Duality Manifest Approach for Double Copy

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KL 1807.08443

Cho, KL 1904.11650

Kim, KL, Monteiro, Nicholson, Veiga 1912.02177

Kim, KL, Berman 2010.08255

SGC2020 @ APCTP

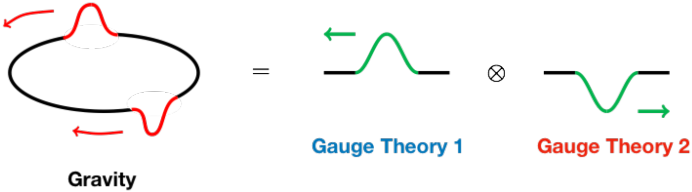
- What does the string theory tell us 'gravity'? : gauge/gravity correspondence
- Open/closed string dualities
    - AdS/CFT correspondence [Maldacena 1997]
    - Double copy [Bern, Carrasco, Johansson 2010,2012]
- Low energy effective theory of string theory:
    - Open string : (super) Yang-Mills theory
    - Closed string : (super) gravity
- An insight into gravity given by string theory, invisible in point particle theory!

- Double copy relation for tree level = KLT relation
- Tree level closed string and open string scattering amplitudes are related via the KLT relation [Kawai, Lewellen, Tye 1986]

$$M_n^{\text{tree}} = A_n^{\text{tree}} \mathcal{K}_n \tilde{A}_n^{\text{tree}}$$

where $\mathcal{K}_n$ is the KLT kernel.

- KLT relation provides the string theory origin of double copy structure.



- Spectrum (in 4D) :

$$\text{graviton}^{\pm 2}(p_i) = \text{gluon}^{\pm 1}(p_i) \otimes \text{gluon}^{\pm 1}(p_i)$$

$$\left.\begin{array}{l}\text{dilaton}\\ \text{axion}\end{array}\right\} = \text{gluon}^{\pm 1}(p_i) \otimes \text{gluon}^{\mp 1}(p_i)$$

- The double copy has the potential to be a new way of quantum gravity

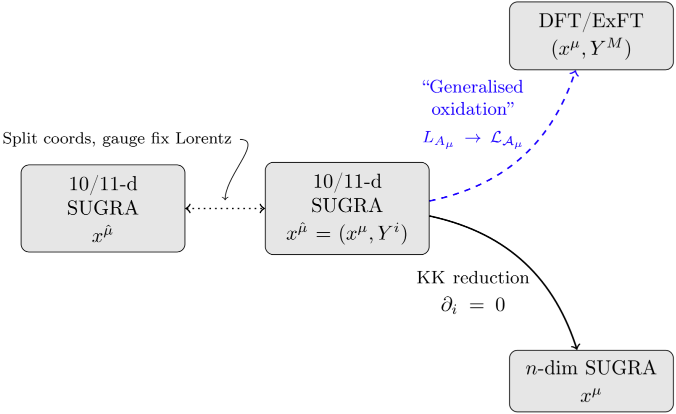
(perturbative) quantum gravity = (Yang-Mills)^2

- After KK reduction, the theories enjoys various global symmetries

  $\rightarrow$ String/M-theory duality [Hull, Townsend 1994]

- Duality groups for GR, half-maximal and maximal SUGRA

G	H	H^*
$GL(d)$	$SO(d)$	$SO(1,d-1)$
$O(d,d)$	$O(d) \times O(d)$	$O(1,d-1) \times O(1,d-1)$
$SL(5)$	$SO(5)$	$SO(2,3)$
$Spin(5,5)$	$Spin(5) \times Spin(5)$	$SO(5,\mathbb{C})$
$E_{6(6)}$	$USp(8)$	$USp(4,4)$
$E_{7(7)}$	$SU(8)$	$SU^*(8)$
$E_{8(8)}$	$SO(16)$	$SO^*(16)$



from the review article by Berman, Blair, 2020

- Tree level scattering amplitude $\longrightarrow$ on-shell, no quantum effects.

