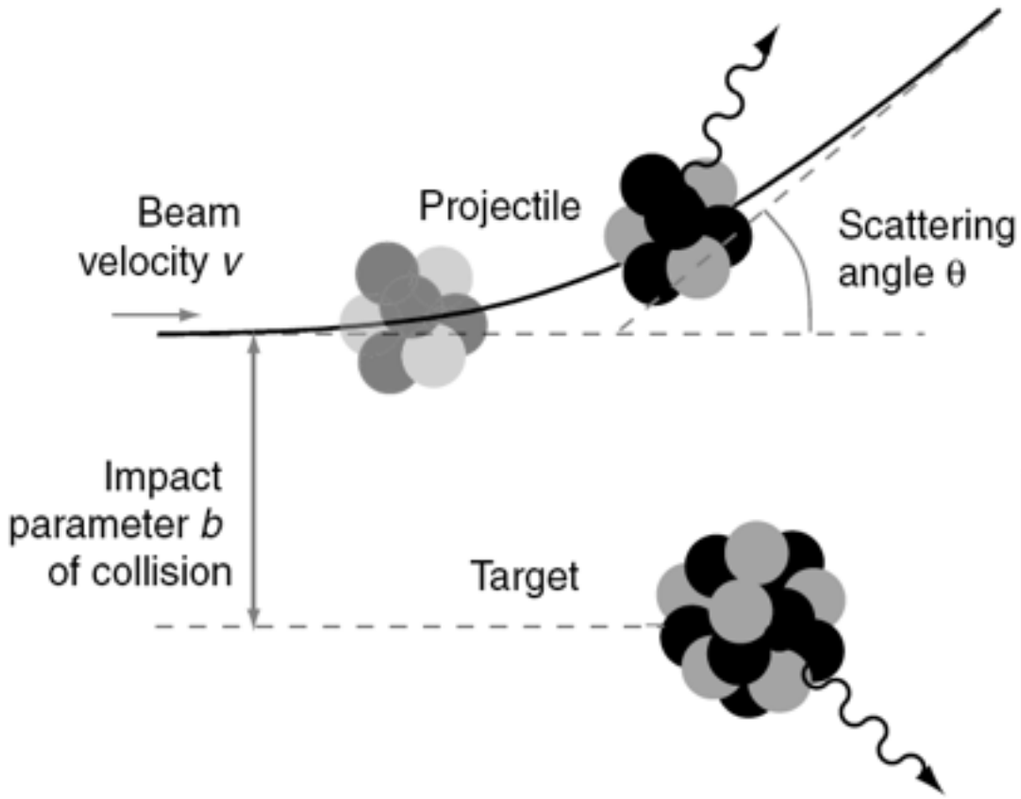
Coupled-channels analyses for ^{9,11}Li + ^{208}Pb fusion reactions with multi-neutron transfer couplings

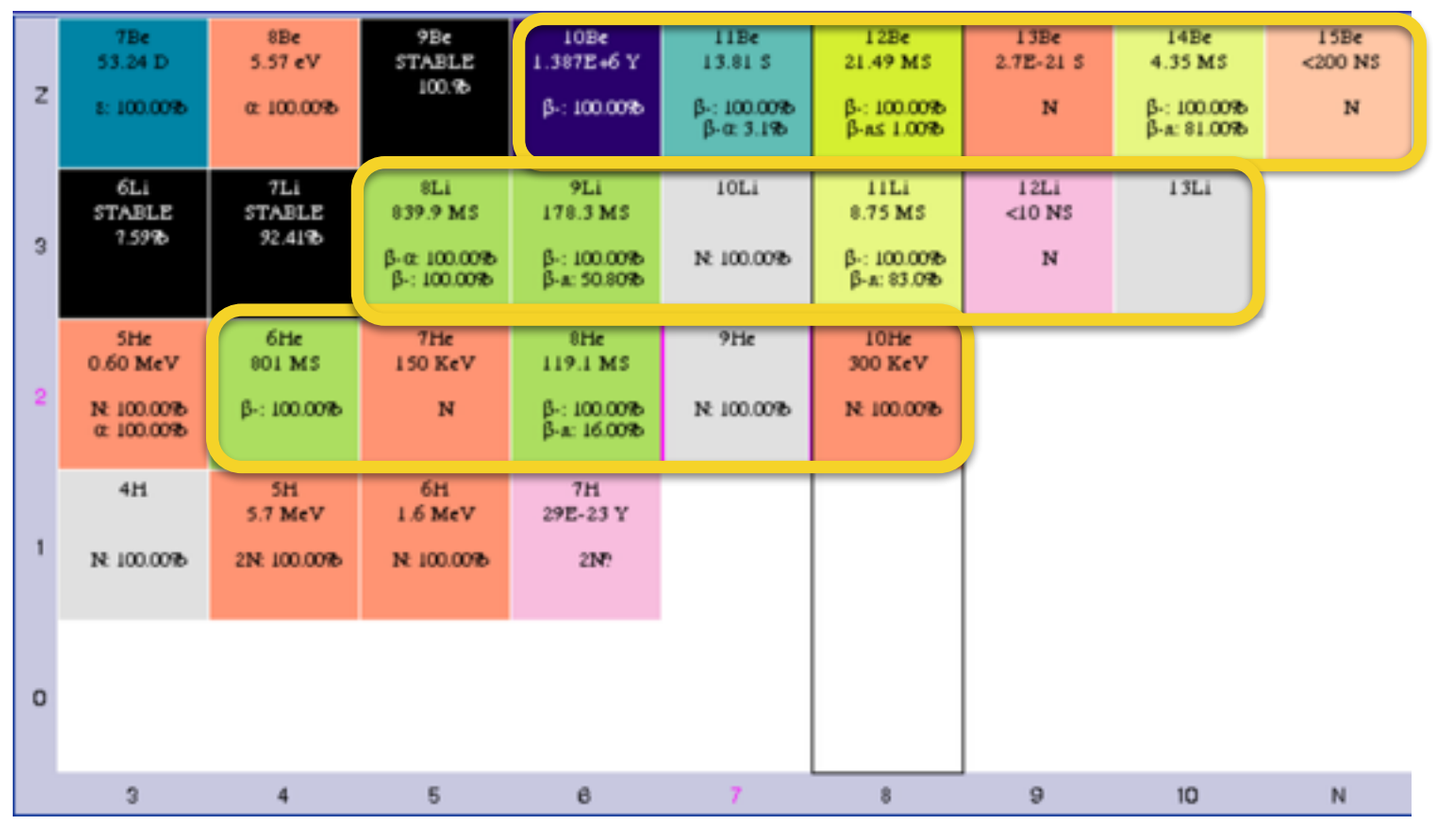
Ki-Seok Choi, Myung-Ki Cheoun, W. Y. So, K.Hagino and K. S. Kim

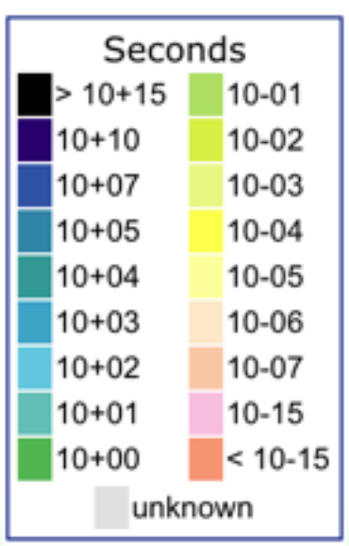
Soongsil University, Korea

Nuclear reaction with unstable nuclei

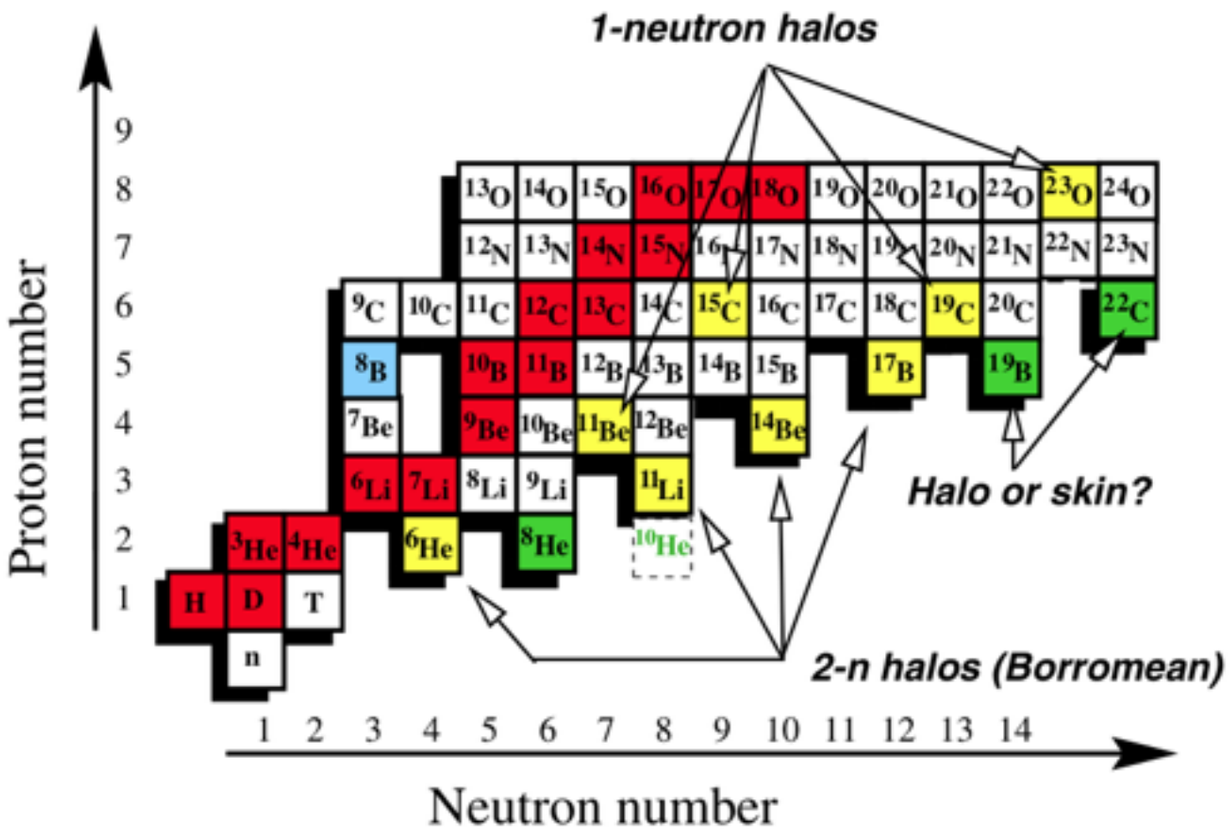


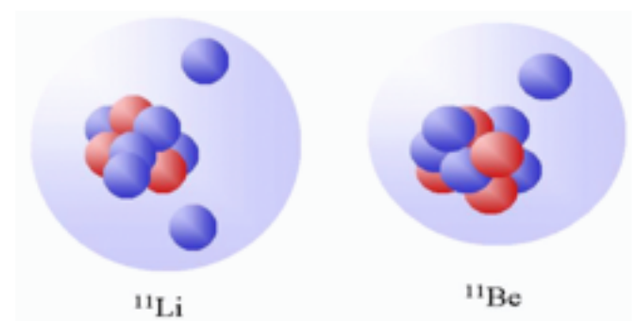
If projectile is unstable?

