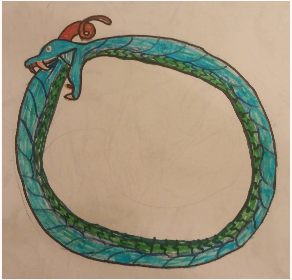
Solving the dark problem at 'short' distance via Stringy Gravity



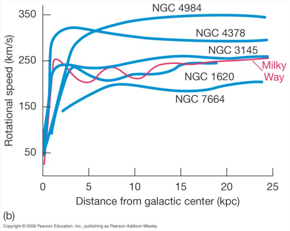
Uroboros : An ancient Egyptian symbol for a serpent which eats its own tail

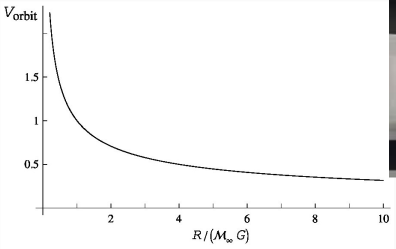
Jeong-Hyuck Park **박 정 혁 (朴 廷 爀)** **Sogang University**

New perspectives on Gravitation & Cosmology, APCTP, Pohang 1st Dec. 2017

Dark Problems

Dark Matter Problem





Vera Rubin

Galaxy rotation curves : observation   Keplerian $\sqrt{MG/R}$ fall-off : GR or Newton

- The galaxy rotation curve is a plot of the orbital velocities of visible stars versus their radial distance from the galactic center.
- While Einstein gravity (GR), with Schwarzschild solution, predicts the Keplerian (inverse square root) monotonic fall-off of the velocities, $V_{\text{orbit}} = \sqrt{MG/R}$, observations however show rather ‘flat’ ($\sim$ 200 km/s) curves after a fairly rapid rise.

- The resolution of the discrepancy may call for ‘dark matter’, or modifications of the law of gravity.

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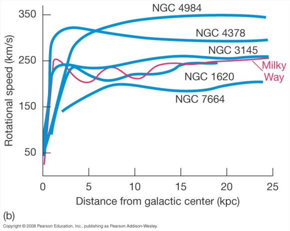

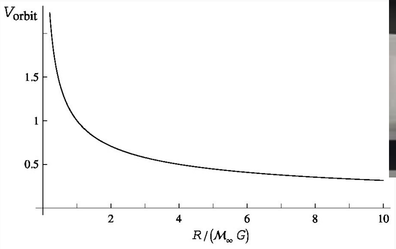

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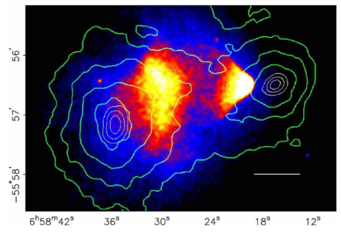

Bullet Cluster

- Bullet cluster consists of two colliding clusters of galaxies.
- Observation shows apparent discrepancy between the center of the gravitational lensing and the center of the visible matter.
- It is claimed to provide the best evidence for the existence of weakly interacting dark matter and against Modified Newtonian Dynamics (MOND)".

- However, it should not rule out every modified gravity if the modification is caused by extra field (dark gravity or dark matter depending on interpretation).
- It is necessary to examine the details of alternative theories of gravity, for the test.

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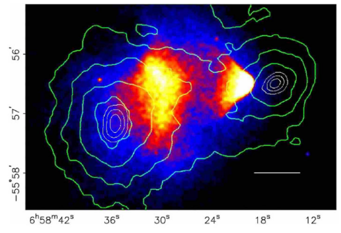

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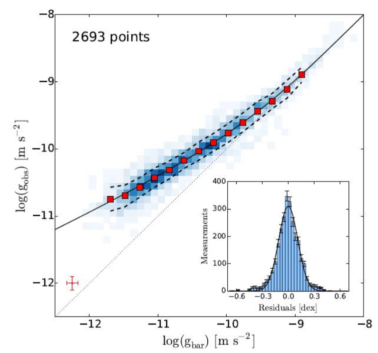

$$g_{obs} = \mathcal{F}(g_{bar}) = \frac{g_{bar}}{1 - e^{-\sqrt{g_{bar}/g_{\dagger}}}}$$