  Its extension to the level of the classical equations of motion?
- Q: Can solutions of the Einstein field equations be represented by solutions of the Yang-Mills equations beyond perturbative level?

$$\text{Solution of GR} \underbrace{\Longleftrightarrow}_{?} \text{Solution of YM}$$

- Graviton $h_{\mu\nu}$ is given by the linearized perturbation of the metric

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

  Recall the spectrum relation. Is it possible to represent $h_{\mu\nu} \sim A_\mu \tilde{A}_\nu$?

- One way is the so called classical double copy based on Kerr-Schild formalism in GR [Monteiro, O'Connell, White, 2014]
- The Kerr-Schild ansatz is an extension of linear perturbation around a background metric $\tilde{g}$.
- Einstein equation is nonlinear PDE $\Longrightarrow$ Hard to solve
- What is the condition

Einstein equation becomes linear?

- Kerr and Schild proposed a metric ansatz which makes Einstein equation a linear equation [Kerr 1963], [Kerr, Schild 1965] .
- Meyers-Perry BH, (A)dS Kerr, (A)dS Kerr-Newman, Black string, branes, Waves in flat and (A)dS spaces (PP-wave, Kundt wave, Shock wave) etc.

- Kerr-Schild ansatz

$$g_{\mu\nu} = \tilde{g}_{\mu\nu} + \kappa\varphi\ell_\mu\ell_\nu$$

$\tilde{g}_{\mu\nu}$: a background metric satisfying Einstein equation

ℓ_μ : null vector

$$\ell_\mu\tilde{g}^{\mu\nu}\ell_\nu = \ell_\mu g^{\mu\nu}\ell_\nu = 0$$

- The main advantage of the Kerr-Schild ansatz is that it preserves some features of the linearized perturbation

$$g^{\mu\nu} = \tilde{g}^{\mu\nu} - \kappa\varphi\ell^\mu\ell^\nu\,, \qquad \det(g) = \det(\tilde{g})$$

- Suppose a vacuum Einstein equation, $R_{\mu\nu} = 0$. We get an on-shell constraint from $R_{\mu\nu}\ell^\mu\ell^\nu = 0 \Longleftrightarrow$ geodesic equation $\ell^\mu\nabla_\mu\ell^\nu = 0$

- Using the null and geodesic condition of ℓ, the Einstein equation reduces to a linear equation

$$R_{\mu\nu} = \kappa\tilde{\nabla}_\rho\Big(\tilde{\nabla}_{(\mu}\big(\varphi\ell_{\nu)}\ell^\rho\big) - \frac{1}{2}\tilde{\nabla}^\rho\big(\varphi\ell_\mu\ell_\nu\big)\Big)$$

- Consider KS ansatz on a flat background, $\tilde{g} = \eta$

$$g_{\mu\nu} = \eta_{\mu\nu} + \varphi \ell_\mu \ell_\nu$$

- Identify the null vector ℓ and φ with gauge field and the biadjoint scala field [Monteiro, O'Connell, White, 2014]

$$A_\mu = \varphi \ell_\mu$$

- Assume that the KS spacetime is stationary (no time dependence) and choose ℓ^μ as $\ell^0 = 1$

$$R_{00} = \frac{1}{2} \nabla^2 \varphi$$
$$R_{0i} = \frac{1}{2} \partial^j \big(\partial_i(\varphi \ell_j) - \partial_j(\varphi \ell_i) \big) = -\frac{1}{2} \partial^j F_{ij}$$

  where $F_{ij} = \partial_i A_j - \partial_j A_i$

- $g_{\mu\nu} = \eta_{\mu\nu} + \varphi^{-1} A_\mu A_\nu$

- Schwarzschild BH in Eddington-Finkelstein coordinate

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{2GM}{r}\ell_\mu \ell_\nu$$

where

$$\ell^\mu = \left(1, \frac{x^i}{r}\right), \quad r^2 = x^i x_i, \quad i = 1 \ldots 3$$

Schwarzschild BH $\sim$ Coulomb potential $\otimes$ Coulomb potential

- Kerr BH in KS coordinate

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{2Mr^3}{r^4 + a^2 z^2} l_\mu l_\nu$$

where

$$\ell_\mu = \left(1, \frac{rx + ay}{r^2 + a^2}, \frac{ry - ax}{r^2 + a^2}, \frac{z}{r}\right), \qquad \frac{x^2 + y^2}{r^2 + a^2} + \frac{z^2}{r^2} = 1$$

