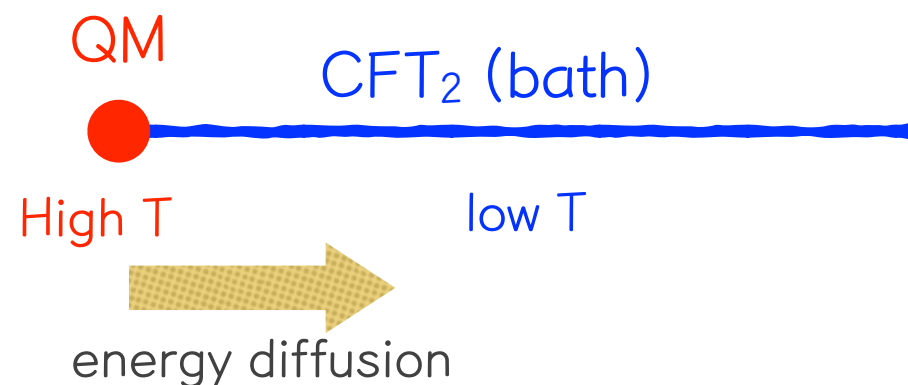


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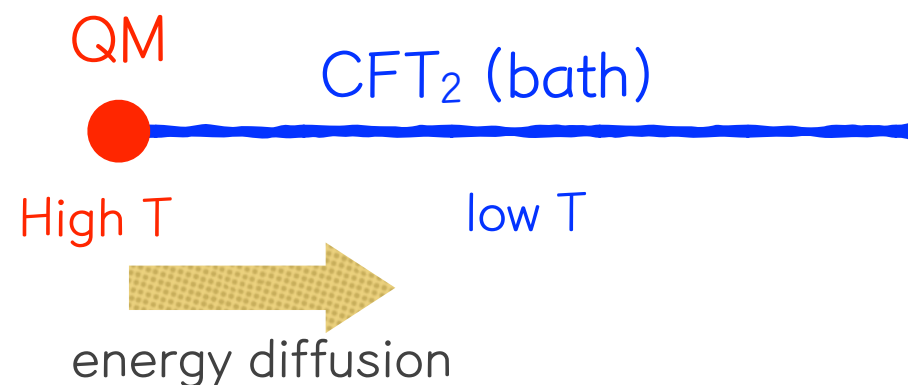
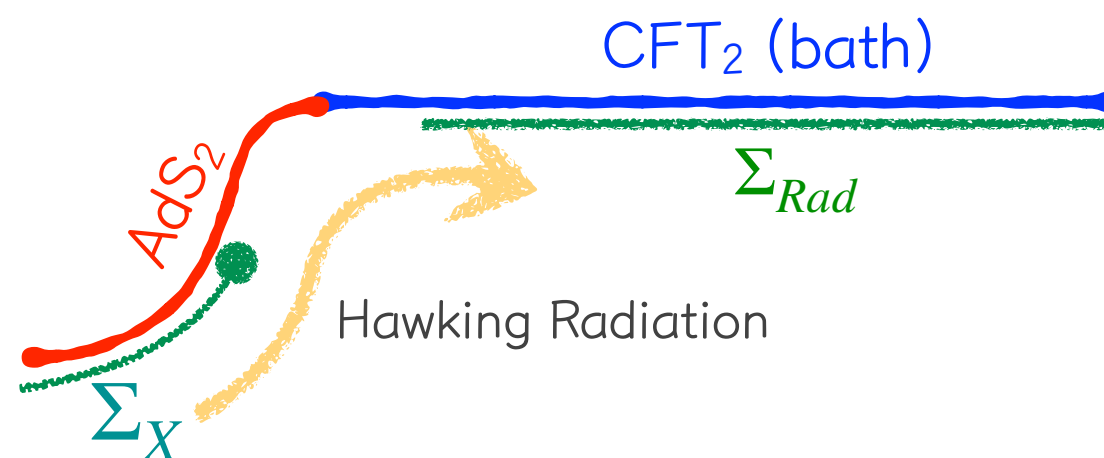


Low-Dimensional Model (Bottom-up)

- * Holography:
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[Almheiri, Engelhardt, Marolf, Maxfield, 2019]

S_{QES} can be calculated by using
conformal symmetry
e.g. twist operator

- * Quantum Mechanics (1+0)dim
+ CFT_2 (1+1)dim



Low-Dimensional Model (Bottom-up)

- * Doubly-holography:
 AdS_3 (1+2)dim

[Almheiri, Mahajan, Maldacena, Zhao, 2019]