McGaugh, Lelli and Schombert
arXiv:1609.05917

- Recent analysis on the rotation curves for 2693 points in 153 galaxies with very different masses, sizes, and gas fractions indicates an intriguing correlation between observations and the Newtonian prediction by baryons.
- This relation seems to strongly indicates a certain modified law of gravity, at least for rotating galaxies.

Dark Energy : Cosmological Constant Problem

The Nobel Prize in Physics 2011

Photo: U. Montan
Saul Perlmutter
Prize share: 1/2

Photo: U. Montan
Brian P. Schmidt
Prize share: 1/4

Photo: U. Montan
Adam G. Riess
Prize share: 1/4

The Nobel Prize in Physics 2011 was divided, one half awarded to Saul Perlmutter, the other half jointly to Brian P. Schmidt and Adam G. Riess *"for the discovery of the accelerating expansion of the Universe through observations of distant supernovae"*.

Disturbing disagreement between the observed values of vacuum energy density (the small value of the cosmological constant) and theoretical large value of zero-point energy in QFT.

Uroboros relation

– While mathematics is about dimensionless numbers, *e.g.* $\mathbb{N}$, $\mathbb{R}$, $\mathbb{C}$, physical quantities are generically dimensionful,

$$[R] = \mathrm{Length}\,, \qquad [M] = \mathrm{Mass}\,, \quad etc.$$

– Physical laws given by mathematical formulas must be consistent with the dimensions of the physical quantities.

– In natural unit, $c \equiv 1$, the orbital velocity is dimensionless. Further, $R/(MG) \equiv \mathbf{x}$ is a dimensionless radial variable normalized by mass.

– The Keplerian orbital velocity reads then in terms of the two dimensionless variables,

$$V_{\mathrm{orbit}} = \sqrt{1/\mathbf{x}}\,.$$

- In any (modified) gravitational theory based on metric, $\mathbf{x}$ is the natural variable of the dimensionless metric, $g_{\mu\nu}(\mathbf{x})$, for spherically symmetric vacuum configurations.

- The orbital velocity of of a circular geodesic motion is then given by

$$V_{\mathrm{orbit}}(\mathbf{x}) = \sqrt{-\tfrac{1}{2}\mathbf{x}\frac{\partial g_{tt}(\mathbf{x})}{\partial \mathbf{x}}}\,.$$

Especially for Schwarzschild metric, we recover the Keplerian orbital velocity.

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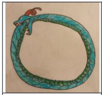
Uroboros idea

	Electron ($R \simeq 0$)	Proton	Hydrogen Atom	Billiard Ball	Earth	Solar System ($1\mathrm{AU}/M_{\odot}G$)	Milky Way (visible)	Galaxy Cluster	Universe ($M \propto R^3$)
$R/(MG)$	0^+	7.1×10^{38}	2.0×10^{43}	2.4×10^{26}	1.4×10^{9}	1.0×10^{8}	1.5×10^{6}	$\sim 10^{5}$	0^+

'Uroboros' spectrum of the dimensionless Radial variable normalized by Mass in natural units.
The orbital speed of rotation curves is also dimensionless, and depends on the single variable, $R/(MG)$.

JHP arXiv:1707.08961

- ⋆ The observations of stars and galaxies far away, or the dark matter/energy problems, are actually revealing the mathematically short-distance nature of Gravity: Long distance divided by far heavier mass gives small values of $\mathbf{x} = R/(MG)$.

- ⋆ Thus, a modified gravity may safely solve the dark problems, provided
  - it agrees with GR for $\mathbf{x} \geq 10^8$ (asymptotically Keplerian) not to be ruled out ;
  - it enhances the gravitational attraction for $10^5 \leq \mathbf{x} \leq 10^6$ to solve DM ;

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  - further, it features repulsive force at shorter scale, $\mathbf{x} \leq 10^4$ to solve DE .