Questions

- Some questions will be addressed in this talk:
- How can we include Kalb-Ramond field $B_{\mu\nu}$ and dilaton ϕ in the Kerr-Schild formalism?

$$\square \otimes \square = \square\!\square \oplus \begin{array}{c}\square\\[-4pt]\square\end{array} \oplus \bullet$$

- M-theory generalization?
- Classical double copy for non Kerr-Schild type geometries? $\rightarrow$ not discuss today (relaxing the null condition consistently)
- Kerr-Schild method for DFT and beyond

- DFT has crucial advantages to describe the double copy.
- Double copy $\Longleftrightarrow$ Left-right decomposition of closed string theory
- Generalized metric is represented by the coset

$$\mathcal{H} \to \frac{O(d,d)}{O(d-1,1) \times O(1,1-d)}$$

and this implies there are two local Lorentz groups $\Longrightarrow \{e_\mu{}^m, \bar{e}_\mu{}^{\bar{m}}\}$

- These are related with local Lorentz groups for left-right sectors of closed string theory. [Arkani-Hamed,Kaplan, 2008], [Hohm, 2011]

$$\eta_{\mu\nu} + h_{\mu\nu} \to h_{m\bar{n}}$$

- Cheung and Remmen derived perturbative DFT action (without dilaton and $B_{\mu\nu}$) around an arbitrary curved background from Einstein-Hilbert action by assuming the two local Lorentz groups. [Cheung, Remmen, 2016]

Kerr-Schild ansatz for DFT/ExFT

- Manifest under T-duality $\to O(d,d)$ tensors
- generalized metric $\mathcal{H}_{MN}$: rank-2 $O(d,d)$ tensor and $O(d,d)$ element

$$\mathcal{H}_{MN}\mathcal{J}^{NP}\mathcal{H}_{PQ} = \mathcal{J}_{MQ}$$

where $\mathcal{J}_{MN}$ is the $O(d,d)$ metric

$$\mathcal{J}_{MN} = \begin{pmatrix} 0 & \delta^{\mu}{}_{\nu} \\ \delta_{\mu}{}^{\nu} & 0 \end{pmatrix}$$

- Parametrization in terms of massless NSNS sector fields, $\{g, B, \phi\}$ (with an assumption that g is invertible)

$$\mathcal{H}_{MN} = \begin{pmatrix} g^{\mu\nu} & -g^{\mu\rho}B_{\rho\nu} \\ B_{\mu\rho}g^{\rho\nu} & g_{\mu\nu} - B_{\mu\rho}g^{\rho\sigma}B_{\sigma\nu} \end{pmatrix}$$

- DFT scalar d : $O(d,d)$ scalar

$$e^{-2d} = \sqrt{-g}e^{-2\phi}$$

- Generalized metric $\mathcal{H}^{M}{}_{N}$ induces chirality:

$$P^{M}{}_{N} = \frac{1}{2}(\delta^{M}{}_{N} + \mathcal{H}^{M}{}_{N})\,, \qquad \bar{P}^{M}{}_{N} = \frac{1}{2}(\delta^{M}{}_{N} - \mathcal{H}^{M}{}_{N})\,,$$

- Chirality = double local Lorentz groups

$$O(1,d-1) \times O(1,d-1)$$

- The chirality represents the left and right moving sectors.

$$\text{Supergravity basis}: (x^{\mu},0), (0,\tilde{x}_{\mu})$$

$$\text{double-copy basis}: \frac{1}{2}\,(x^{\mu}+\tilde{x}^{\mu}, x_{\mu}+\tilde{x}_{\mu})\,, \frac{1}{2}\,(x^{\mu}-\tilde{x}^{\mu}, \tilde{x}_{\mu}-x_{\mu})$$

- We introduce an ansatz for the generalized metric

$$\mathcal{H}_{MN} = \mathcal{H}_{0MN} + \kappa\varphi\big(K_M\bar{K}_N + \bar{K}_M K_N\big)$$

  Recall that KS ansatz in GR is given by $g_{\mu\nu} = \eta_{\mu\nu} + \varphi l_\mu l_\nu$

- K and $\bar{K}$ are null vectors satisfying the chirality conditions

$$K_M K^M = 0\,, \qquad \bar{K}_M \bar{K}^M = 0\,,$$

$$P_{0MN}K^N = K_M\,, \qquad \bar{P}_{0MN}\bar{K}^N = \bar{K}_M\,, \qquad K_M\bar{K}^M = 0\,,$$

- We refer this form as Kerr-Schild ansatz for the generalized metric. This ansatz satisfies the $O(d,d)$ constraint automatically without any approximation or truncation.