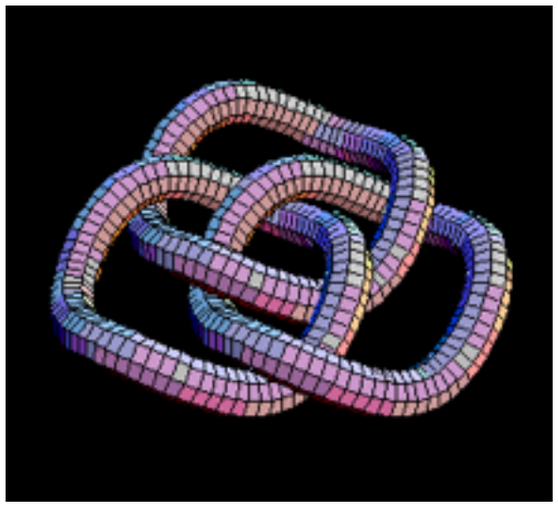




Halo nuclei

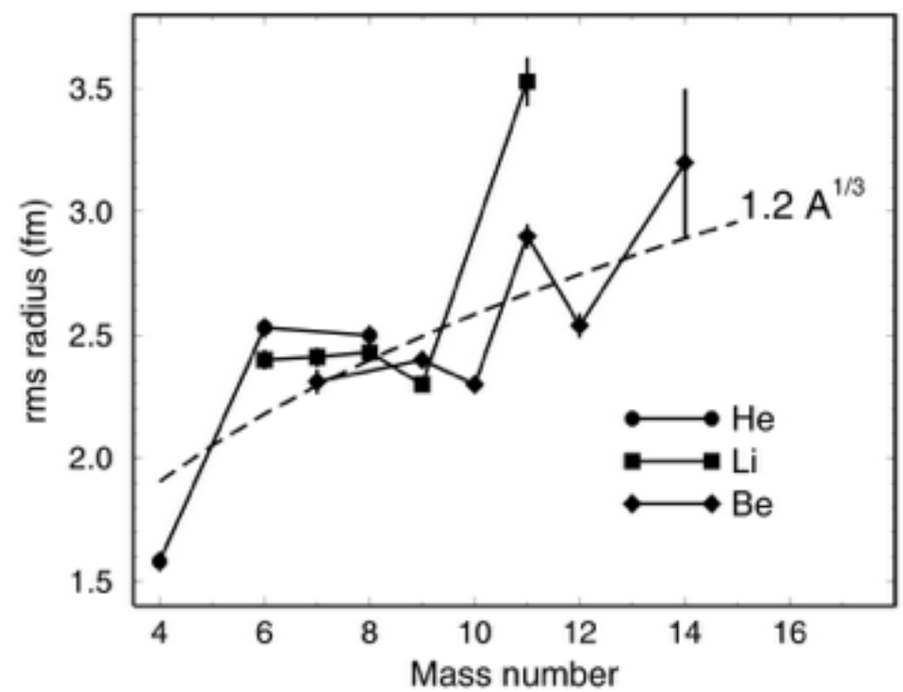






Borromean rings

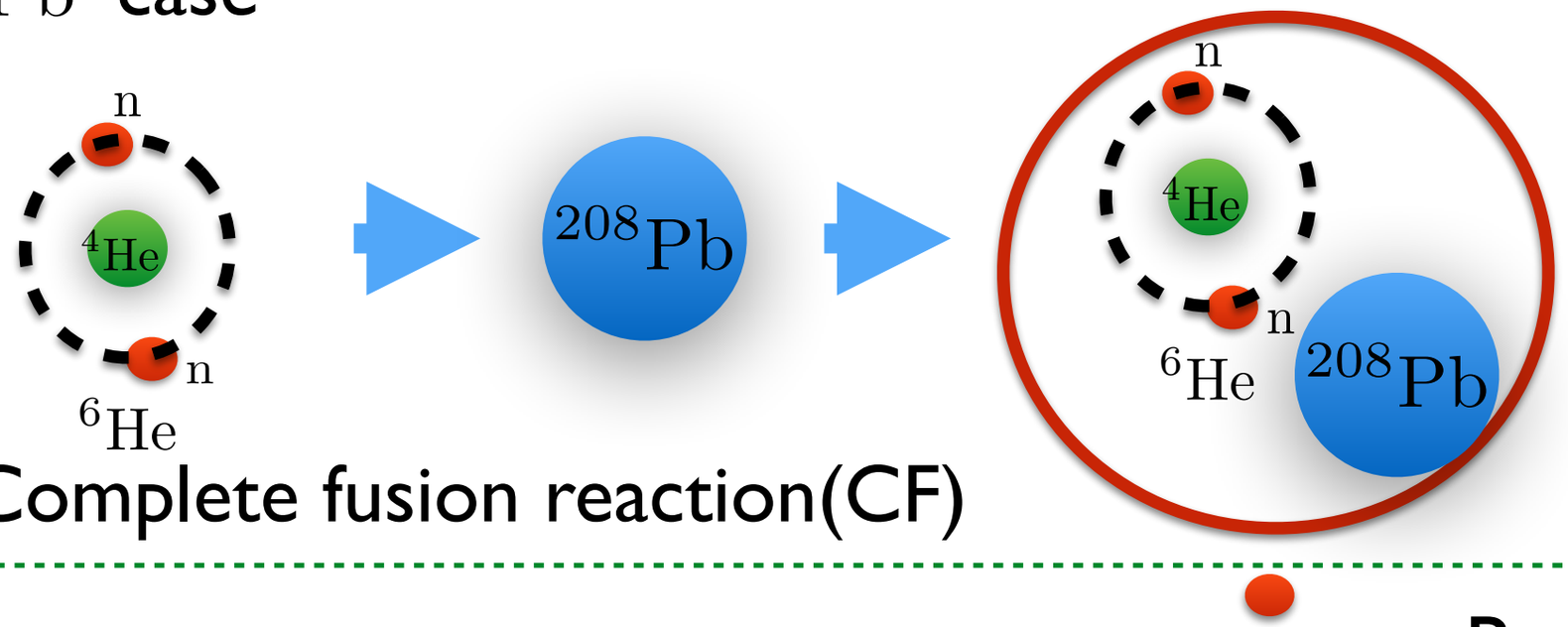
Nucleus	rms matter radii (fm)	
^{4}He	1.57 ± 0.04	1.58 ± 0.04
^{6}He	2.48 ± 0.03	2.71 ± 0.04
^{9}Li	2.32 ± 0.02	2.30 ± 0.02
^{11}Li	3.12 ± 0.16	3.53 ± 0.10
^{12}Be	2.59 ± 0.06	2.54 ± 0.05
^{14}Be	3.16 ± 0.38	3.20 ± 0.30



1. I. Tanihata et al : Phys. Lett . B 206, 592 (1998)
2. J.S. AL-Khalili et al : Phys. Rev. C 54 1843 (1996)

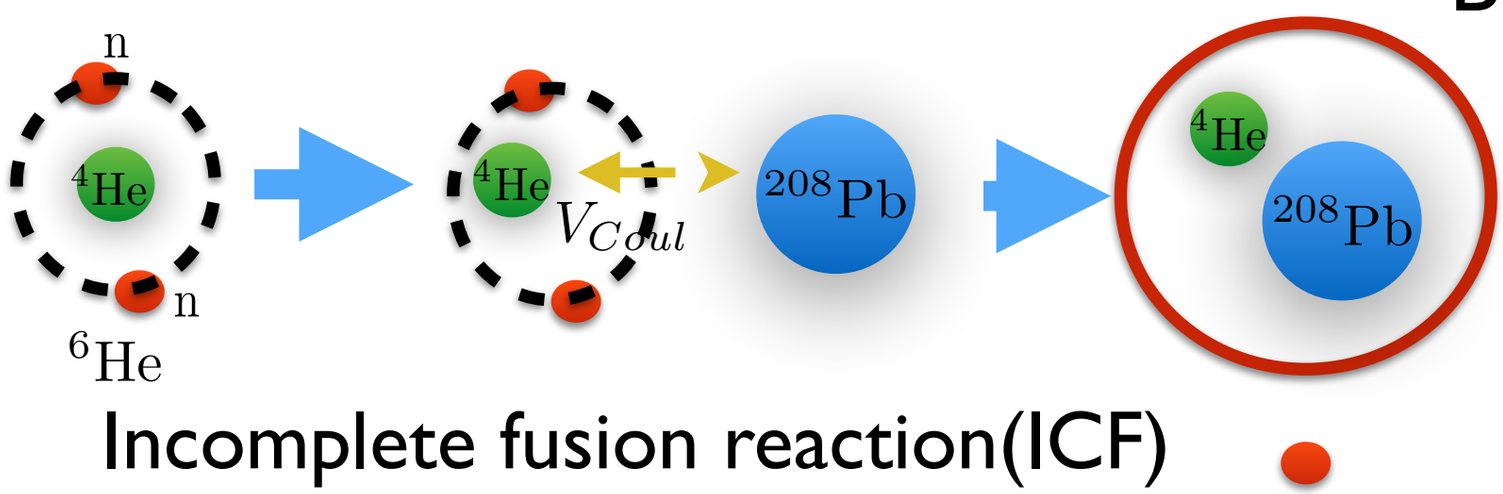
Fusion reaction

$^{6}\mathrm{He} + ^{208}\mathrm{Pb}$ case



Complete fusion reaction(CF)

Break-up



Incomplete fusion reaction(ICF)

Total fusion reaction(TF) = CF+ICF

Wong's Formula

penetrability of parabola

$$P(E) = \frac{1}{1 + \exp\left[\frac{2\pi}{\hbar\Omega}(V_b - E)\right]}$$

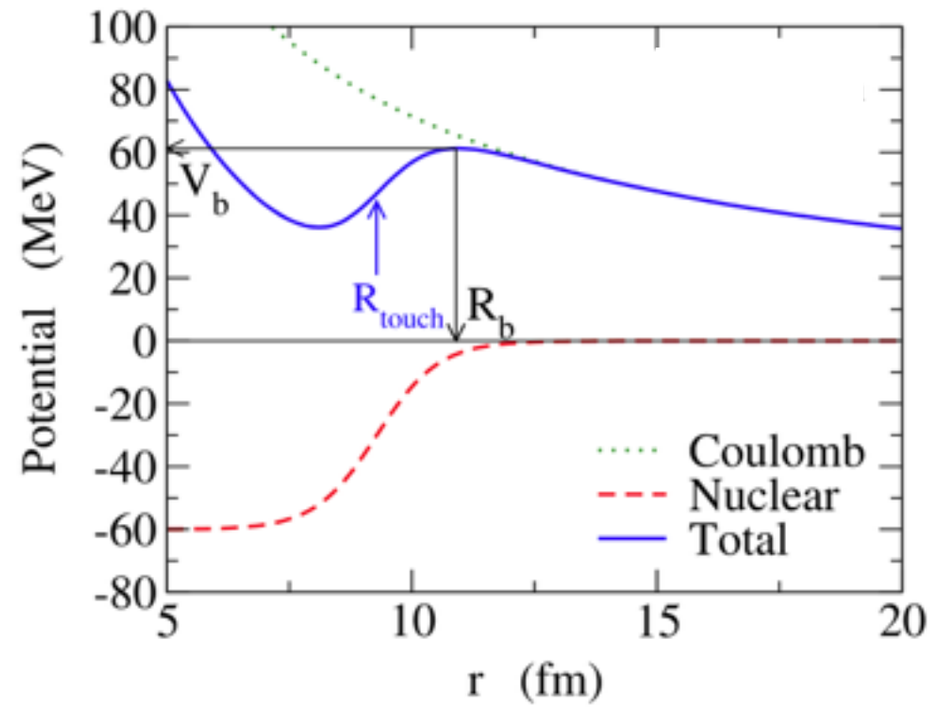
$$\Omega = \sqrt{-\frac{V_N''(R_b) + 2Z_P Z_T e^2/R_b^3}{\mu}}$$:curvature