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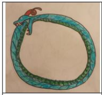	Electron ($R \simeq 0$)	Proton	Hydrogen Atom	Billiard Ball	Earth	Solar System ($1\mathrm{AU}/M_\odot G$)	Milky Way (visible)	Galaxy Cluster	Universe ($M \propto R^3$)
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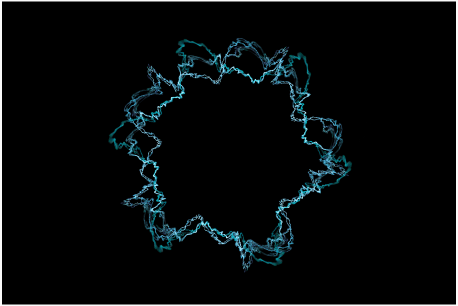
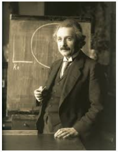
Stringy Gravity



Stringy Gravity is the ‘unambiguous’ extension of General Relativity, dictated entirely by the Symmetry Principle from string theory.

String theory predicts not GR but its own gravity.

Ever since Einstein formulated General Relativity (GR), by employing the differential geometry *a la* Riemann, the Riemannian metric, $g_{\mu\nu}$, has been privileged to be the only geometric and hence gravitational field.



$$\nabla_{\lambda} g_{\mu\nu} = 0\,.$$

- All other fields are then meant to be 'extra' matters.
- The coupling of GR to matters, *e.g.* to the Standard Model, are then 'minimally' determined through the explicitly appearing metric and covariant derivatives in Lagrangians, which ensure both diffeomorphisms and local Lorentz symmetry.

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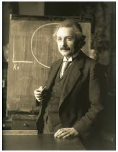

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- On the other hand, string theory suggests us to put a skew-symmetric two-form gauge potential, $B_{\mu\nu}$, and a scalar dilaton, ϕ, on an equal footing along with the metric,

$$\int \mathrm{d}^D x \, \sqrt{-g} e^{-2\phi} \left(R_g + 4\partial_\mu\phi\partial^\mu\phi - \tfrac{1}{12} H_{\lambda\mu\nu} H^{\lambda\mu\nu} \right) \qquad \text{where} \qquad H = \mathrm{d}B \,,$$

  - Forming the massless sector of closed strings, they are ubiquitous in all string theories.
  - Further, a nontrivial symmetry of string theory, called T-duality which forms the group, $\mathbf{O}(D, D)$, transforms them to one another: Namely, $\{g_{\mu\nu}, B_{\mu\nu}, \phi\}$ forms a multiplet of T-duality. Buscher

- **String theory suggests us to view the whole massless sector of closed strings as the gravitational unity, or $\{g_{\mu\nu}, B_{\mu\nu}, \phi\}$ as the gravitational trinity.**
  - Riemannian geometry is for 'particle' theory.
  - 'String' theory requires a novel differential geometry, beyond Riemann.

- The conventional treatment of the closed string massless sector is 'organized' in terms of Riemannian geometry:

$$\int \mathrm{d}^D x \, \sqrt{-g} e^{-2\phi} \left(R_g + 4\partial_\mu\phi\partial^\mu\phi - \tfrac{1}{12} H_{\lambda\mu\nu} H^{\lambda\mu\nu} \right) .$$

  - In this conventional description, the Riemannian metric provides the background geometry, while the dilaton and the B-field are viewed as *'matter'* living on it.
  - The diffeomorphisms and the B-field gauge symmetry are manifest, *e.g.*

$$\delta B_{\mu\nu} = \mathcal{L}_\xi B_{\mu\nu} + \partial_\mu \Lambda_\nu - \partial_\nu \Lambda_\mu \,, \qquad \delta H_{\lambda\mu\nu} = \mathcal{L}_\xi H_{\lambda\mu\nu} \,.$$