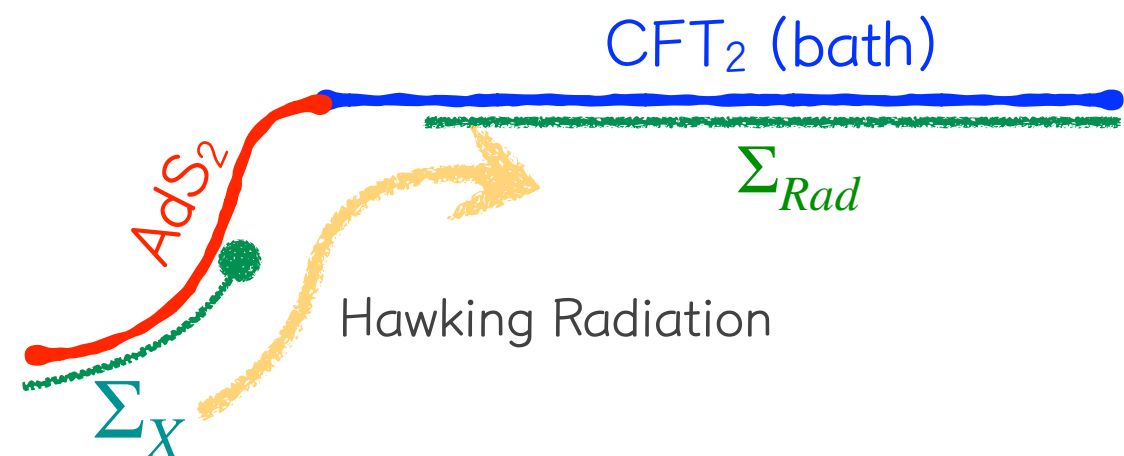
Also, recall Kusuki's lecture



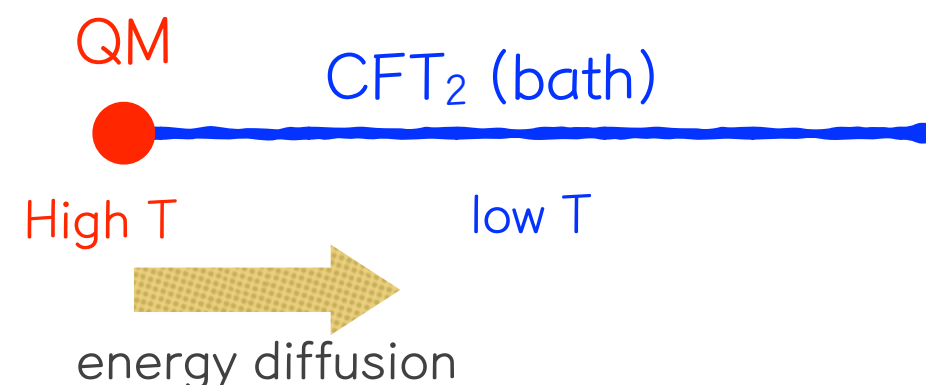
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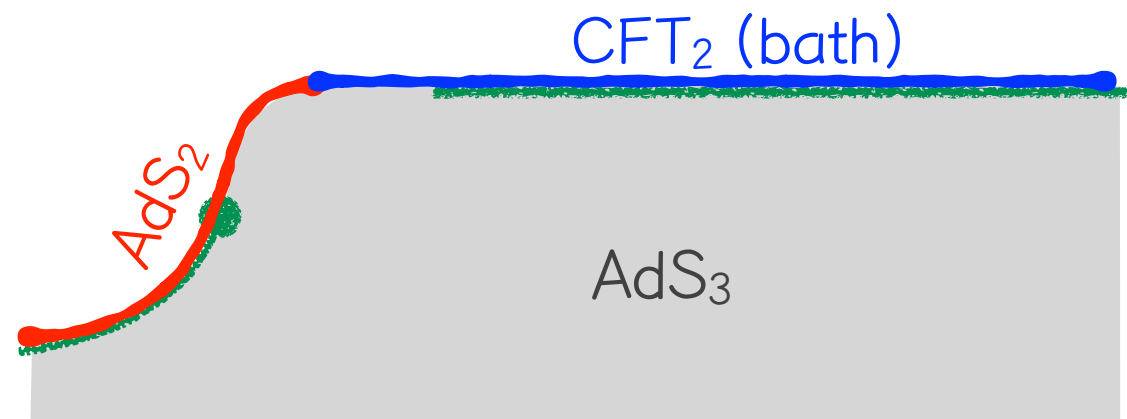


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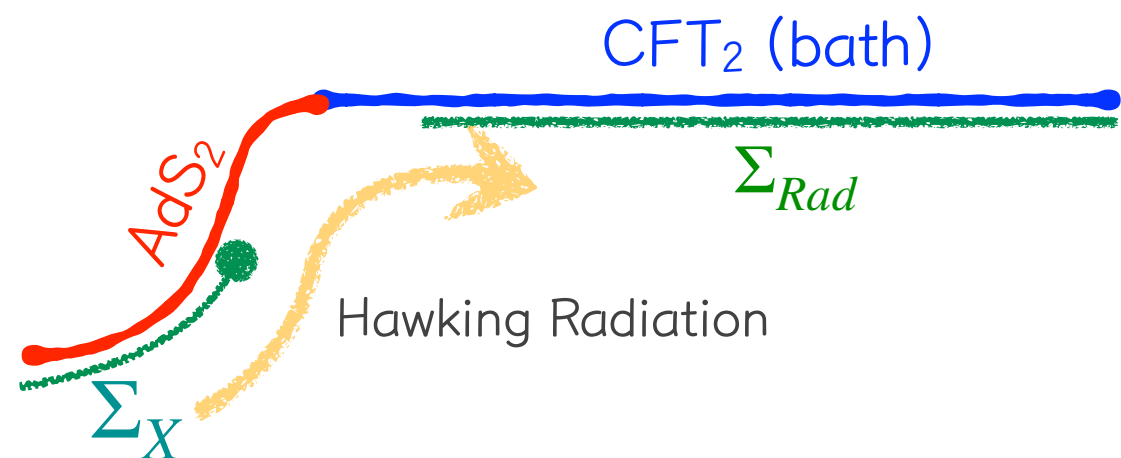
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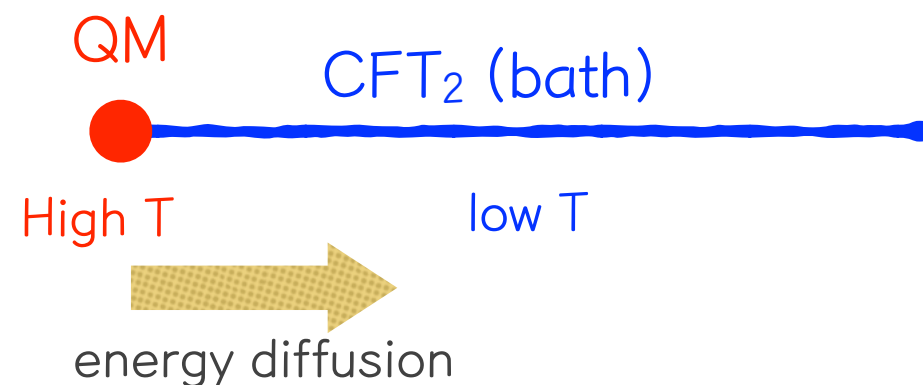


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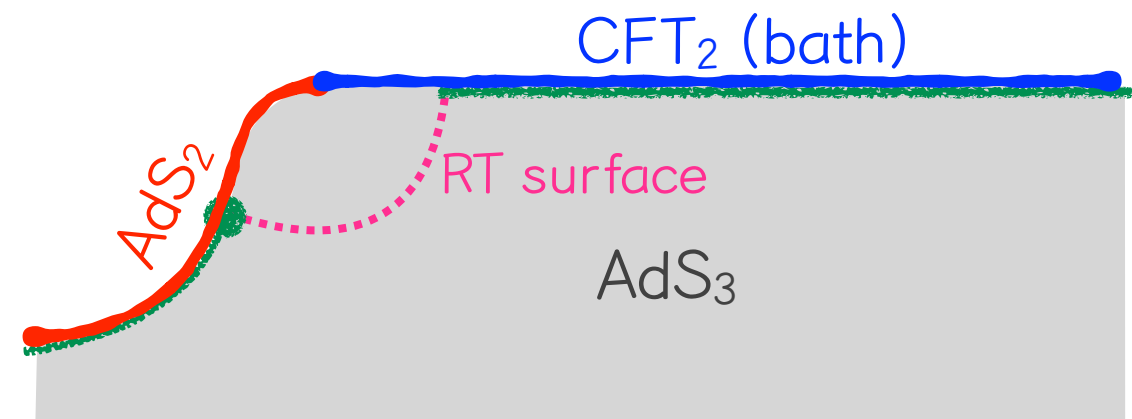
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S_{QES} can be calculated by using
RT surface