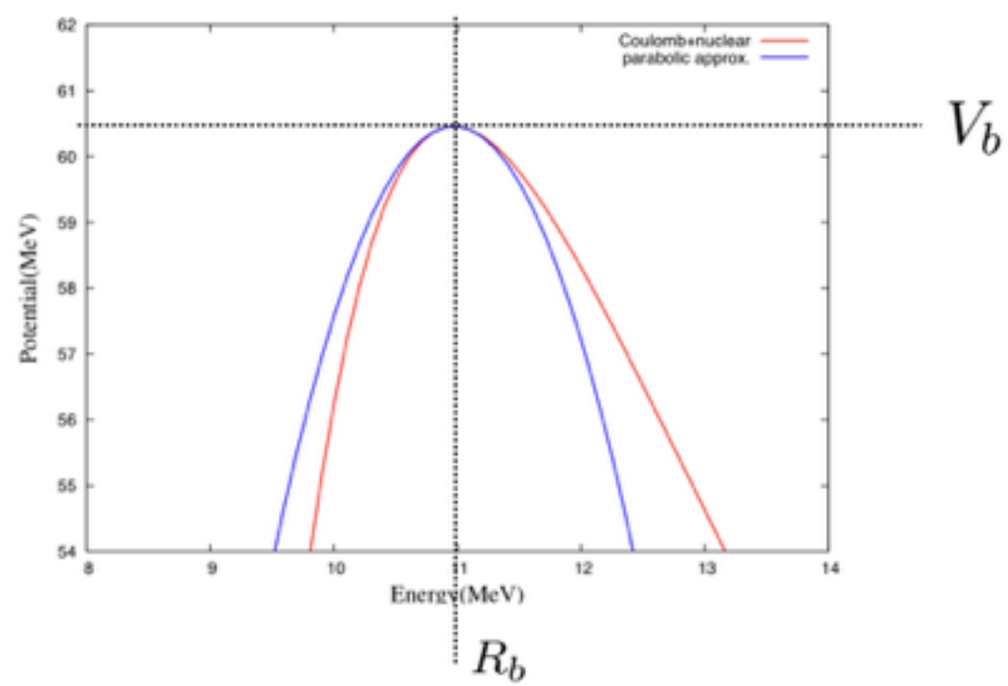
approximated penetrability of parabola with l dependance

$$P_l(E) = P_{l=0}\left(E - \frac{l(l+1)\hbar^2}{2\mu R_b^2}\right)$$

$$\sigma_{\text{fus}}(E) = \frac{\pi}{k^2}\int_0^\infty dl\ (2l+1)P_l(E)$$

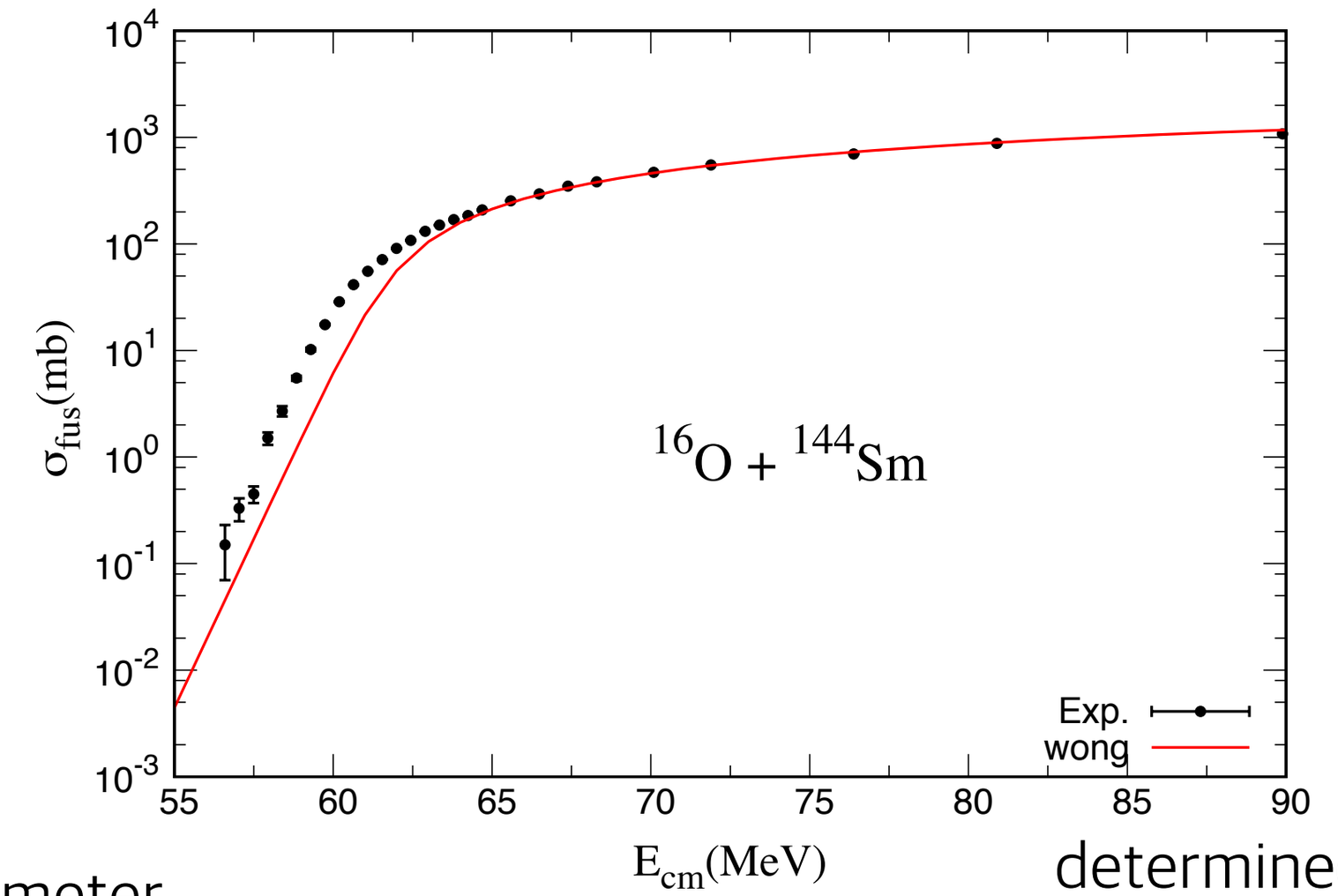
$$\sigma_{\text{fus}}(E) = \frac{\hbar\Omega}{2E}R_b^2 \ln\left[1 + \exp\left(\frac{2\pi}{\hbar\Omega}(E - V_b)\right)\right]$$





C. Y. Wong, Phys. ReV. Lett. 31, 766 (1973)

Example : $^{16}O+^{144}Sm$ fusion reaction



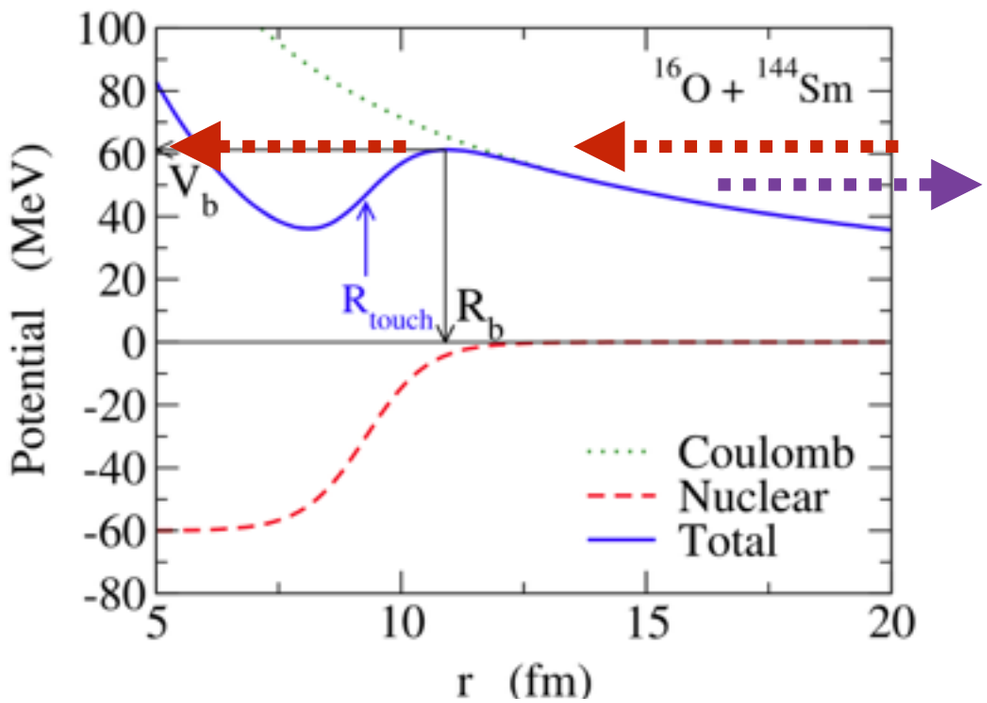
parameter

determined by potential

V_0	r_0	a_0	Ω	r_b	V_b
105.1	1.1	0.75	4.19	10.97	60.46

J.R.Leigh et al : phys. rev. C 52, 3151 (1995)

One-dimensional potential model (Barrier penetration model(BPM))



$$V(r) = V_N(r) + V_C(r)$$

$$V_C(r) = \frac{Z_P Z_T e^2}{r}$$

$$V_N(r) = -\frac{V_0}{1 + \exp[(r - R_0)/a]}$$

: incoming wave boundary condition (IWBC)

$$u_l(r) = \sqrt{\frac{k}{k_l(r)}}\, \mathcal{T}_l \exp\left(-i \int_{r_{\rm abs}}^{r} k_l(r')dr'\right), \qquad r \le r_{\rm abs}$$

$$k_l(r) = \sqrt{\frac{2\mu}{\hbar^2}\left(E - V(r) - \frac{l(l+1)\hbar^2}{2\mu r^2}\right)}.$$