    But, $\mathbf{O}(D,D)$ T-duality symmetry mixing the massless sector is secretly hidden.
  - There is also much ambiguity to occur, when we try to couple the closed string massless sector, especially ϕ and $B_{\mu\nu}$, to other matters, or the Standard Model, *e.g.* the choice of the frame, string (Jordan) or Einstein.
- Thus, Riemannian geometry fails to provide the unifying geometric description of the closed string massless sector.
- Recently, through developments of so-called "Double Field Theory" (Siegel, Hull, Zwiebach) and its underlying stringy differential geometry beyond Riemann (Imtak Jeon, Kanghoon Lee & JHP), Stringy Gravity has been established.
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* **The formalism has been successfully applied to Superstring Theory to construct**
  - *$D=10$ Maximally Supersymmetric Stringy Gravity* Jeon-Lee-JHP-Suh 2012

$$\mathcal{L}=e^{-2d}\Big[\,\tfrac{1}{8}S_0+\tfrac{1}{2}\mathrm{Tr}(\mathcal{F}\bar{\mathcal{F}})+i\bar{\rho}\mathcal{F}\rho'+i\bar{\psi}_{\bar{p}}\gamma_q\mathcal{F}\bar{\gamma}^{\bar{p}}\psi'^q+i\tfrac{1}{2}\bar{\rho}\gamma^p\mathcal{D}_p\rho-i\tfrac{1}{2}\bar{\rho}'\bar{\gamma}^{\bar{p}}\mathcal{D}_{\bar{p}}\rho'$$
$$-i\bar{\psi}^{\bar{p}}\mathcal{D}_{\bar{p}}\rho-i\tfrac{1}{2}\bar{\psi}^{\bar{p}}\gamma^q\mathcal{D}_q\psi_{\bar{p}}+i\bar{\psi}'^p\mathcal{D}_p\rho'+i\tfrac{1}{2}\bar{\psi}'^p\bar{\gamma}^{\bar{q}}\mathcal{D}_{\bar{q}}\psi'_p\,\Big]$$

  - *Doubled-yet-gauged Green-Schwarz superstring* JHP 2016

$$\mathcal{S}=\frac{1}{4\pi\alpha'}\int\mathrm{d}^2\sigma\;\;-\tfrac{1}{2}\sqrt{-h}h^{ij}\Pi_i^M\Pi_j^N\mathcal{H}_{MN}-\epsilon^{ij}D_iX^M\left(\mathcal{A}_{jM}-i\Sigma_{jM}\right)\;,$$

where with $i,j=0,1$, we set $\Pi_i^M:=D_iX^M-i\Sigma_i^M$ and $\Sigma_i^M:=\bar{\theta}\gamma^M\partial_i\theta+\bar{\theta}'\bar{\gamma}^M\partial_i\theta'$.

They enjoy all the desired symmetries :

- **$\mathbf{O}(D,D)$ T-duality**
- **Doubled-yet-gauged diffeomorphisms**
- **twofold Lorentz symmetry, Spin$(1,9)_L\times$ Spin$(9,1)_R$** $\Rightarrow$ **Unification of IIA & IIB**
- **Maximal 16+16 SUSY & kappa symmetries (full order)**
- **Worldsheet diffeomorphisms plus Weyl symmetry**
- **Coordinate gauge symmetry :** $X^M\;\sim\;X^M+\Delta^A,\;\;\Delta^A\partial_A=0$

* **String theory is better formulated on doubled-yet-gauged spacetime.**

- *Coupling to the Standard Model* — Choi-JHP 2015 [PRL]

$D=4$ **Stringy Gravity naturally, or minimally, couples to the Standard Model in particle physics, dictated by Symmetry Principle:**

- **O**$(4,4)$ T-duality
- Twofold local Lorentz symmetry, **Spin**$(1,3)_L \times$ **Spin**$(3,1)_R$
- Doubled-yet-gauged diffeomorphisms
- $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$ gauge symmetry