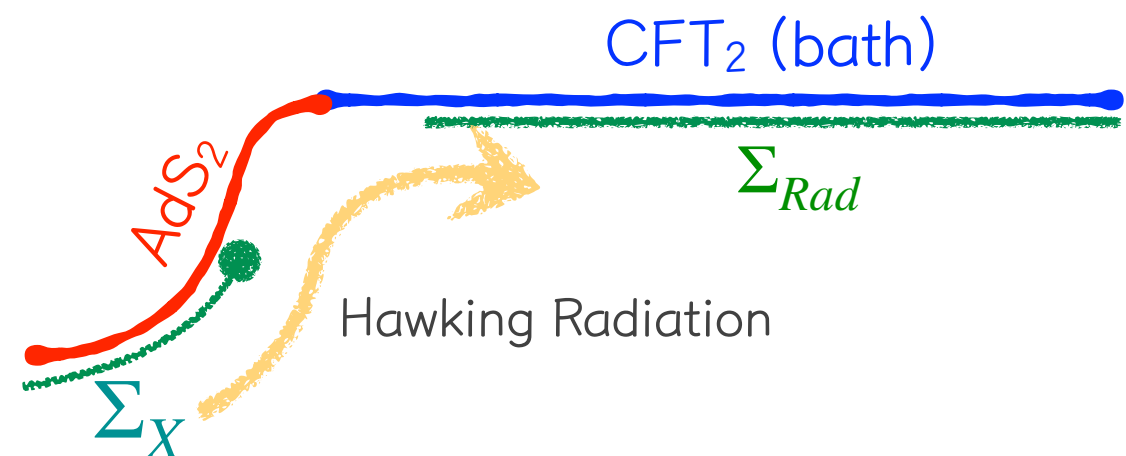


- * Holography:

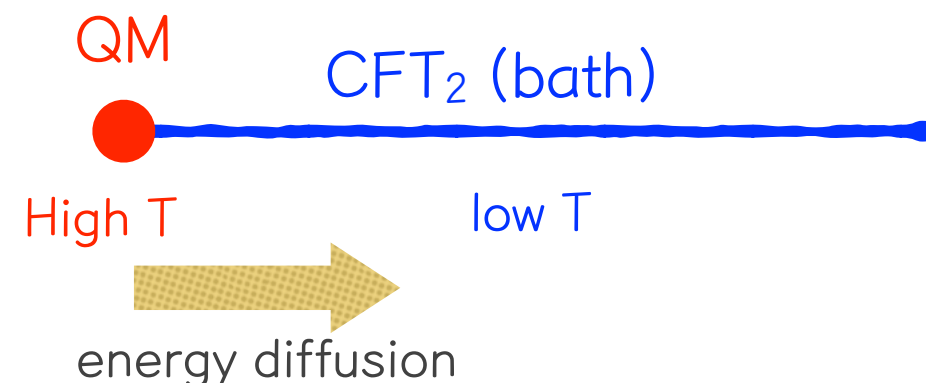
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Doubly-holography: 3D Janus Black Hole

Eternal Janus Black Hole Solutions

- * Einstein gravity coupled with massless scalar field

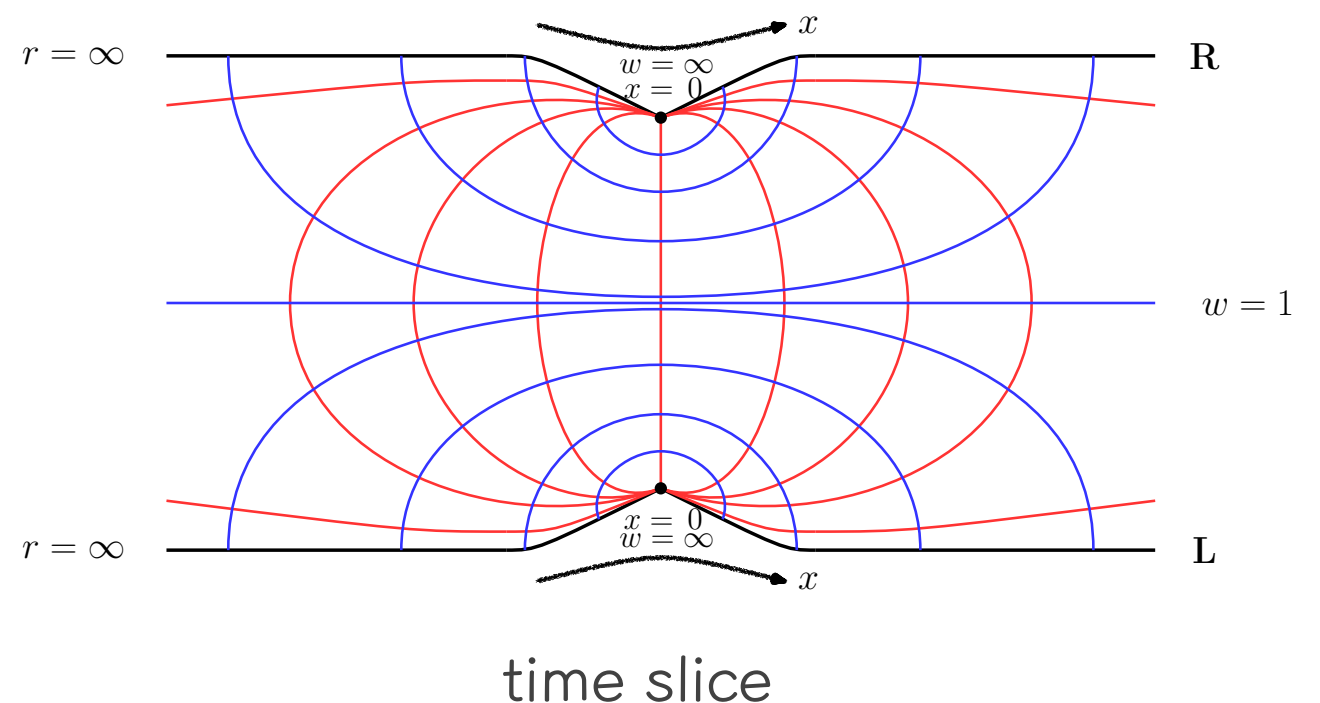
$$I = \frac{1}{16\pi G} \int d^3x \left[R - g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{2}{\ell^2} \right]$$

- * Three-dimensional two-sided Janus black hole solution

$$ds^2 = \ell [dy^2 + f(y) ds_{AdS_2}^2]$$

$$f(y) = \frac{1}{2} (1 + \sqrt{1 - 2\gamma^2} \cosh 2y)$$

$$\phi(y) = \frac{1}{\sqrt{2}} \log \left[\frac{1 + \sqrt{1 - 2\gamma^2} + \sqrt{2}\gamma \tanh y}{1 + \sqrt{1 - 2\gamma^2} - \sqrt{2}\gamma \tanh y} \right]$$



Holographic Dual

- * Janus black hole is holographically dual to **Interface CFT₂** (ICFT₂).