$$H_l^{(-)}(kr) - S_l\, H_l^{(+)}(kr), \qquad r \to \infty$$

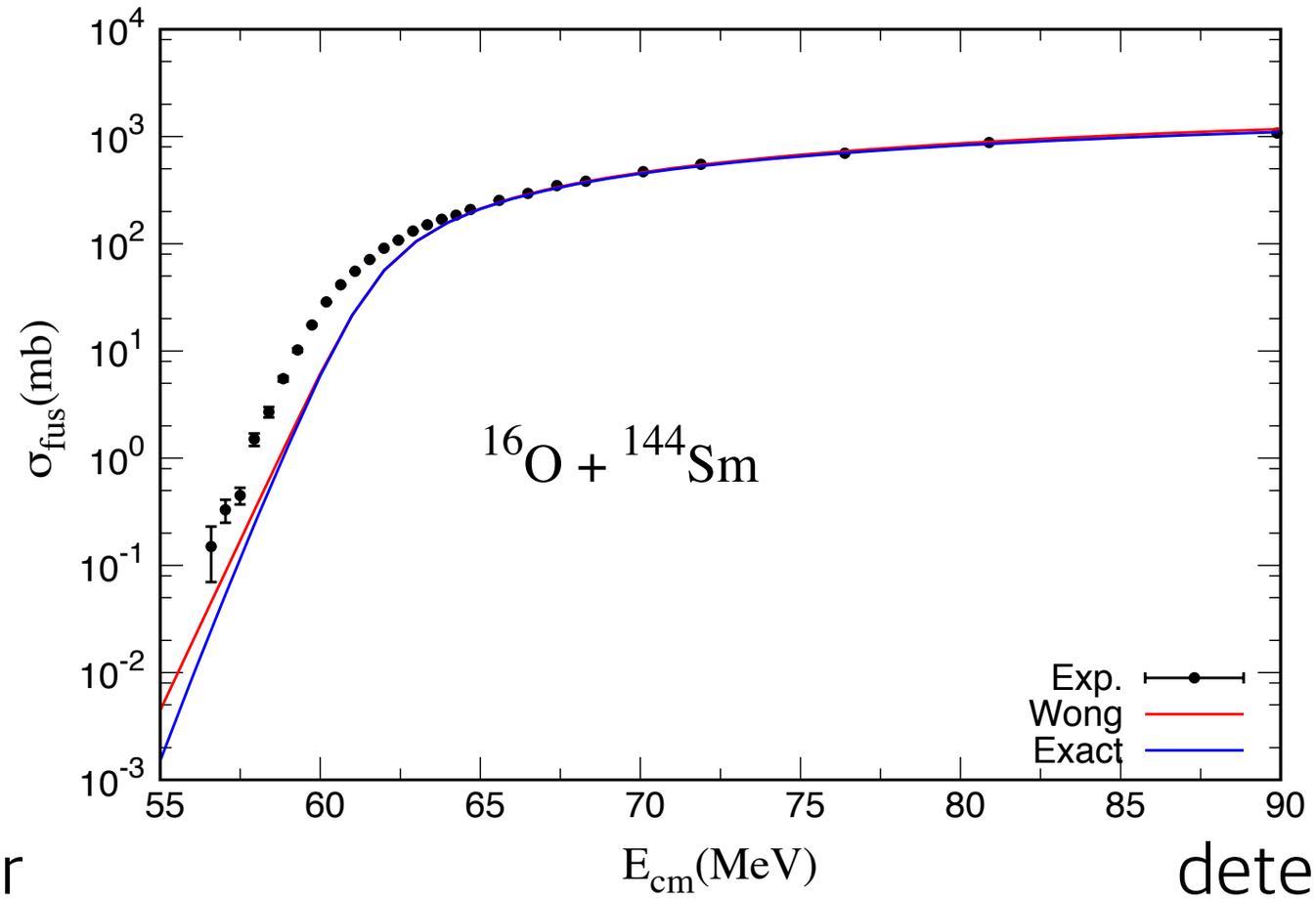
$$k = \sqrt{2\mu E/\hbar^2}$$

$$P_l(E) = 1 - |S_l|^2 = |\mathcal{T}_l|^2$$

$$\sigma_{\rm fus}(E) = \frac{\pi}{k^2}\sum_l (2l+1)P_l(E).$$

One-dimensional potential model

Example : $^{16}O+^{144}Sm$ fusion reaction



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J.R.Leigh et al : phys. rev. C 52, 3151 (1995)

Fusion reaction with Coupled-Channel method in BPM

(Barrier penetration model)

Coupled-channel equations

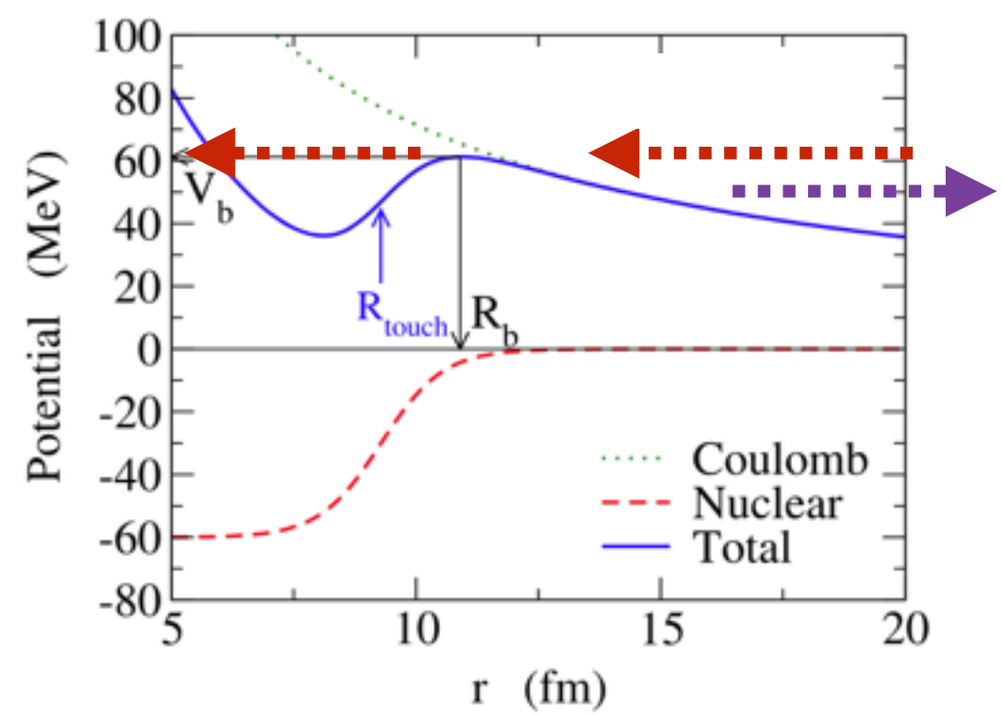
$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2}+\frac{J(J+1)\hbar^2}{2\mu r^2}+V_N^{(0)}(r)+\frac{Z_P Z_T e^2}{r}+\epsilon_n-E\right]\psi_n(r)+\sum_m V_{nm}(r)\psi_m(r)=0,$$

$$V_N^{(0)}(r)=-\frac{V_0}{1+\exp((r-R_0)/a)}, \qquad R_0=r_0(A_P^{1/3}+A_T^{1/3}).$$

$$\psi_n(r)\to T_n\exp\left(-i\int_{r_{\min}}^{r}k_n(r')\,dr'\right), \quad r\leqslant r_{\min},$$

$$\to H_J^{(-)}(k_n r)\delta_{n,0}+R_n H_J^{(+)}(k_n r), \quad r>r_{\max},$$

$$k_n(r)=\sqrt{\frac{2\mu}{\hbar^2}\left(E-\epsilon_n-\frac{J(J+1)\hbar^2}{2\mu r^2}-V_N(r)-\frac{Z_P Z_T e^2}{r}-V_{nn}(r)\right)}$$



K.Hagino et al : CPC 123, 143 (1999)

Fusion reaction with Coupled-Channel method

Vibrational coupling

$$V_N(r,\hat{O}) = -\frac{V_0}{1+\exp((r-R_0-\hat{O})/a)}$$

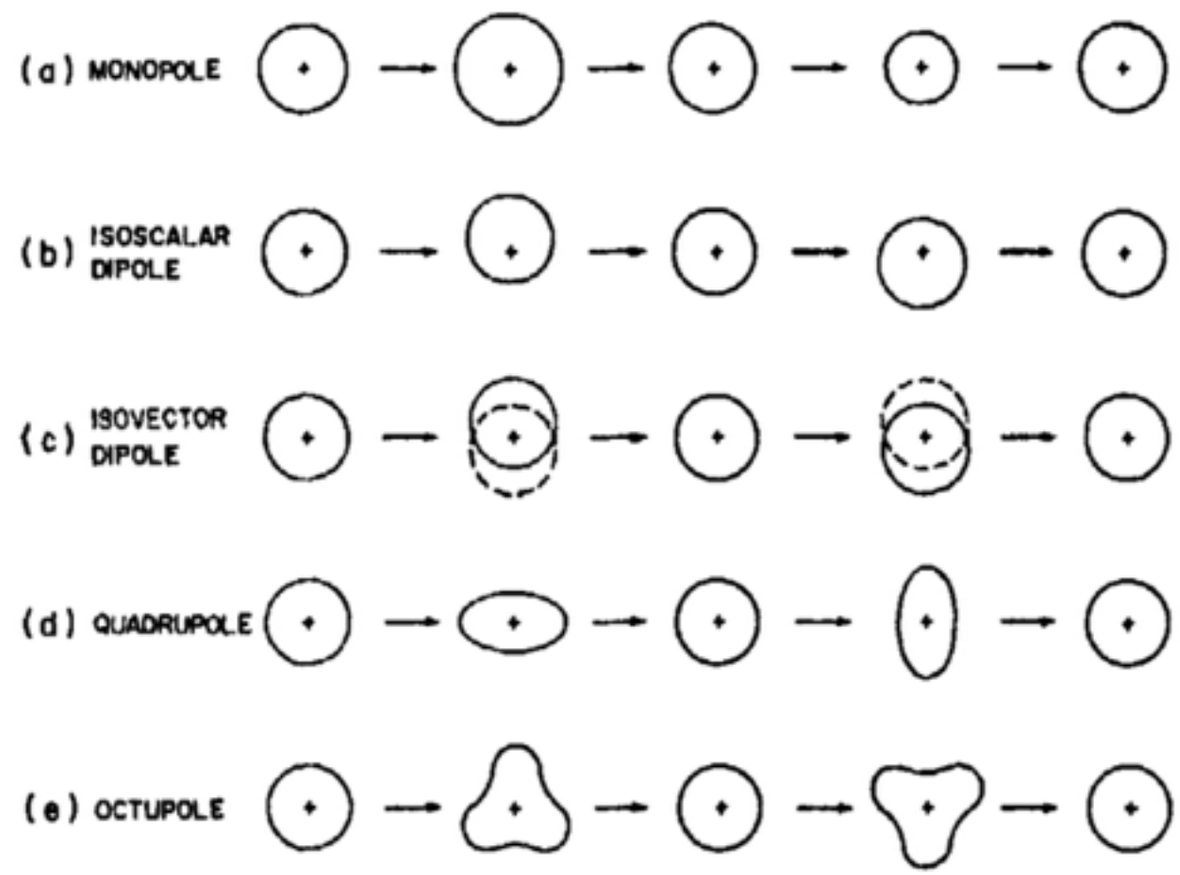
$$\hat{O}|\alpha\rangle = \lambda_\alpha|\alpha\rangle$$

$$\hat{O} = \frac{\beta_\lambda}{\sqrt{4\pi}} R_T\left(a^\dagger_{\lambda 0} + a_{\lambda 0}\right)$$

$$\hat{O}_{nm} = \frac{\beta_\lambda}{\sqrt{4\pi}} R_T\left(\sqrt{m}\,\delta_{n,m-1} + \sqrt{n}\,\delta_{n,m+1}\right)$$

$$\begin{aligned} V^{(N)}_{nm} &= \langle I0|V_N(r,\hat{O})|I'0\rangle - V^{(0)}_N(r)\delta_{n,m} \\ &= \sum_\alpha \langle I0|\alpha\rangle\langle\alpha|I'0\rangle V_N(r,\lambda_\alpha) - V^{(0)}_N(r)\delta_{n,m} \end{aligned}$$

$$V^{(C)}_{nm}(r) = \frac{\beta_\lambda}{\sqrt{4\pi}}\frac{3}{2\lambda+1} Z_P Z_T e^2 \frac{R_T^\lambda}{r^{\lambda+1}}\left(\sqrt{m}\,\delta_{n,m-1} + \sqrt{n}\,\delta_{n,m+1}\right)$$



K.Hagino et al : CPC 123, 143 (1999)