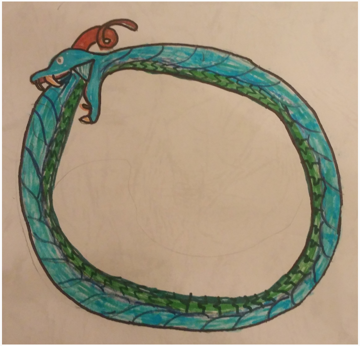
$$\mathcal{L}_{\rm SM} = e^{-2d}\left[\begin{array}{l}\frac{1}{16\pi G_N}S_0\\ +\sum_{\mathcal{V}} P^{AB}\bar{P}^{CD}\mathrm{Tr}(\mathcal{F}_{AC}\mathcal{F}_{BD}) + \sum_{\psi}\bar{\psi}\gamma^{a}\mathcal{D}_{a}\psi + \sum_{\psi'}\bar{\psi}'\bar{\gamma}^{\bar{a}}\mathcal{D}_{\bar{a}}\psi'\\ -\mathcal{H}^{AB}(\mathcal{D}_A\phi)^{\dagger}\mathcal{D}_B\phi - V(\phi) + y_d\,\bar{q}\cdot\phi\,d + y_u\,\bar{q}\cdot\tilde{\phi}\,u + y_e\,\bar{l}'\cdot\phi\,e'\end{array}\right]$$

While coupling to SM, one has to decide the spin group for each fermion, as it a prediction of Stringy Gravity that **the spin group is intrinsically twofold:**

Spin$(1,3)_L$ **vs.** **Spin**$(3,1)_R$.

Conjecture: **the quarks and the leptons may belong to the distinct spin groups.**

Gravitational Implication



Stringy Gravity modifies GR at 'short' distance in terms of x $= R/(MG)$, and may solve the DM/DE problems in Uroboros manner.

Sungmoon Ko, JHP & Minwoo Suh 1606.09307 [JCAP]

- *Partial darkness of Stringy Gravity*

i) Point-like particles couple to the string metric only,

$$\int \mathrm{d}\tau \left[e^{-1} D_\tau x^A D_\tau x^B \mathcal{H}_{AB}(x) - \tfrac{1}{4} m^2 e \right] \quad \Longrightarrow \quad \int \mathrm{d}\tau \; - m\sqrt{-\dot{x}^\mu \dot{x}^\nu g_{\mu\nu}}\,.$$

ii) Each SM fermion couples to Stringy Gravity as

$$e^{-2d}\,\bar{\psi}\gamma^A \mathcal{D}_A \psi \equiv \frac{1}{\sqrt{2}}\sqrt{-g}\,\bar{\chi}\gamma^\mu \left(\partial_\mu \chi + \frac{1}{4}\omega_{\mu pq}\gamma^{pq}\chi + \frac{1}{24}H_{\mu pq}\gamma^{pq}\chi\right)$$

where $\chi \equiv e^{-\phi}\psi$. This field redefinition removes the string dilaton, ϕ, completely.

– Like F1, χ can source the H-flux, and seems to remember it stringy origin!

iii) Each SM gauge boson couples to Stringy Gravity as

$$e^{-2d}\,\mathrm{Tr}\left(P^{AB}\bar{P}^{CD}\mathcal{F}_{AC}\mathcal{F}_{BD}\right) \equiv -\tfrac{1}{4}\sqrt{-g}e^{-2\phi}\,\mathrm{Tr}\left(g^{\kappa\lambda}g^{\mu\nu}F_{\kappa\mu}F_{\lambda\nu}\right)$$

- While ϕ and B-field are 'dark' to point particles, the self-interaction of the massless closed string sector, together with its coupling to the Standard Model, should let Stringy Gravity modify General Relativity.

  This motivates us to look for the most general spherically symmetric vacua of Stringy Gravity, especially $D = 4$, in analogy with the Schwarzschild solution in GR.