- Chirality condition $\Longrightarrow$ the K_M and $\bar{K}_M$ are parametrized in terms of the d-dimensional vectors l^μ and $\bar{l}^\mu$

$$K_M = \frac{1}{\sqrt{2}} \begin{pmatrix} l^\mu \\ l_\mu \end{pmatrix} , \qquad \bar{K}_M = \frac{1}{\sqrt{2}} \begin{pmatrix} \bar{l}^\mu \\ -\bar{l}_\mu \end{pmatrix} .$$

- Null condition $\Longrightarrow$ l and $\bar{l}$ are null vectors

$$l^\mu \tilde{g}_{\mu\nu} l^\nu = l^\mu l_\mu = 0 \,, \qquad \bar{l}^\mu \tilde{g}_{\mu\nu} \bar{l}^\nu = \bar{l}^\mu \bar{l}_\mu = 0 \,, \qquad l \cdot \bar{l} \neq 0$$

- More than one pair of null vectors?
- It is strictly forbidden in the Lorentzian signature metric! (Theory of quadratic form)

- Using the parametrization of generalized metric, we have

$$(g^{-1})^{\mu\nu} = (\tilde{g}^{-1})^{\mu\nu} + \kappa\varphi l^{(\mu}\bar{l}^{\nu)} \,,$$

$$g_{\mu\nu} = \tilde{g}_{\mu\nu} - \frac{\kappa\varphi}{1+\frac{1}{2}\kappa\varphi(l\cdot\bar{l})} l_{(\mu}\bar{l}_{\nu)} \,,$$

$$B_{\mu\nu} = \tilde{B}_{\mu\nu} + \frac{\kappa\varphi}{1+\frac{1}{2}\kappa\varphi(l\cdot\bar{l})} l_{[\mu}\bar{l}_{\nu]} \,,$$

$$\det g = (\det\tilde{g})\left(1+\frac{1}{2}\kappa\varphi(l\cdot\bar{l})\right)^{-2}$$

- Though the generalised metric $\mathcal{H}$ is linear in κ, g and B are nonlinear.
- If we identify l^{μ} and $\bar{l}^{\mu}$ and ignore the B field, then it reduces to the conventional Kerr-Schild ansatz,

$$g^{\mu\nu} = \tilde{g}^{\mu\nu} + \kappa\varphi l^{\mu}l^{\nu} \,, \qquad g_{\mu\nu} = \tilde{g}_{\mu\nu} - \kappa\varphi l_{\mu}l_{\nu} \,.$$

Linearised perturbation of generalised metric

- KS ansatz: linearised perturbation is exact.
- (generalised) metric is realised by cosets G/H

$$\text{GR :} \qquad \phi_{ij} \in \frac{GL(d)}{O(d)}$$

$$\text{Half maximal (DFT) :} \qquad \mathcal{H}_{MN} \in \frac{O(d,d)}{O(d)\times O(d)}$$

$$\text{Maximal (ExFT)} \qquad \mathcal{M}_{MN} \in \frac{E_{d(d)}}{H_d}$$

- Number of components of the generalised metric $>$ d.o.f of supergravity fields

$$\delta\mathcal{H} \neq \{\delta g \,, \delta B\}$$

- We need a projection operator ${P_{MN}}^{PQ}$ to maintain the coset structure

$$\delta\mathcal{M}_{MN} = {P_{MN}}^{PQ}\delta\mathcal{M}_{PQ}$$

- We assume that the generalised coordinate X^M includes the time direction to define a null vector.
- Kerr-Schild ansatz for GR and ExFT

$$\mathcal{M}_{MN} = \mathcal{M}_{0MN} + \kappa\varphi P_{0MN}{}^{PQ} K_P K_Q$$
$$\left(\mathcal{M}^{-1}\right)^{MN} = \left(\mathcal{M}_0^{-1}\right)^{MN} - \kappa\varphi P_{0MN}{}^{PQ} K^P K^Q$$