- ✓ Janus deformation induces interface d.o.f. at $x = 0$.

$$\dim(H_{interface}) \sim \log \frac{1}{\sqrt{1-2\gamma^2}}$$

- ✓ Interface d.o.f. breaks conformal symmetry (partially).
c.f. BCFT

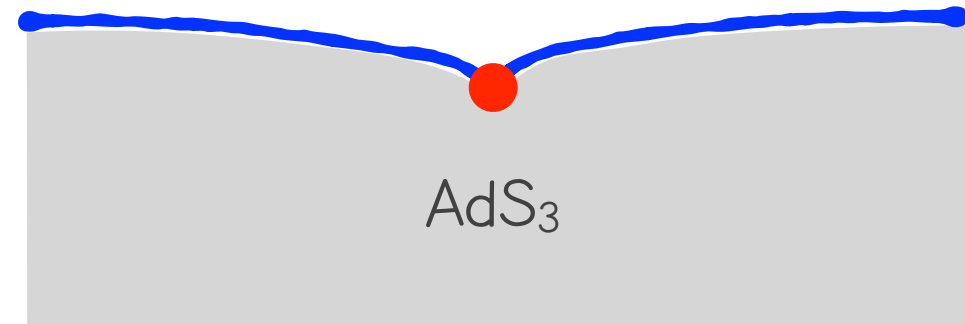
- * Eternal Janus black hole is dual to **TFD of ICFT₂**.

- ✓ ICFT_L and ICFT_R lives on the boundary of eternal Janus black hole, respectively.
- ✓ **Maximally entangled** state of ICFT_L and ICFT_R.

$$|TFD(t)\rangle = \frac{1}{\sqrt{Z}} \sum_n e^{-\frac{\beta}{2}E_n - 2itE_n} |n\rangle_L \otimes |n\rangle_R$$

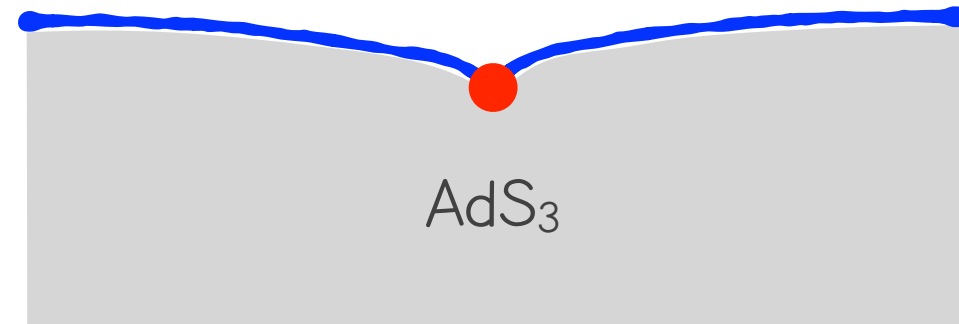
Top-down Approach

* Eternal Janus Black Hole
 AdS_3 (1+2)dim

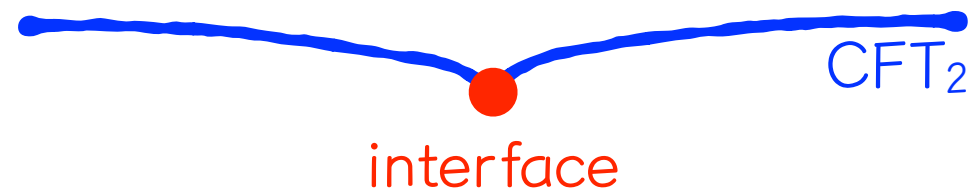


Top-down Approach

- * Eternal Janus Black Hole
 AdS_3 (1+2)dim



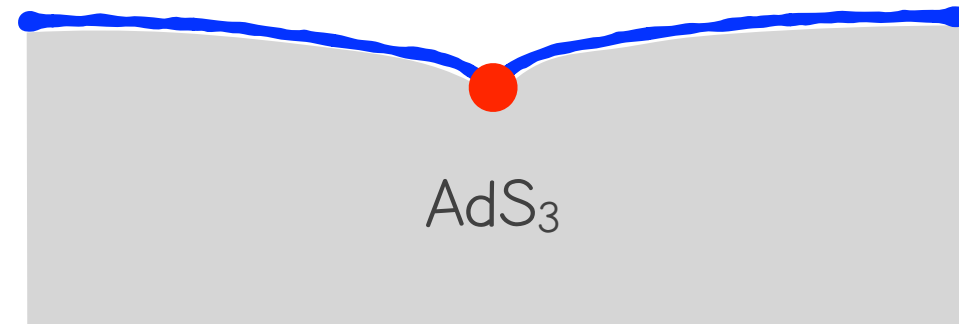
- * “Doubly-holography”
Interface (QM) (1+0)dim
+ CFT_2 (1+1)dim



This is usual holography for Janus black hole.

Top-down Approach

- * Eternal Janus Black Hole
 AdS_3 (1+2)dim

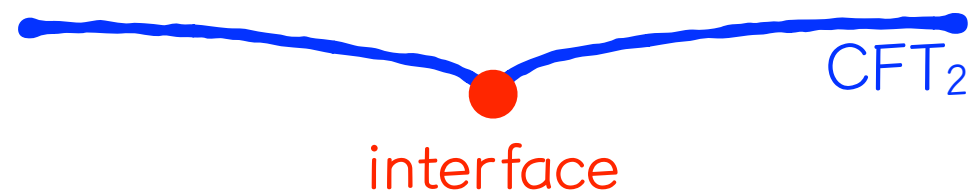


- * AdS_2 (1+0)dim + CFT_2 (1+1)dim

?