- *Asymptotically flat spherical vacuum:*

$$e^{2\phi} = \gamma_+ \left(\frac{r-\alpha}{r+\beta}\right)^{\frac{b}{\sqrt{a^2+b^2}}} + \gamma_- \left(\frac{r-\alpha}{r+\beta}\right)^{\frac{-b}{\sqrt{a^2+b^2}}}, \qquad B_{(2)} = h\cos\vartheta \, \mathrm{d}t \wedge \mathrm{d}\varphi\,,$$

$$\mathrm{d}s^2 = e^{2\phi}\left[-\left(\frac{r-\alpha}{r+\beta}\right)^{\frac{a}{\sqrt{a^2+b^2}}}\mathrm{d}t^2 + \left(\frac{r-\alpha}{r+\beta}\right)^{\frac{-a}{\sqrt{a^2+b^2}}}\left(\mathrm{d}r^2 + (r-\alpha)(r+\beta)\mathrm{d}\Omega^2\right)\right],$$

where a, b, h ($h^2 \leq b^2$) are three free parameters and

$$\alpha = \frac{a}{a+b}\sqrt{a^2+b^2}\,, \qquad \beta = \frac{b}{a+b}\sqrt{a^2+b^2}\,, \qquad \gamma_\pm = \frac{1}{2}\left(1 \pm \sqrt{1 - h^2/b^2}\right).$$

In particular, the special case of $b = h = 0$ corresponds to the Schwarzschild geometry.

– This is a rederivation of the solution by Burgess-Myers-Quevedo (1994) who generated the above solution by applying S-duality to the scalar-gravity solution of Fischer (1948), Janis-Newman-Winicour (1968). It solves the familiar action,

$$\int \mathrm{d}^4x\, \sqrt{-|g|}\, e^{-2\phi}\left(R + 4\,|\mathrm{d}\phi|^2 - \frac{1}{12}\,|\mathrm{d}B|^2\right).$$

– Equivalently, it solves the EOMs of $D = 4$ pure Stringy Gravity: the “stringy Einstein” curvatuture vanishes.

Thus, within the framework of Stringy Gravity, it should be identified as **the vacuum solution**, in analogy with the Schwarzschild solution in GR.

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$$\mathrm{d}s^2 = e^{2\phi}\left[-\left(\frac{r-\alpha}{r+\beta}\right)^{\frac{a}{\sqrt{a^2+b^2}}} \mathrm{d}t^2 + \left(\frac{r-\alpha}{r+\beta}\right)^{\frac{-a}{\sqrt{a^2+b^2}}} \left(\mathrm{d}r^2 + (r-\alpha)(r+\beta)\mathrm{d}\Omega^2\right)\right],$$

where a, b, h ($h^2 \leq b^2$) are three free parameters and

$$\alpha = \frac{a}{a+b}\sqrt{a^2+b^2}\,, \qquad \beta = \frac{b}{a+b}\sqrt{a^2+b^2}\,, \qquad \gamma_\pm = \frac{1}{2}\left(1 \pm \sqrt{1 - h^2/b^2}\right).$$

In particular, the special case of $b = h = 0$ corresponds to the Schwarzschild geometry.

- This is a rederivation of the solution by Burgess-Myers-Quevedo (1994) who generated the above solution by applying S-duality to the scalar-gravity solution of Fischer (1948), Janis-Newman-Winicour (1968). It solves the familiar action,

$$\int \mathrm{d}^4x\, \sqrt{-|g|}\, e^{-2\phi}\left(R + 4\,|\mathrm{d}\phi|^2 - \tfrac{1}{12}\,|\mathrm{d}B|^2\right).$$

- Equivalently, it solves the EOMs of $D = 4$ pure Stringy Gravity: the "stringy Einstein" curvatuture vanishes.

  Thus, within the framework of Stringy Gravity, it should be identified as **the vacuum solution**, in analogy with the Schwarzschild solution in GR.