  and for DFT

$$\mathcal{H}_{MN} = \mathcal{H}_{0MN} + \varphi P_{0MN}{}^{PQ} K_P K_Q$$

- $\mathcal{M}_0$ and $\mathcal{H}_0$ are background generalised metrics.
  The backgrounds do not have to be flat,
- K is null with respect to $\mathcal{M}_0$ for GR and ExFT and $\mathcal{J}$ for DFT

$$K_M \left(\mathcal{M}_0^{-1}\right)^{MN} K_N = 0\,, \qquad K_M \mathcal{J}^{MN} K_N = 0$$

- However, the null condition on K is not sufficient for the linear structure of the inverse metric.
- To make this so we need to impose an additional condition for the fluctuation piece Q_{MN}

$$Q_{MP}Q^{PQ} = 0$$

where

$$Q_{MN} = \varphi P_{0MN}{}^{PQ} K_P K_Q$$

- Dimension of the maximal null space N (Witt index)

  - GR (Lorentzian metric signature) :1 (very powerful)

  - DFT and ExFT cases are bigger than 1 (too big)
- Nilpotency of Q constrains $N \to$ reduced null space $\hat{N}$

- GR case, the projector is trivial, $\left(P^{\mathrm{GR}}\right)_{\mu\nu}{}^{\rho\sigma} = \delta_{\mu}^{(\rho}\delta_{\nu}^{\sigma)}$, thus it reduces to the usual KS ansatz in GR

$$g_{\mu\nu} = \tilde{g}_{\mu\nu} + \kappa\varphi K_{\mu}K_{\nu}$$
$$\left(g^{-1}\right)^{\mu\nu} = \left(\tilde{g}^{-1}\right)^{\mu\nu} - \kappa\varphi K^{\mu}K^{\nu}$$

- DFT projection operator

$$\left(P^{\mathrm{DFT}}\right)_{MN}{}^{PQ} = 2P_{(M}{}^{P}\bar{P}_{N)}{}^{Q}$$

where

$$P_{MN} = \frac{1}{2}\left(\mathcal{J}_{MN} + \mathcal{H}_{MN}\right), \quad \bar{P}_{MN} = \frac{1}{2}\left(\mathcal{J}_{MN} - \mathcal{H}_{MN}\right)$$

- Introduce chiral vectors for a given null vector K_M

$$L_M = P_{0M}{}^N K_N \,, \qquad \bar{L}_M = \bar{P}_{0M}{}^N K_N$$

Until in this stage, L and $\bar{L}$ don't have to be null vectors (L and $\bar{L}$ are orthogonal)

- $Q_{MN} = \varphi \left(L_M \bar{L}_N + L_N \bar{L}_M \right)$
- The nilpotency condition Q can be rephrased by each chiral vectors are null vector
- The D-dimensional null space reduces to two dimensional subspace, chiral and antichiral spaces.
- We can apply this framework to the heterotic DFT and ExFTs

Equations of Motion and Double Copy

- In GR, equations of motion is written in terms of curvature tensor

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0$$

- In Riemannian geometry, Riemann tensor is given by commutator of covariant derivative

$$[\nabla_\mu, \nabla_\nu] V_\rho = R_{\mu\nu\rho\sigma} V^\sigma$$

- DFT covariant derivative and curvature?

- Generalized Lie derivative: Recast the diffeomorphism and one-form gauge transform of $B_{\mu\nu}$ in an $O(D,D)$ covariant way.
- "Semi" covariant derivative with respect to the gen. diffeomorphism [Jeon,KL,Park, 2011]

$$\nabla_M V_N = \partial_M V_N + \Gamma_{MNP} V^P$$

- EoM are given by the generalized curvature tensor and scalar

$$\mathcal{R}_{MN} = 0\,, \qquad \mathcal{R} = 0$$

d-dimensional form:

$\mathcal{R}_{(\mu\nu)} = 0$	EoM for metric
$\mathcal{R}_{[\mu\nu]} = 0$	EoM for $B_{\mu\nu}$
$\mathcal{R} = 0$	EoM for DFT dilaton