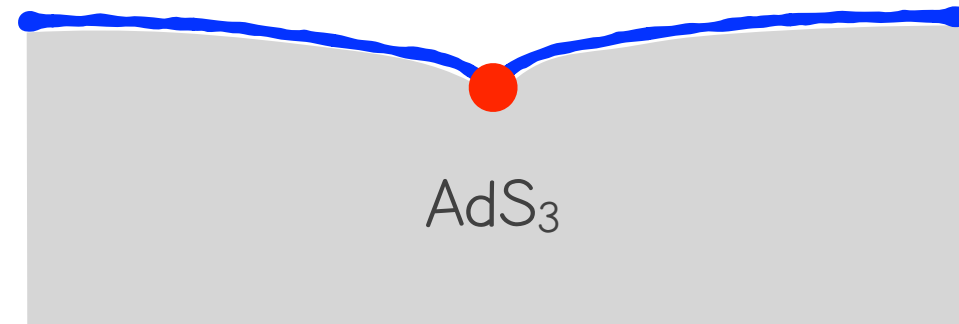
Emergence of 2D gravity is not clear for general γ .

- * “Doubly-holography”
Interface (QM) (1+0)dim
+ CFT_2 (1+1)dim



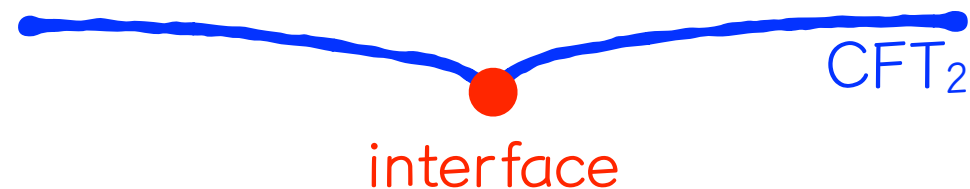
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* Eternal Janus Black Hole
 AdS_3 (1+2)dim



* AdS_2 (1+0)dim + CFT_2 (1+1)dim

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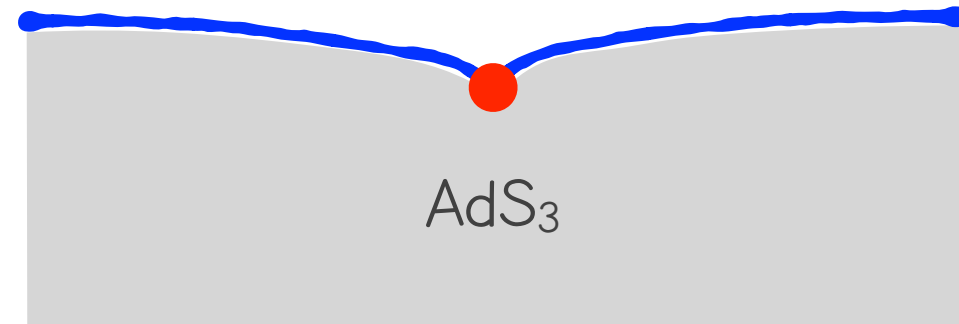


This is usual holography for Janus black hole.

* Eternal Janus Black Hole
AdS₃ (1+2)dim

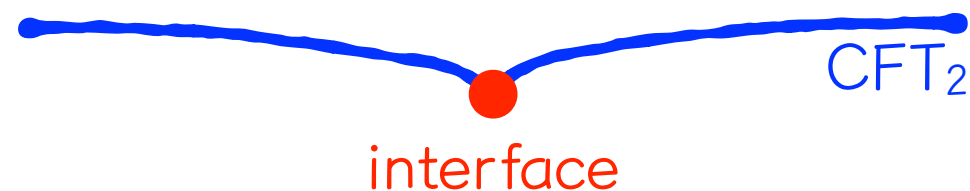
As γ approaches to $\frac{1}{\sqrt{2}}$, interface d.o.f increases.

Shadow region gets larger.



* AdS₂ (1+0)dim + CFT₂ (1+1)dim

* “Doubly-holography”
Interface (QM) (1+0)dim
+ CFT₂ (1+1)dim

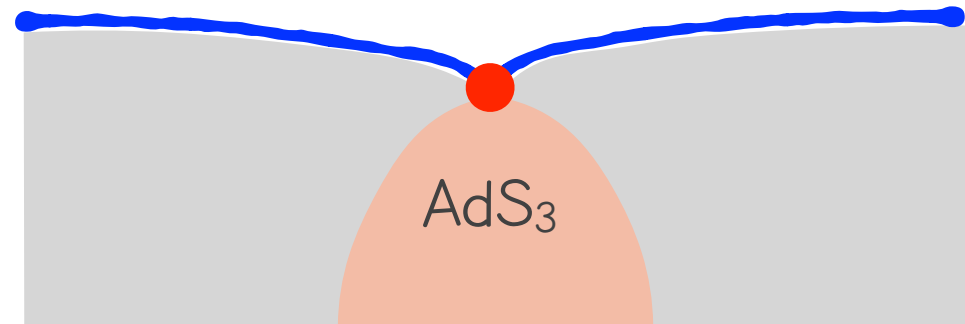


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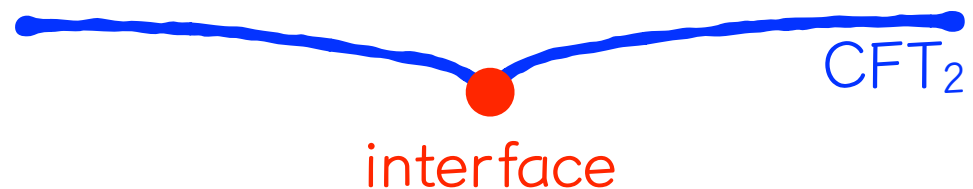
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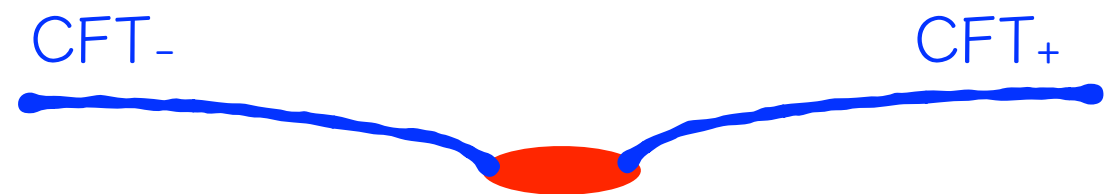
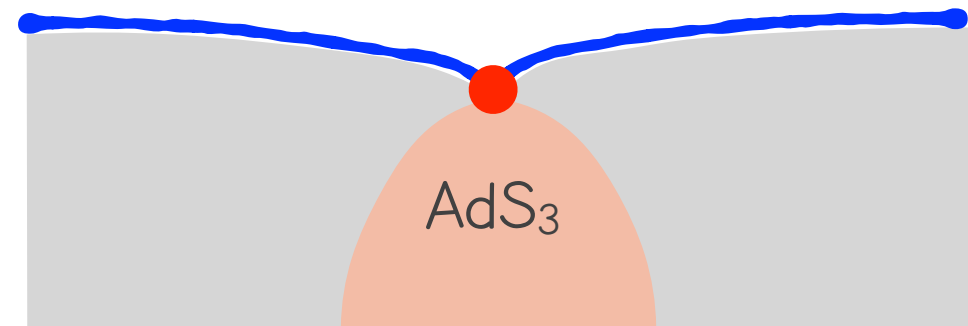


This is usual holography for Janus black hole.