- *Orbital velocity*

Given the exact spherical solution, we define 'proper' radius, $R := \sqrt{g_{\vartheta\vartheta}(r)}$, which converts the string metric into a canonical form,

$$\mathrm{d}s^2 = g_{tt}\mathrm{d}t^2 + g_{RR}\mathrm{d}R^2 + R^2\left(\mathrm{d}\vartheta^2 + \sin^2\vartheta\,\mathrm{d}\varphi^2\right).$$

We then compute the 'orbital velocity' of circular geodesics,

$$V_{\rm orbit} = \left|R\frac{\mathrm{d}\varphi}{\mathrm{d}t}\right| = \left[-\tfrac{1}{2}R\frac{\mathrm{d}g_{tt}}{\mathrm{d}R}\right]^{\frac{1}{2}},$$

as a function of $R/(MG)$ which is a dimensionless radial variable normalized by 'asymptotic' mass (Komar mass[1]),

$$MG := \lim_{R\to\infty}(RV^2_{\rm orbit}) = \tfrac{1}{2}\left(a + b\sqrt{1-h^2/b^2}\right).$$

⋆ **Stringy Gravity reduces to Newton Gravity at spatial infinity,**

$$g_{tt} \;\to\; -1 + \frac{2MG}{R}, \qquad V_{\rm orbit} \;\to\; \sqrt{\frac{MG}{R}} \qquad \textbf{as} \quad R \to \infty.$$

⋆ Yet, **Stringy Gravity modifies GR at 'short' distance, in terms of $R/(MG)$.** Generically ($b \neq 0$), the orbital velocity is not monotonic: it features a maximum.

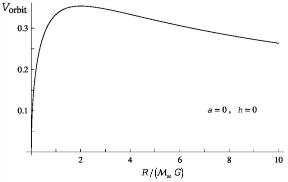
[1] *c.f.* ADM mass *a la* Wald, $\mathcal{Q}[\partial_t] = \frac{1}{4}\left[a + \left(\frac{a-b}{a+b}\right)\sqrt{a^2+b^2}\right]$ JHP-Rey-Rim-Sakatani, Blair 2015

- *Rotation curves*

– Specifically, if $b = 0$ (and hence $h = 0$), the solution reduces to the Schwarzschild metric, resulting in the Keplerian orbital velocity, $V_{\rm orbit} = \sqrt{\frac{MG}{R}}$.

– As long as $b \neq 0$, **rotation curves feature a maximum** and thus non-Keplerian over a finite range, while becoming asymptotically Keplerian at infinity.

For example, if $a = h = 0$ and $b = 2MG$, we reproduce the renowned orbital velocity formula, $V_{\rm orbit} = \sqrt{\frac{MGR}{(R+2MG)^2}}$, by Hernquist :

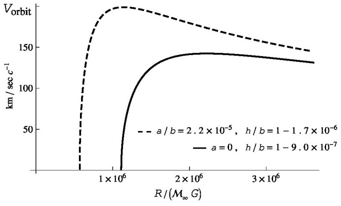


The orbital velocity in Hernquist model assumes its maximum, $\frac{1}{2\sqrt{2}}$, about 35% of the speed of light, at $R = 2MG$.

However, this value seems too high compared to observations of galaxies.

– **More interesting cases turn out to include nontrivial H-flux ($h \neq 0$ and hence $b \neq 0$).**

– By tuning the variable, it is possible to make the maximal velocity arbitrarily small, such as about $150\,\mathrm{km/s}\,c^{-1}$, comparable to observations:



Rotation curves in Stringy Gravity (dimensionless, nonexhaustive).

For sufficiently small $R/(MG)$, the gravitational force can be repulsive.

Rescaling the horizontal axis, $R/(MG) \rightarrow R$, rotation curves oscillate.