- For simplicity consider a flat background,

$$\mathcal{H}_{0\,MN} = \begin{pmatrix} \eta^{\mu\nu} & 0 \\ 0 & \eta_{\mu\nu} \end{pmatrix}, \qquad d_0 = \text{const.}$$

- One may solve the chirality condition

$$K_M = \frac{1}{\sqrt{2}} \begin{pmatrix} l^{\mu} \\ l_{\mu} \end{pmatrix}, \qquad \bar{K}_M = \frac{1}{\sqrt{2}} \begin{pmatrix} \bar{l}^{\mu} \\ -\bar{l}_{\mu} \end{pmatrix}$$

- An on-shell condition from the DFT equations of motion, $K^K \bar{K}^L \mathcal{R}_{KL} = 0$,

$$\bar{K}^M \partial_M K_P = 0\,, \qquad K^M \partial_M \bar{K}_P = 0\,, \qquad K^P \partial_P f = 0\,, \qquad \bar{K}^P \partial_P f = 0\,.$$

Recall that in GR, $R_{\mu\nu}\ell^{\mu}\ell^{\nu} \implies$ geodesic condition, $l^{\mu}\nabla_{\mu}l^{\nu} = l^{\mu}\partial_{\mu}l^{\nu} = 0$

Equations of motion

- In terms of d-dimensional vector indices, the field equations reduces to

$$\Box(\varphi l_\mu \bar{l}_\nu) - \partial^\rho \partial_\mu (\varphi l_\rho \bar{l}_\nu) - \partial^\rho \partial_\nu (\varphi l_\mu \bar{l}_\rho) + \partial_\mu \partial_\nu (\varphi l \cdot \bar{l}) = 0\,.$$

- Note that $\mathcal{R}_{\mu\nu}$ is not symmetric tensor:

  - symmetric part $\rightarrow$ eom of g

  - antisymmetric part $\rightarrow$ eom of B

- It is interesting that the generalized KS ansatz for $g_{\mu\nu}$ and $B_{\mu\nu}$ is not linear in κ, l^μ and $\bar{l}^\mu$, but the field equations are linear in these fields.
- General than the usual KS ansatz in GR
- Curved background generalization is straightforward.

- The KLT and BCJ relations indicate that not only the pure Einstein equation, but also the field equations of entire massless NS-NS sector should be related to the gauge theory.
- Suppose that the full geometry admits at least one Killing vector ξ^{μ}.
- We can locally choose a coordinate system $x^{\mu} = \{x^{i}, y\}$ such that the Killing vector is a constant, $\xi^{\mu} = \partial x^{\mu}/\partial y = \delta^{\mu}_{y}$.

- Classical double copy is achieved by contracting the constant Killing vector ξ^μ with the generalized Ricci tensor

$$\mathcal{R}_{\mu\nu} = \frac{\kappa}{4}\partial^\rho\Big[\,\partial^\rho(\varphi l_\mu \bar{l}_\nu) - \partial_\mu(\varphi l_\rho \bar{l}_\nu) - \partial_\nu(\varphi l_\mu \bar{l}_\rho)\,\Big] = 0$$

- Since $\mathcal{R}_{\mu\nu}$ is not symmetric tensor, we get three independent equations as follows:

$$\xi^\nu \mathcal{R}_{\mu\nu} = \frac{\kappa}{4}\Big[\,\partial^\rho\partial_\rho(\varphi l_\mu) - \partial^\rho\partial_\mu(\varphi l_\rho)\,\Big] = 0\,,$$
$$\xi^\mu \mathcal{R}_{\mu\nu} = \frac{\kappa}{4}\Big[\,\partial^\rho\partial_\rho(\varphi \bar{l}_\nu) - \partial^\rho\partial_\nu(\varphi \bar{l}_\rho)\,\Big] = 0\,,$$

- Identify φl_μ and $\varphi \bar{l}_\mu$ with gauge fields

$$A_\mu = \varphi l_\mu\,, \qquad \bar{A}_\mu = \varphi \bar{l}_\mu$$

- Then $\xi^\nu \mathcal{R}_{\mu\nu}$ and $\xi^\mu \mathcal{R}_{\mu\nu}$ reduce to a pair of Maxwell equations

$$\partial^\mu F_{\mu\nu} = 0\,, \qquad \partial^\mu \bar{F}_{\mu\nu} = 0\,,$$

- This shows that the generalized KS type solution can be written in terms of the solutions of the two independent Maxwell equations.