Fusion reaction with Coupled-Channel method

Rotational coupling

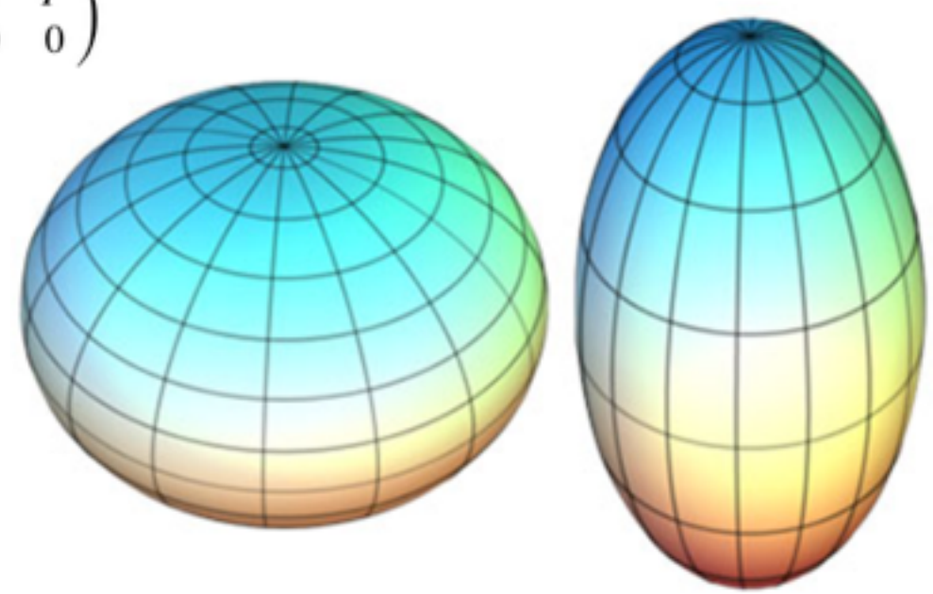
$$R_0 \to R_0 + \hat{O} = R_0 + \beta_2 R_T Y_{20} + \beta_4 R_T Y_{40}$$

$$V_N(r, \hat{O}) = -\frac{V_0}{1 + \exp((r - R_0 - \hat{O})/a)} \qquad \hat{O}|\alpha\rangle = \lambda_\alpha |\alpha\rangle$$

$$\hat{O}_{II'} = \sqrt{\frac{5(2I+1)(2I'+1)}{4\pi}}\beta_2 R_T \begin{pmatrix} I & 2 & I' \\ 0 & 0 & 0 \end{pmatrix}^2 + \sqrt{\frac{9(2I+1)(2I'+1)}{4\pi}}\beta_4 R_T \begin{pmatrix} I & 4 & I' \\ 0 & 0 & 0 \end{pmatrix}^2$$

$$\begin{aligned} V_{nm}^{(N)} &= \langle I0|V_N(r, \hat{O})|I'0\rangle - V_N^{(0)}(r)\delta_{n,m} \\ &= \sum_\alpha \langle I0|\alpha\rangle\langle\alpha|I'0\rangle V_N(r, \lambda_\alpha) - V_N^{(0)}(r)\delta_{n,m} \end{aligned}$$

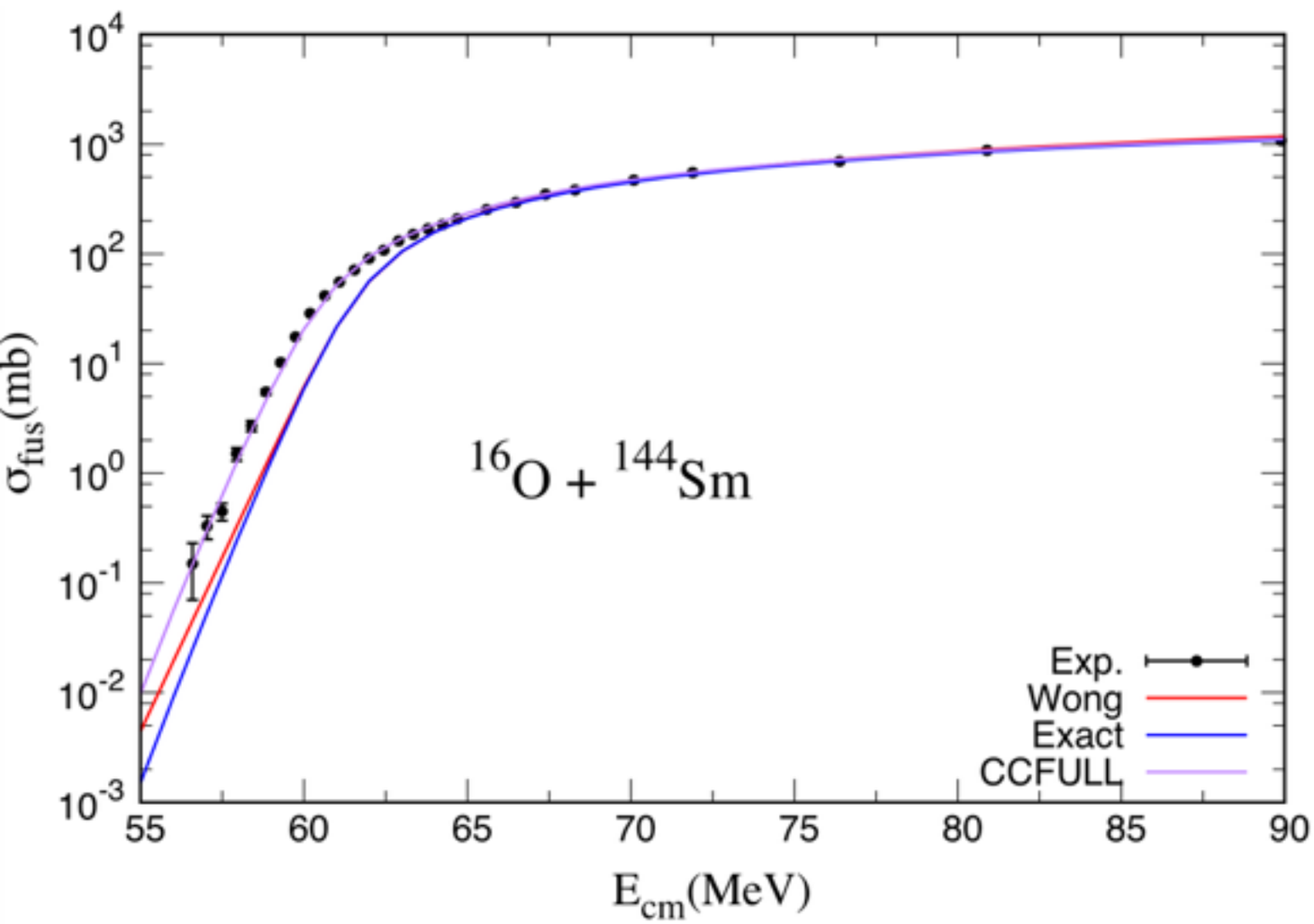
$$\begin{aligned} V_{nm}^{(C)} = &\frac{3Z_P Z_T}{5}\frac{R_T^2}{r^3}\sqrt{\frac{5(2I+1)(2I'+1)}{4\pi}}\left(\beta_2 + \frac{2}{7}\sqrt{\frac{5}{\pi}}\beta_2^2\right)\begin{pmatrix} I & 2 & I' \\ 0 & 0 & 0 \end{pmatrix}^2 \\ &+ \frac{3Z_P Z_T}{9}\frac{R_T^4}{r^5}\sqrt{\frac{9(2I+1)(2I'+1)}{4\pi}}\left(\beta_4 + \frac{9}{7}\beta_2^2\right)\begin{pmatrix} I & 4 & I' \\ 0 & 0 & 0 \end{pmatrix}^2. \end{aligned}$$



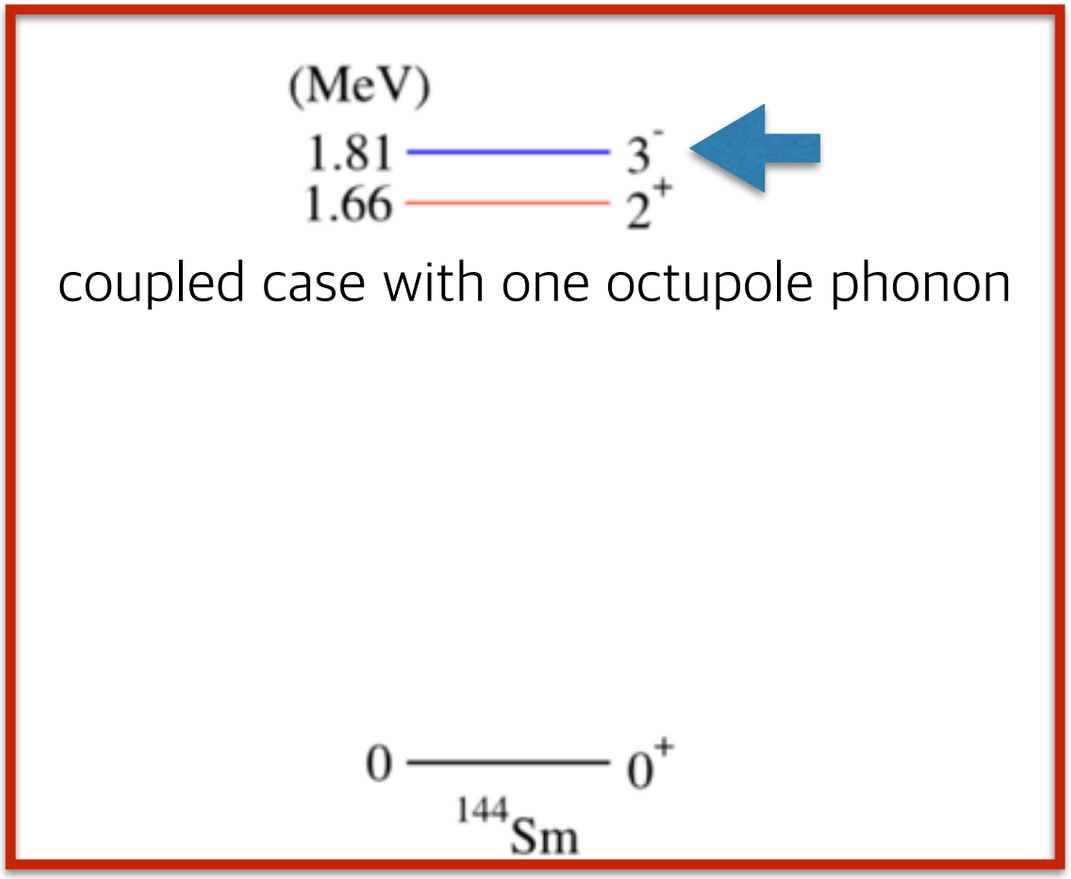
Oblate and Prolate nuclei

K.Hagino et al : CPC 123, 143 (1999)

Example : $^{16}O+^{144}Sm$ fusion reaction



$$[B(E\lambda;\uparrow)]^{1/2} = \frac{3Z_i e}{4\pi}\beta_\lambda^C R_C^\lambda$$



coupled case with one octupole phonon

$$V_N(r,\hat{O}) = -\frac{V_0}{1+\exp((r-R_0-\hat{O})/a)}$$

$$\hat{O}_{nm} = \frac{\beta_\lambda}{\sqrt{4\pi}} R_T\left(\sqrt{m}\,\delta_{n,m-1} + \sqrt{n}\,\delta_{n,m+1}\right)$$

Parameters

V_0	r_0	a_0	r_t	β
105.1	1.1	0.75	1.06	0.205

K.Hagino et al : CPC 123, 143 (1999)
J.R.Leigh et al : phys. rev. C 52, 3151 (1995)