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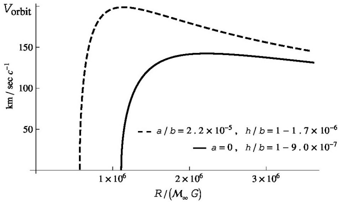

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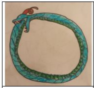
- *Uroboros spectrum of $R/(MG)$*

	Electron ($R \simeq 0$)	Proton	Hydrogen Atom	Billiard Ball	Earth	Solar System ($1\mathrm{AU}/M_\odot G$)	Milky Way (visible)	Galaxy Cluster	Universe ($M \propto R^3$)
$R/(MG)$	0^+	7.1×10^{38}	2.0×10^{43}	2.4×10^{26}	1.4×10^{9}	1.0×10^{8}	1.5×10^{6}	$\sim 10^{5}$	0^+

'Uroboros' spectrum of the dimensionless Radial variable normalized by Mass in natural units. The orbital speed of rotation curves is also dimensionless, and depends on the single variable, $R/(MG)$.

– The observations of stars and galaxies far away, or the dark matter and the dark energy problems, are revealing the short-distance nature of gravity!

– The repulsive gravitational force at very short-distance, $R/(MG) \to 0^+$, may be responsible for the acceleration of the Universe.

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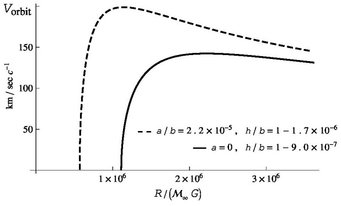

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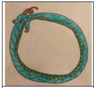	Electron ($R \simeq 0$)	Proton	Hydrogen Atom	Billiard Ball	Earth	Solar System ($1\mathrm{AU}/M_\odot G$)	Milky Way (visible)	Galaxy Cluster	Universe ($M \propto R^3$)
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Thank you.

This talk is based on works in collaborations with Imtak Jeon, Kanghoon Lee, Soo-Jong Rey, Yuho Sakatani, Yoonji Suh, Wonyoung Cho, Jose Fernández-Melgarejo, Woohyun Rim, Sung Moon Ko, Charles Melby-Thompson, Rene Meyér, Minwoo Suh, Kang-Sin Choi, Chris Blair, Emanuel Malek and Xavier Bekaert.

- *Differential geometry with a projection: Application to double field theory* — 1011.1324 JHEP
- **Stringy differential geometry, beyond Riemann** — **1105.6294 PRD**
- *Incorporation of fermions into double field theory* — 1109.2035 JHEP
- *Ramond-Ramond Cohomology and O(D,D) T-duality* — 1206.3478 JHEP
- *Supersymmetric Double Field Theory: Stringy Reformulation of Supergravity* — 1112.0069 PRD
- **Stringy Unification of IIA and IIB Supergravities under $\mathcal{N}=2$ $D=10$ Supersymmetric Double Field Theory** — **1210.5078 PLB**
- *Supersymmetric gauged Double Field Theory: Systematic derivation by virtue of 'Twist'* — 1505.01301 JHEP
- **Comments on double field theory and diffeomorphisms** — **1304.5946 JHEP**
- *Covariant action for a string in doubled yet gauged spacetime* — 1307.8377 NPB
- **Green-Schwarz superstring on doubled-yet-gauged spacetime** — **1609.04265 JHEP**
- *Double field formulation of Yang-Mills theory* — 1102.0419 PLB
- **Standard Model as a Double Field Theory** — **1506.05277 PRL**
- **The rotation curve of a point particle in stringy gravity** — **1606.09307 JCAP**
- **O**(D,D) *Covariant Noether Currents and Global Charges in Double Field Theory* — 1507.07545 JHEP
- *Dynamics of Perturbations in Double Field Theory & Non-Relativistic String Theory* — 1508.01121 JHEP
- *Higher Spin Double Field Theory: A Proposal* — 1605.00403 JHEP
- *U-geometry: SL(5)* $\Rightarrow$ **U-gravity: SL(N)** — 1302.1652 JHEP/**1402.5027 JHEP**
- *M-theory and Type IIB from a Duality Manifest Action* — 1311.5109 JHEP