* Eternal Janus Black Hole
AdS₃ (1+2)dim

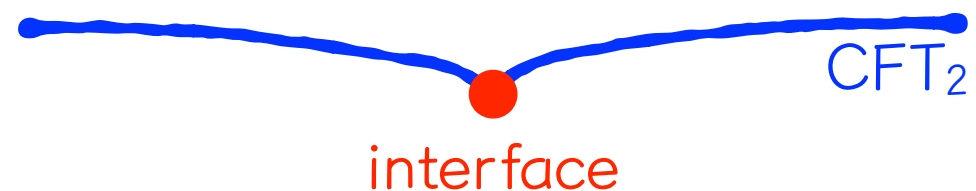
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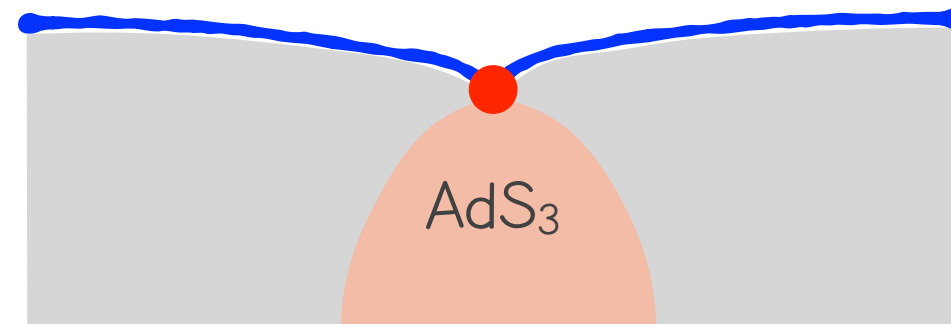


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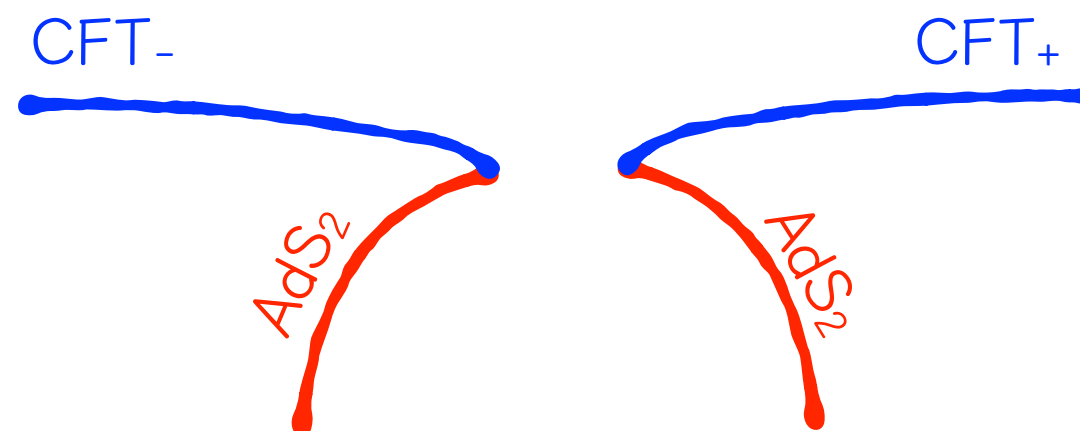
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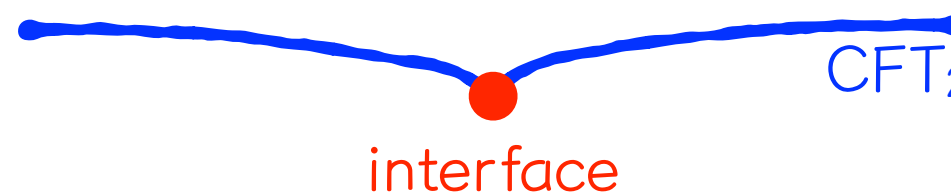


* AdS₂ (1+0)dim + CFT₂ (1+1)dim

$x > 0$ and $x < 0$ regions are “almost” decoupled.