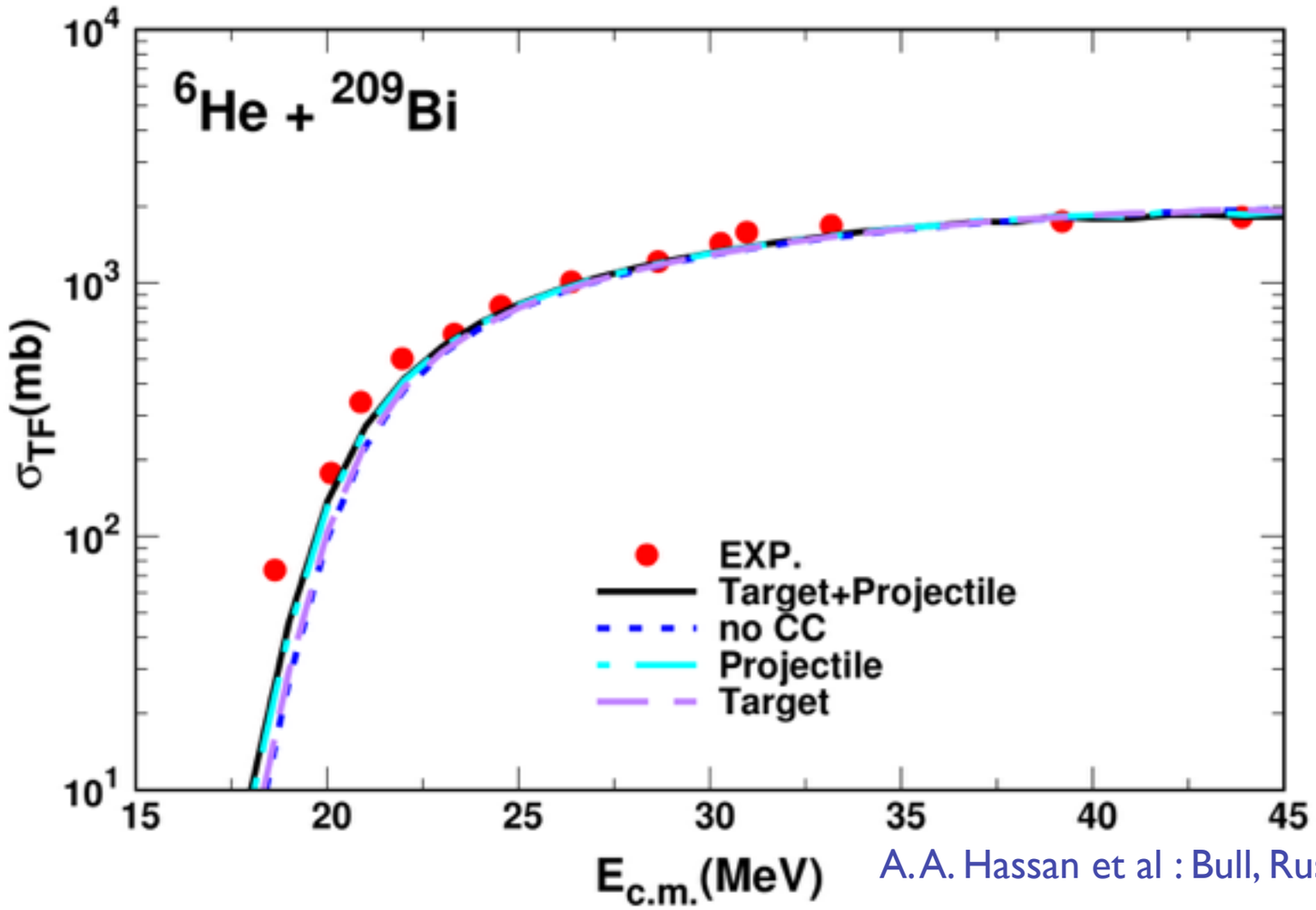
AKyuz-Winther Potential(Global potential)

$$V_N(r) = -\frac{V_0}{1+\exp[(r-R_0)/a]},$$

$$\begin{aligned}
V_0 &= 16\pi\gamma\bar{R}a, \\
R_0 &= R_P + R_T, \\
R_i &= 1.20A_i^{1/3} - 0.09 \text{ fm}, \qquad (i = P, T) \\
\bar{R} &= R_P R_T/(R_P + R_T), \\
\gamma &= 0.95\left[1 - 1.8\left(\frac{N_P - Z_P}{A_P}\right)\left(\frac{N_T - Z_T}{A_T}\right)\right] \text{ MeV fm}^{-2}, \\
1/a &= 1.17\left[1 + 0.53\left(A_P^{-1/3} + A_T^{-1/3}\right)\right] \text{ fm}^{-1},
\end{aligned}$$

R. A. Broglia and A. Winther, Heavy-Ion Reactions (Addison-Wesley, New York, 1991).

Total fusion cross section of $^{6}He + ^{209}Bi$

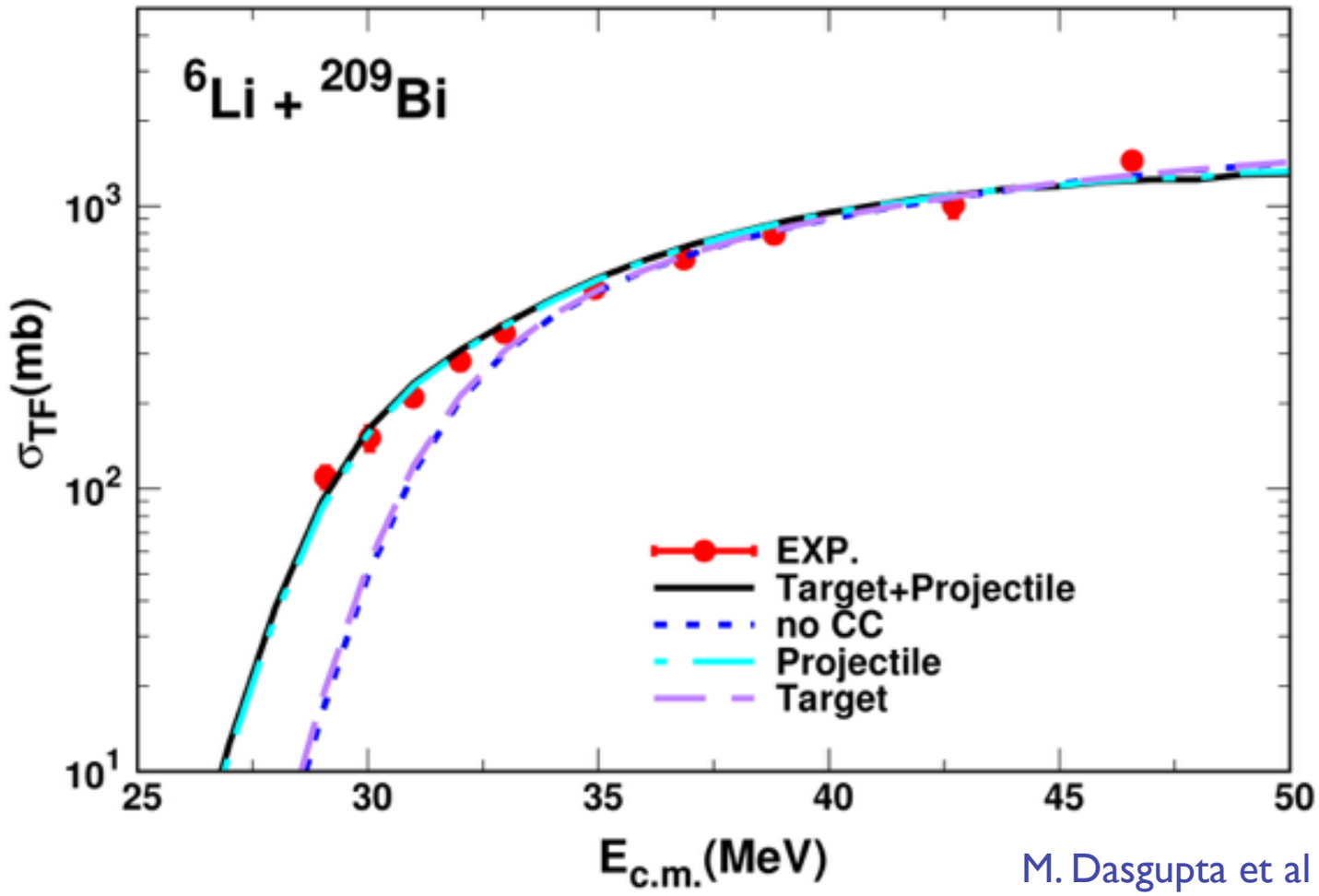


A.A. Hassan et al : Bull, Rus. Acad. Sci. Phys. 70, 1558 (2006)

System	$V_N^{(0)}$			Target excitation		Projectile excitation	
	V_0 (MeV)	r_0 (fm)	a_0 (fm)	ε_x(Eλ; Band) (MeV)	β_λ	ε_x(Eλ; Band) (MeV)	β_2
$^{6}He+^{209}Bi$	41.746	1.177	0.619	0.9(E2; Vi.),2.6(E3; Vi.)	0.0056, 0.0937	1.8(E2; Vi.)	0.70

M. Martin : Nuclear Data Sheets 63, 723 (1991)
E. Crema, P. R. S. Gomes, and L. C. Chamon : Phys. Rev. C 75, 037601 (2007)

Total fusion cross section of $^{6}Li + ^{209}Bi$

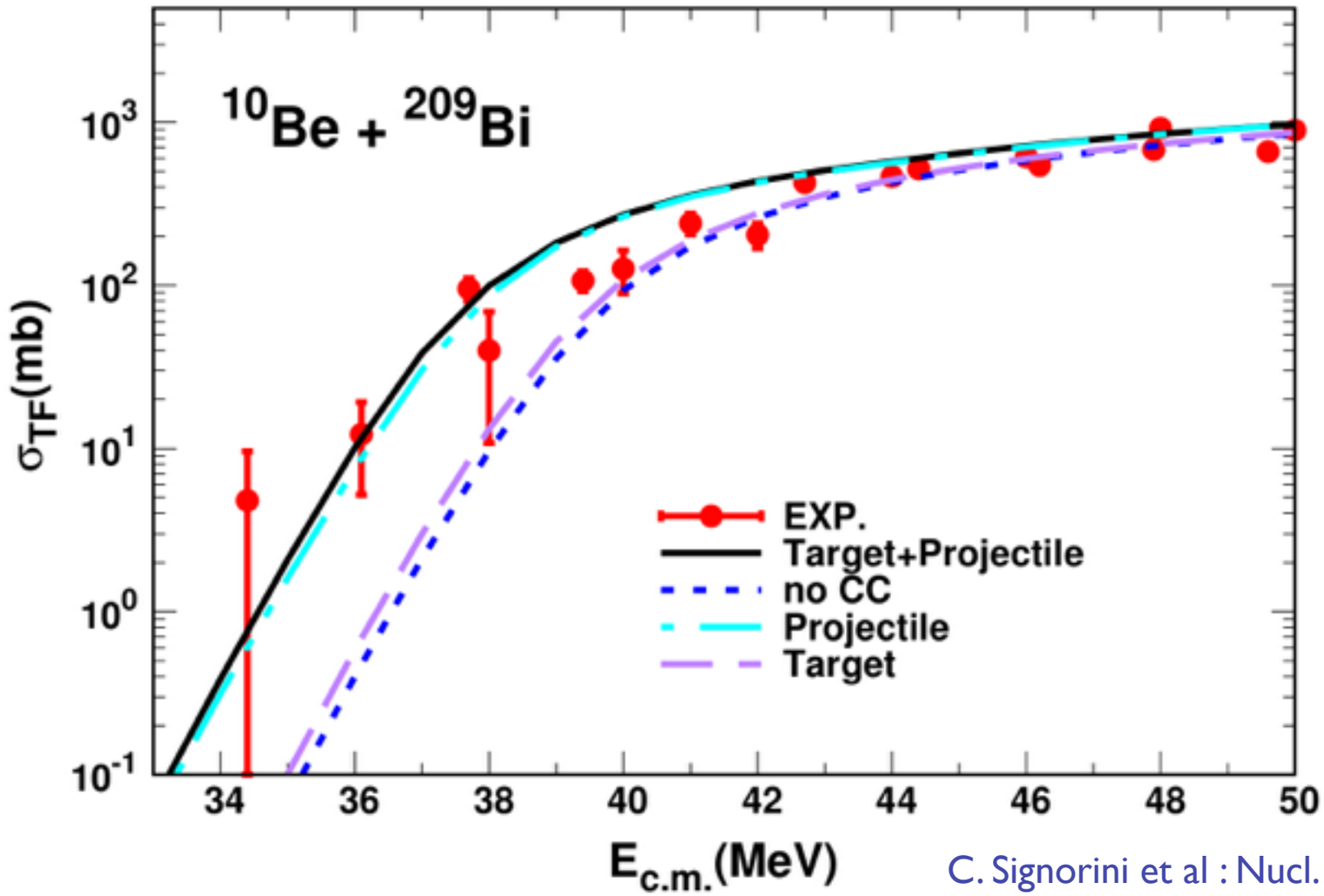


M. Dasgupta et al : Phys. Rev. C 70, 024606 (2004)

System	$V_N^{(0)}$			Target excitation		Projectile excitation	
	V_0 (MeV)	r_0 (fm)	a_0 (fm)	ε_x(Eλ; Band) (MeV)	β_λ	ε_x(Eλ; Band) (MeV)	β_2
$^{6}Li+^{209}Bi$	47.625	1.177	0.619	0.9(E2; Vi.),2.6(E3; Vi.)	0.0056, 0.0937	2.186(E2; Ro.)	1.37