Extensions

- $O(D,D)$ duality group is enhanced to $O(D,D+n)$, where n is the number of vector multiplet
- Generalised metric is parametrized in terms of the heterotic supergravity fields, g, B, ϕ and A
- Even though the parametrisation of the generalised metric is changed, the form of the action is the same as the usual DFT. [Hohm, Kwak 2011], [Grana, Marques 2012]

- Heterotic supergravity: relaxed null condition [Cho, Lee 2019]

$$\mathcal{H}_{\hat{M}\hat{N}} = \mathcal{H}_{0\hat{M}\hat{N}} + \kappa\varphi\big(K_{\hat{M}}\bar{K}_{\hat{N}} + K_{\hat{N}}\bar{K}_{\hat{M}}\big)\,,$$

In terms of the heterotic supergravity fields

$$\begin{aligned}
g^{\mu\nu} &= \tilde{g}^{\mu\nu} + \kappa\varphi l^{(\mu}\bar{l}^{\nu)}\,,\\
g_{\mu\nu} &= \tilde{g}_{\mu\nu} - \frac{\kappa\varphi}{1+\frac{\kappa\varphi}{2}(l\cdot\bar{l})} l_{(\mu}\bar{l}_{\nu)} + \frac{1}{4}\Big(\frac{\kappa\varphi}{1+\frac{\kappa\varphi}{2}(l\cdot\bar{l})}\Big)^2 (\bar{l}\cdot\bar{l}) l_\mu l_\nu\,,\\
B_{\mu\nu} &= \tilde{B}_{\mu\nu} + \frac{\kappa\varphi}{1+\frac{\kappa\varphi}{2}(l\cdot\bar{l})}\Big(l_{[\mu}\bar{l}_{\nu]} - \sqrt{\tfrac{\alpha'}{2}}\tilde{A}_{[\mu}{}^{\alpha} l_{\nu]} j_\alpha\Big)\,,\\
A_{\mu\alpha} &= \tilde{A}_{\mu\alpha} + \frac{1}{\sqrt{2\alpha'}}\frac{\kappa\varphi}{1+\frac{\kappa\varphi}{2}(l\cdot\bar{l})} l_\mu j_\alpha\,,
\end{aligned}$$

where l is a null vector, but $\bar{l}$ is not.

- It is possible to couple $U(1)$ gauge fields.

- EoM is still linear!
- The KS double copy is given by

$$\text{heteroticSUGRA} = \text{Maxwell} \otimes \text{Maxwell} + \text{scalar}$$

recall that

$$\text{heterotic string} = \text{10d open superstring} \otimes \text{26d open bosonic string on } T^{16}$$

- Not all the geometries are Kerr-Schild geometry. Is it possible relaxing the null condition in KS ansatz? (JNW example)
- New ansatz - partially relaxed KS form in DFT ($\bar{K}_M \bar{K}^M \neq 0$) [Kim, KL, Monteiro, Nicholson, Veiga,2019]

$$\mathcal{H}_{MN} = \mathcal{H}_{0MN} + \varphi\big(K_M \bar{K}_N + K_N \bar{K}_M\big) - \frac{1}{2}\varphi^2 \bar{K}^2 K_M K_N \,,$$

$$d = d_0 + f \,,$$

- We lost all the nice features in KS formalism, but...

$$g_{\mu\nu} = \tilde{g}_{\mu\nu} - \frac{\varphi}{1 + \frac{\varphi}{2}(l \cdot \bar{l})} l_{(\mu} \bar{l}_{\nu)} \,,$$

$$g^{\mu\nu} = \tilde{g}^{\mu\nu} + \varphi l^{(\mu} \bar{l}^{\nu)} + \frac{\varphi^2 \bar{l}^2}{4} l^\mu l^\nu \,,$$

$$B_{\mu\nu} = \tilde{B}_{\mu\nu} + \frac{\varphi}{1 + \frac{\varphi}{2}(l \cdot \bar{l})} l_{[\mu} \bar{l}_{\nu]}$$