* “Doubly-holography”
Interface (QM) (1+0)dim
+ CFT₂ (1+1)dim



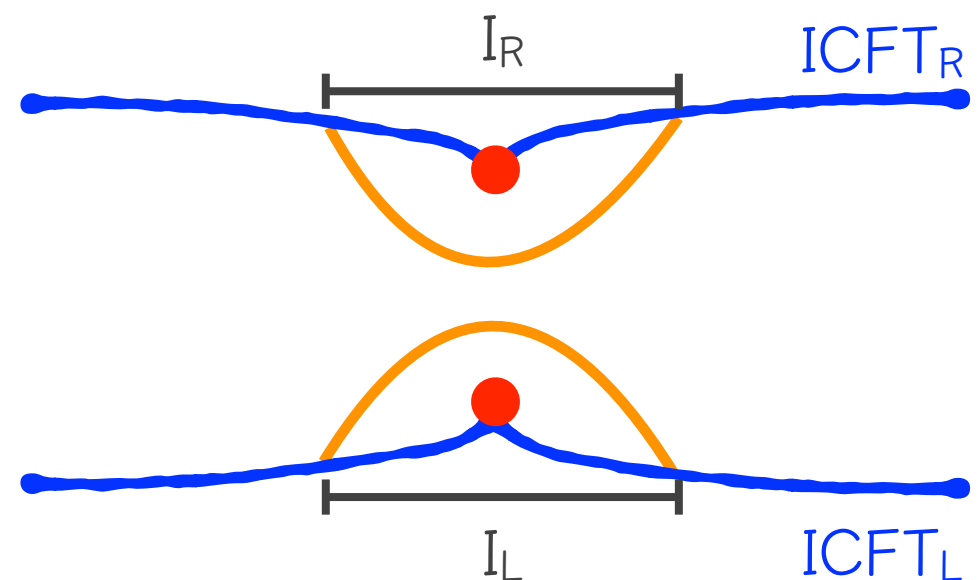
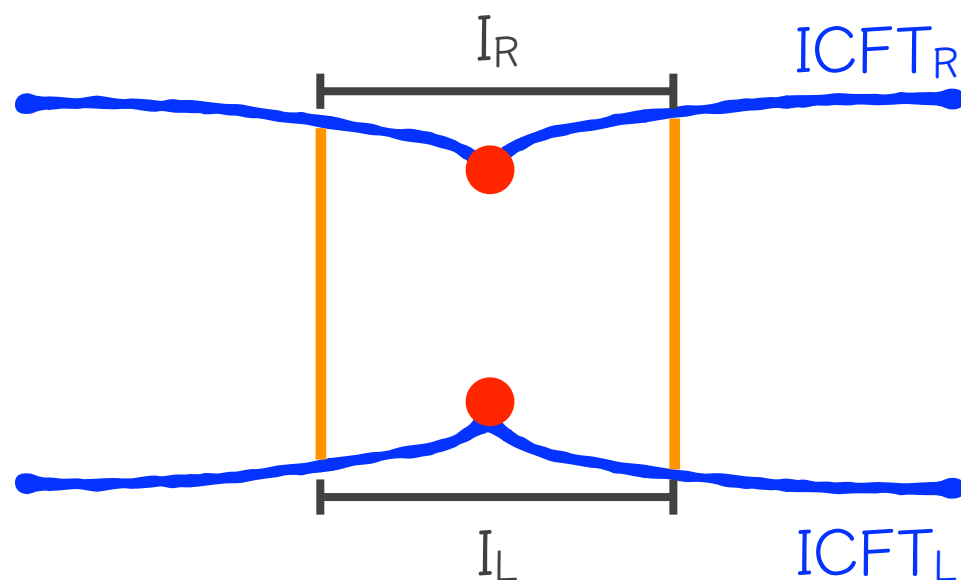
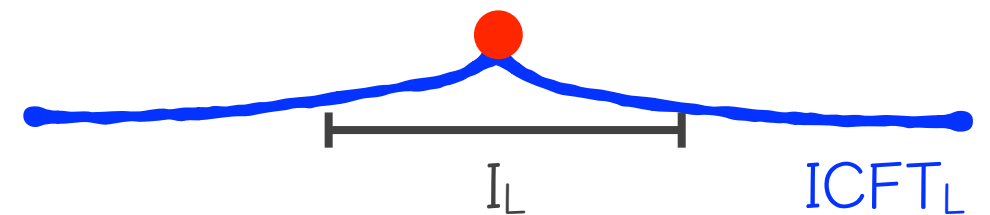
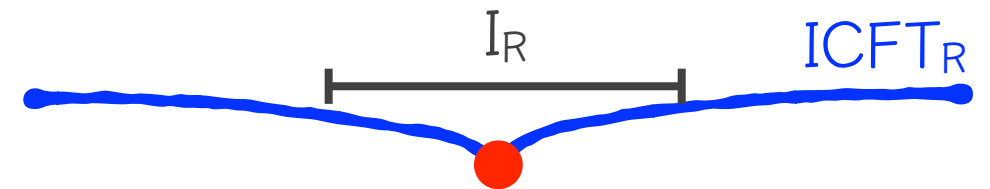
This is usual holography for Janus black hole.

Entanglement Entropy

* We will study the entanglement entropy of $I_R \cup I_L$.

- ✓ I_R and I_L is interval in ICFT_R and ICFT_L , respectively
- ✓ In AdS_3 , this can be evaluated by **RT surface**.
- ✓ This is **classical** calculation.

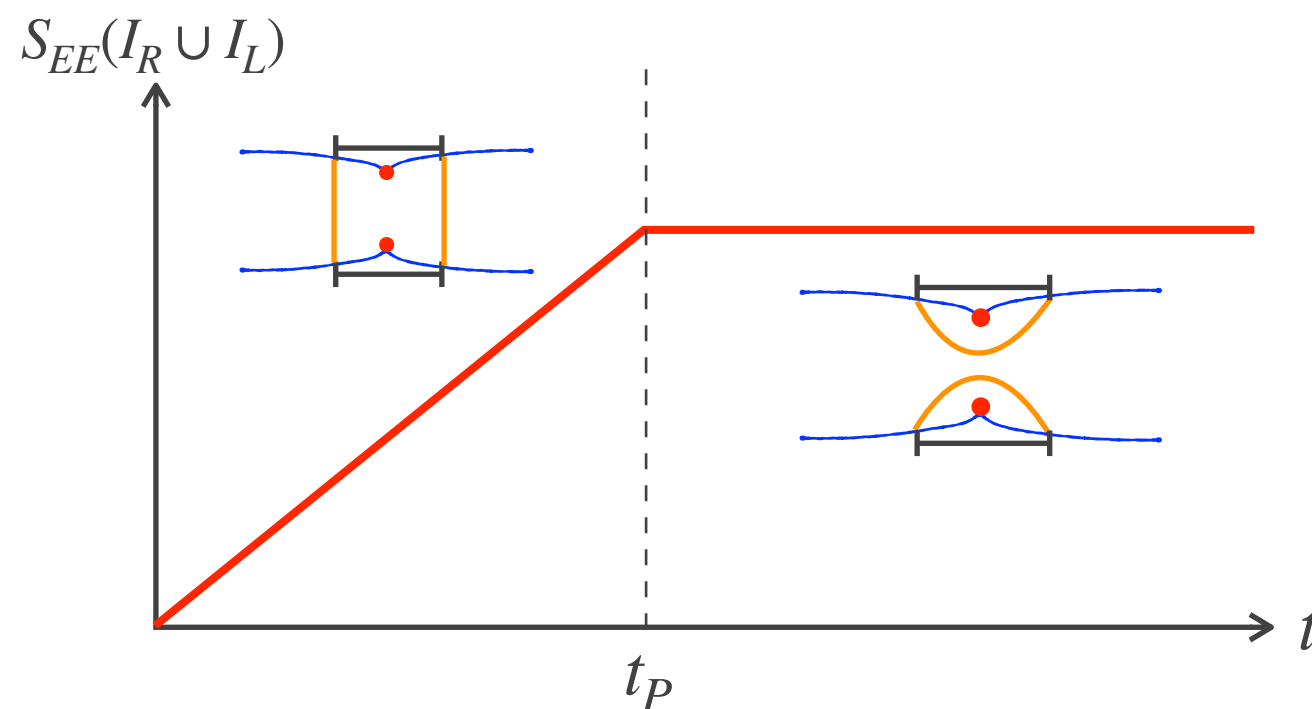
* There are two phases for RT surface.