M. Martin : Nuclear Data Sheets 63, 723 (1991)
Y.W.Wu et al : Phys. Rev. C 68, 044605 (2007)

Total fusion cross section of $^{10}Be + ^{209}Bi$

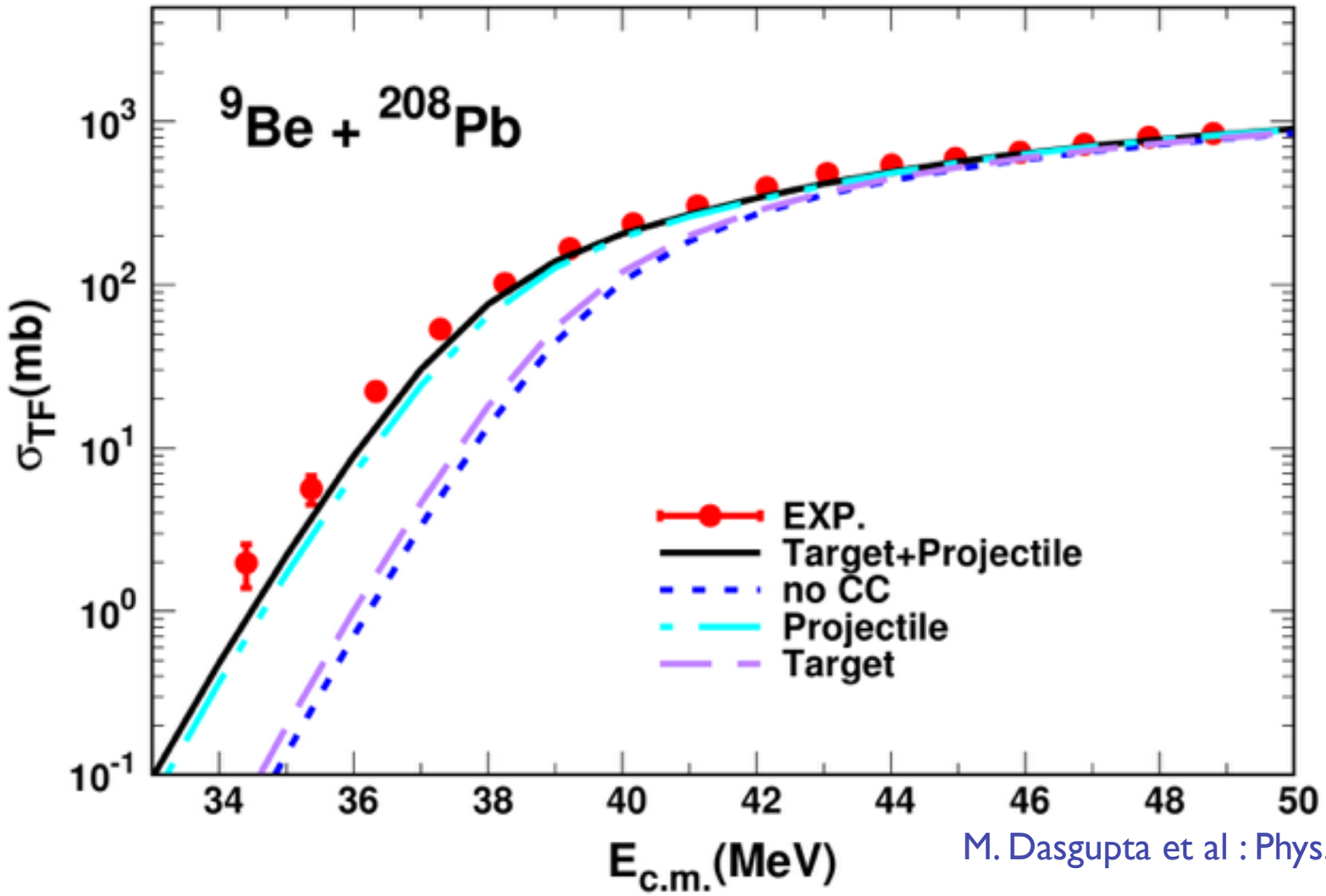


C. Signorini et al : Nucl. Phys. A 735, 329 (2004)

System	$V_N^{(0)}$			Target excitation		Projectile excitation	
	V_0 (MeV)	r_0 (fm)	a_0 (fm)	ε_x(Eλ; Band) (MeV)	β_λ	ε_x(Eλ; Band) (MeV)	β_2
$^{10}Be+^{209}Bi$	51.123	1.178	0.640	0.9(E2; Vi.),2.6(E3; Vi.)	0.0056, 0.0937	3.37(E2; Vi.)	1.14

M. Martin : Nuclear Data Sheets 63, 723 (1991)
C. Signorini et al : Nucl. Phys. A 735, 329 (2004)

Total fusion cross section of $^{9}Be + ^{208}Pb$

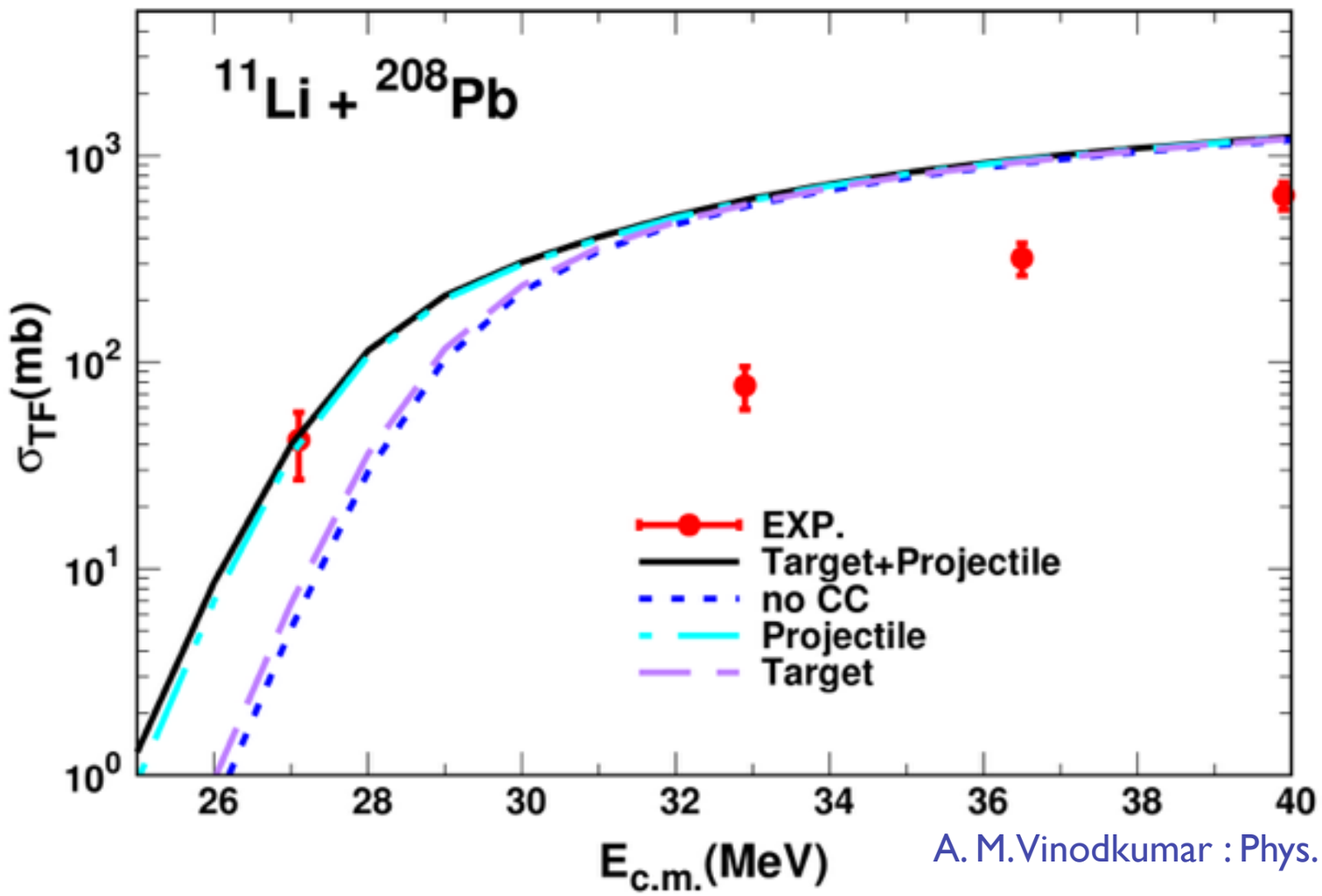


M. Dasgupta et al : Phys. Rev. C 70, 024606 (2004)

System	$V_N^{(0)}$			Target excitation		Projectile excitation	
	V_0 (MeV)	r_0 (fm)	a_0 (fm)	ε_x(Eλ; Band) (MeV)	β_λ	ε_x(Eλ; Band) (MeV)	β_2
$^{9}Be+^{208}Pb$	52.104	1.178	0.636	2.616(E3; Vi.)	0.111	2.43(E2; Ro.)	0.875