- JNW geometry is the most general static spherically symmetric solution in Einstein-dilaton theory

$$\mathrm{d}s^2 = e^{2\phi}\left[-\left(1-\frac{r_0}{r}\right)^{\frac{a}{r_0}}\mathrm{d}t^2 + \left(1-\frac{r_0}{r}\right)^{\frac{-a}{r_0}}\left(\mathrm{d}r^2 + r(r-r_0)\,\mathrm{d}\Omega_2^2\right)\right] ,$$

$$e^{2\phi} = \left(1-\frac{r_0}{r}\right)^{\frac{b}{r_0}} , \qquad r_0 = \sqrt{a^2+b^2} ,$$

- The line element can be transformed from (t,r)-coordinate into (T,R)-coordinate

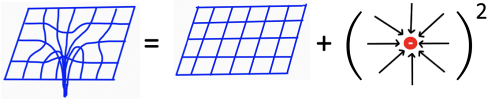
$$\mathrm{d}s^2 = -\mathrm{d}T^2 + \mathrm{d}R^2 + R^2\,\mathrm{d}\Omega_2^2 + V\,l\,\bar{l} ,$$

$$l = \mathrm{d}T + \mathrm{d}R , \qquad \bar{l} = \mathrm{d}T + \Omega(R)\,\mathrm{d}R ,$$

- l and $\bar{l}$ are not standard Kerr-Schild vectors

- Consider JNW case:

$$\mathrm{JNW} \sim (\text{ left-moving Coulomb }) \times (\text{right-moving Coulomb })$$



- Introduce a Killing vector ξ and assume that

$$\xi^\mu l_\mu = 1\,, \qquad \xi^\mu k_{\mu\nu} = 0$$

- Contracting ξ with the EoMs

$$\partial^\sigma \partial_\sigma (\varphi l_\mu) - \partial^\sigma \partial_\mu (\varphi l_\sigma) = 0$$
$$\partial^\lambda \partial_{[\lambda} (\varphi k_{\nu\rho]}) = 0$$

- double copy for M-theory would be

Maxwell theory $\otimes$ 2-form gauge theory

Examples

- Chiral null model (Horowitz,Tseytlin)
- F1-NS5 system
- Charged black string solution
- Charged BH in Einstein-Maxwell-Dilaton theory
- Charged heterotic black string solution
- M2-brane & F1/D1 string

- Double copy and DFT shares the same origin
- We may find DFT for those non-gravitational theories

Double copy	Starting theories	Refs.	Variants and notes
DBI theory	• NLSM • (S)YM theory	[125, 126, 285, 298–301]	• $\mathcal{N} \leq 4$ possible • also obtained as $\alpha' \to 0$ limit of abelian Z-theory
Volkov-Akulov theory	• NLSM • SYM theory (external fermions)	[125, 302–308]	• restriction to external fermions from supersymmetric DBI
Special Galileon theory	• NLSM • NLSM	[125, 285, 301, 306, 309]	• theory is also characterized by its soft limits
DBI + (S)YM theory	• NLSM + ϕ^3 • (S)YM theory	[125, 126, 156, 285, 298–300, 306, 310]	• $\mathcal{N} \leq 4$ possible • also obtained as $\alpha' \to 0$ limit of semi-abelianized Z-theory
DBI + NLSM theory	• NLSM • YM + ϕ^3 theory	[125, 126, 156, 285, 298–300]	

Table 6: List of non-gravitational theories constructed as double copies.

- **Double copy** for classical solutions is possible.
- Perturbative double copy: generic but messy.
- Exact double copy: fully non-linear but not generic.
- Double field theory convenient setting for double copy.
- Exact double copy for M-theory: KS formalism for $SL(5)$ ExFT.

Much more to explore

- Scattering amplitude computation in the DFT language. Extension double copy structure to curved backgrounds.
- Extension to non-Abelian gauge theory?
- Including RR sector, Introducing U(1) gauge fields using Kaluza-Klein reduction, Gauged supergravity extension via Scherk-Schwarz reduction.
- M-theory extension: Other exceptional field theories ($SO(5,5)$, E_6, E_7 and E_8)
- Finding the most general solutions in a flat or curved backgrounds and their physical interpretations. Applications to AdS/CFT?

Thank you