Page Curve

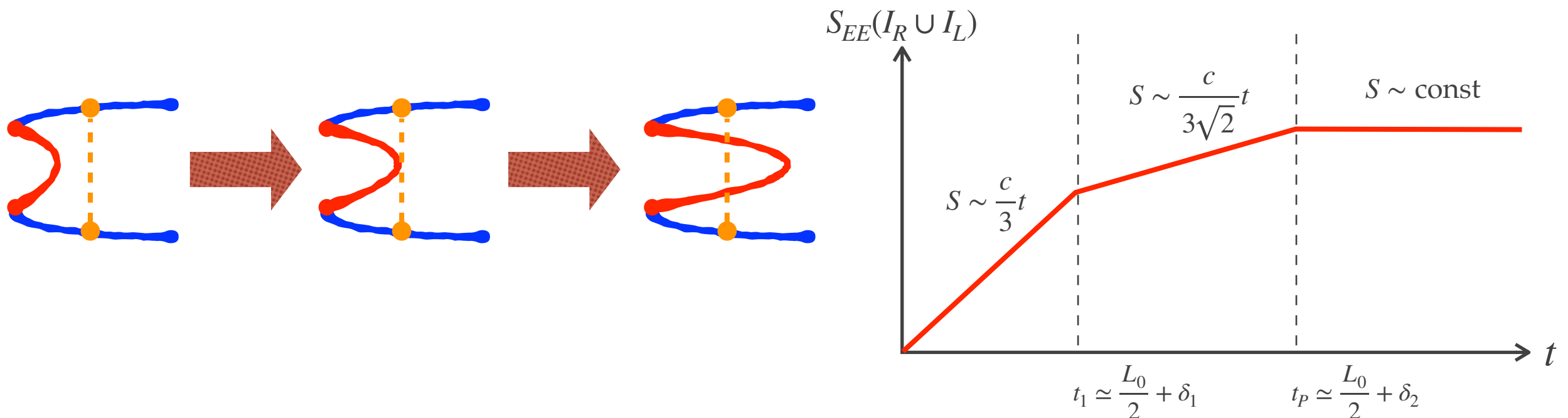
* Transition between two RT surfaces leads to Page curve for eternal Janus black hole

- ✓ Page time: $t_p = \frac{L_0}{2} + \dots$ where $I_L = I_R = \left[-\frac{L_0}{2}, \frac{L_0}{2}\right]$
- ✓ Before Page time, the **connected RT surface** gives EE which increases in time.
- ✓ After Page time, the **disconnected RT surface** leads to time-independent EE. This is the sum of thermal EE of each interval I_R and I_L .



New Phase before Page Time

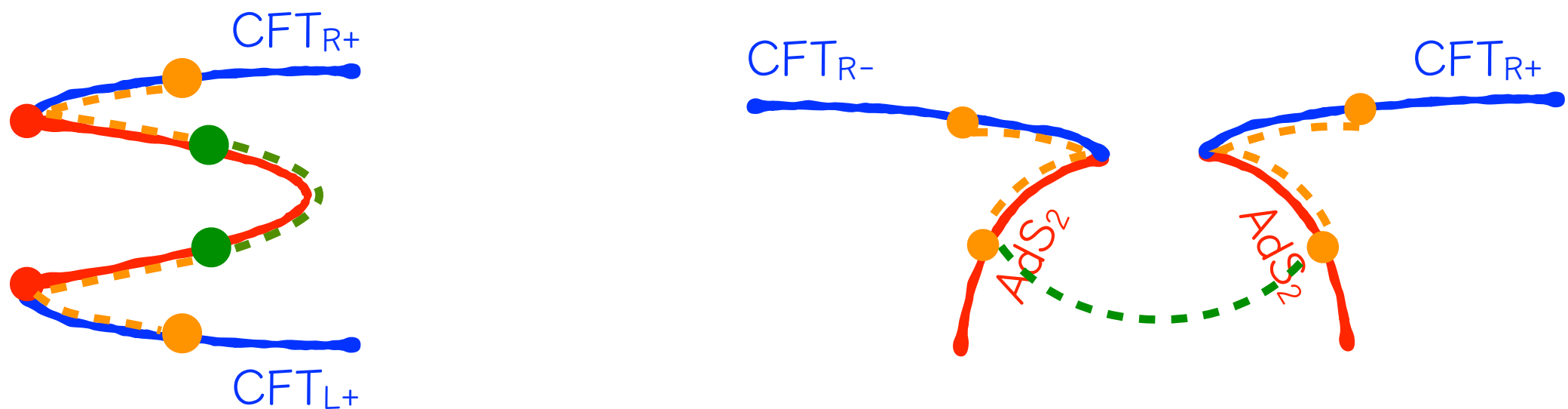
- * As time passes, the shadow region gets larger. For small interface d.o.f., a transition to disconnected RT surface occurs before the shadow region touches the connected RT surface.
- * When the interface d.o.f. is large enough, we have enough time window to exhibit new behavior before Page transition (disconnected RT surface).
i.e. **the shadow region intersects with connected RT surface.**



$$\delta_1 \ll \delta_2 \sim S_{\text{interface}}$$

From Lower-Dim Point of View

- * For large interface d.o.f., we have “2D gravity” description.
- * We can calculate QES from the point of view of 2D gravity + CFT_2 .



- ✓ can confirm the results from 3D calculations.
- * Also, we calculate EE from the point of view of ICFT_2 (QM+ CFT_2)
 - ✓ consistent with 3D calculations.

Conclusion

- * Eternal Janus black hole is a top-down example of Island conjecture.
 - ✓ “2D gravity” description and doubly-holography is available.
 - ✓ 1D, 2D and 3D calculations of EE are consistent.
- * Existence of Island is still not proven yet.
 - ✓ We accepted assumption of the existence of Island (or, QES). But, this does not prove it.
 - ✓ The essence of Island might lie in black hole complementary.

Outlook

- * Is there other observables to test Island conjecture?
- * How can we extract information from Island?
- * What is the origin of the Island?
- * Can we calculate the same thing in higher dimension?
e.g. SYM/AdS₅

Thank You