R. H. Spear : At. Data Nucl. Data Tables 42, 55 (1989)
V.V. Parkar et al : Phys. Rev. C 82, 054601 (2010)

Total fusion cross section of $^{11}Li + ^{208}Pb$

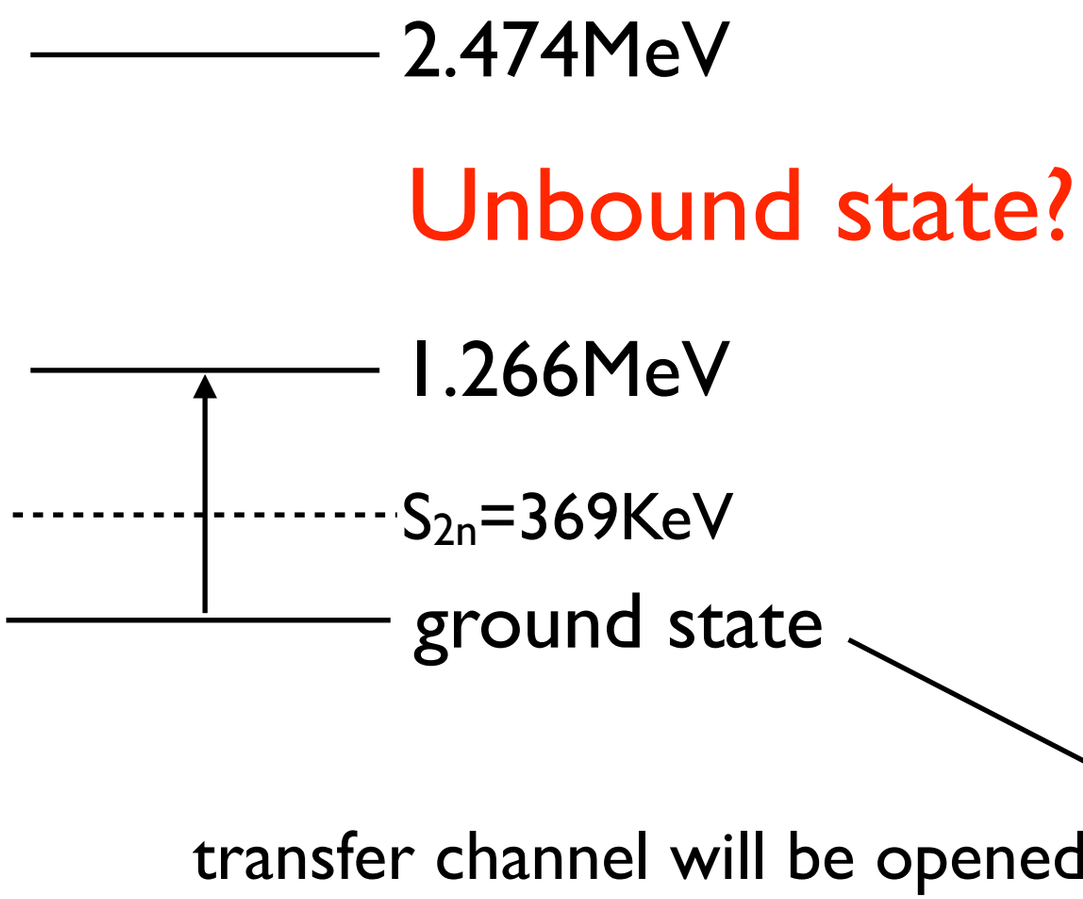


A. M. Vinodkumar : Phys. Rev. C 87, 044603 (2006)

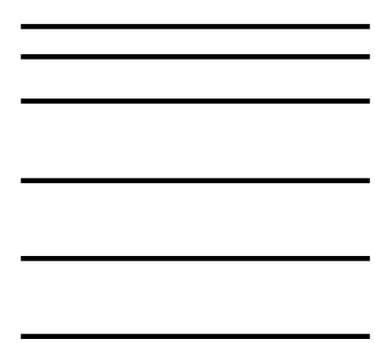
System	$V_N^{(0)}$ V_0 (MeV)	r_0 (fm)	a_0 (fm)	Target excitation ε_x(Eλ; Band) (MeV)	β_λ	Projectile excitation ε_x(Eλ; Band) (MeV)	β_2
$^{11}Li+^{208}Pb$	47.938	1.178	0.644	2.616(E3; Vi.),	0.111	1.266(E2; Vi.)	1.0

R. H. Spear : At. Data Nucl. Data Tables 42, 55 (1989)
J. Kelley et al : Nucl. Phys. A 880, 88 (2012)

^{11}Li level information



E_{level} (keV)	XREF	Jπ	$T_{1/2}$
0	ABCDE GH	3/2-	8.75 ms *14* % β^- = 100
1266 *42*	A EF HI		0.53 MeV *15*
2474 *54*	A C EF HI		1.26 MeV *30*
3.70E3 *13*	F		< 200 keV
4.86E3 *60*	A C H		< 100 keV
6.23E3 *60*	A C H		< 100 keV
11.3E3	A		



dominant channel : ^{9}Li + ^{210}Pb

J. Kelley et al : Nucl. Phys. A 880, 88 (2012)

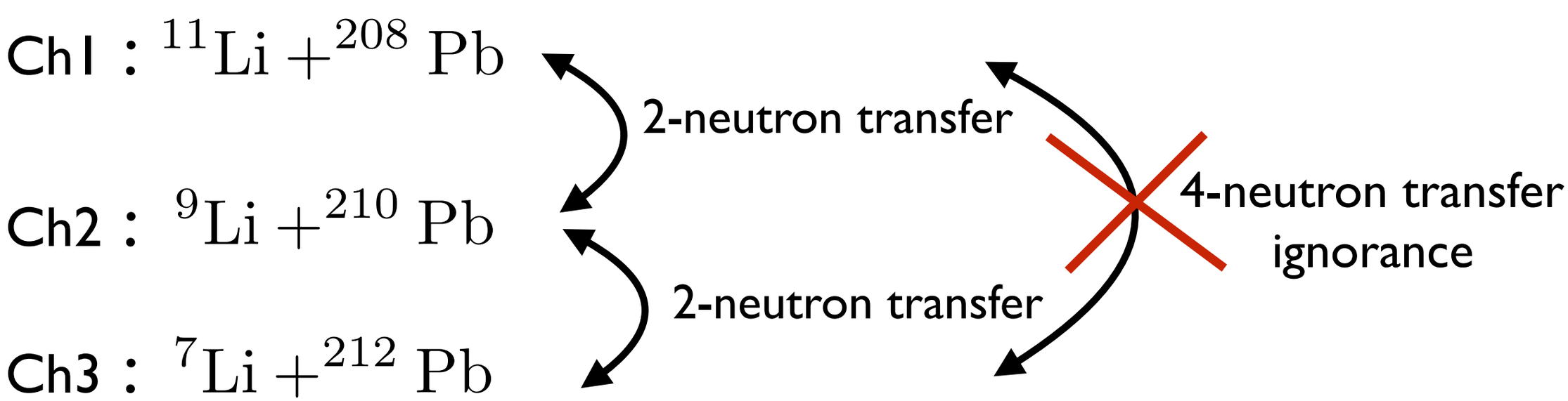
Modification of coupled channel equation

Z									
		6B 2P	7B 1.4 MeV P α	8B 770 MS εα: 100.00% ε: 100.00%	9B 0.54 KeV 2α: 100.00% P: 100.00%	10B STABLE 19.9%	11B STABLE 80.1%	12B 20.20 MS β-: 100.00% B3A: 1.58%	13B 17.33 MS β-: 100.00% β-n: 0.26%
4		5Be P	6Be 92 KeV α: 100.00% P: 100.00%	7Be 53.22 D ε: 100.00%	8Be 5.57 eV α: 100.00%	9Be STABLE 100.%	10Be 1.51E+6 Y β-: 100.00%	11Be 13.76 S β-: 100.00% β-α: 3.10%	12Be 21.47 MS β-: 100.00% β-n: 0.50%
3	3Li P	4Li 6.03 MeV P: 100.00%	5Li ≈1.5 MeV α: 100.00% P: 100.00%	6Li STABLE 7.59%	7Li STABLE 92.41%	8Li 839.9 MS β-α: 100.00% β-: 100.00%	9Li 178.3 MS β-: 100.00%	10Li N: 100.00%	11Li 8.75 MS β-: 100.00% β-n: 86.60%
2		3He STABLE 0.000134%	4He STABLE 99.999866%	5He 0.60 MeV N: 100.00% α: 100.00%	6He 806.7 MS β-: 100.00%	7He 150 KeV N	8He 119.1 MS β-: 100.00% β-n: 16.[illegible]%	9He N: 100.00%	10He 300 KeV N: 100.00%
1	1H STABLE 99.9885%	2H STABLE 0.0115%	3H 12.32 Y β-: 100.00%	4H N: 100.00%	5H 5.7 MeV 2N: 100.00%	6H 1.6 MeV N: 100.00%	7H 500 YS 2N:		
	0	1	2	3	4	5	6	7	N

also unstable!
and many parameters!

Modification of coupled channel equation

$$\begin{pmatrix} K+V_1-E & F_{1\to2}(r) & 0 \\ F_{1\to2}(r) & K+V_2-(E-Q_2) & F_{2\to3}(r) \\ 0 & F_{2\to3}(r) & K+V_2-(E-Q_2-Q_{23}) \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} = 0$$



^{9}Li + ^{208}Pb fusion reaction with inelastic channel

$$\begin{pmatrix} K+V_1(r)-E & F_{1\to2}(r) & 0 \\ F_{1\to2}(r) & K+V_2(r)-(E+Q_{12}) & F_{2\to3}(r) \\ 0 & F_{2\to3}(r) & K+V_3(r)-(E+Q_{12}+Q_{23}) \end{pmatrix} \begin{pmatrix} \psi_1(r) \\ \psi_2(r) \\ \psi_3(r) \end{pmatrix} = 0,$$

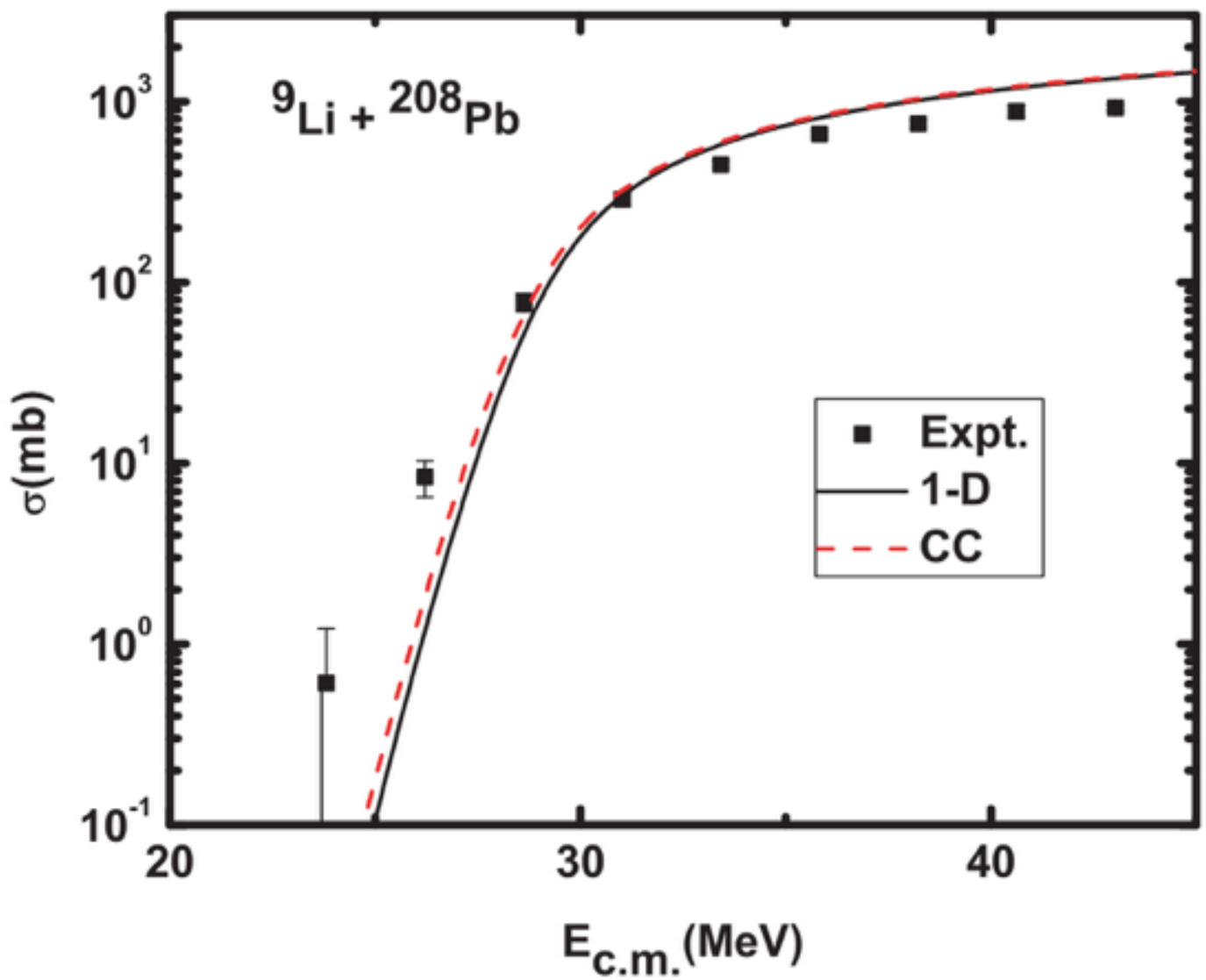
$$\boxed{^{9}\mathrm{Li} + ^{208}\mathrm{Pb}}$$

$$\begin{pmatrix} K+V_2(r)-E & F_{2\to3}(r) \\ F_{2\to3}(r) & K+V_3(r)-(E+Q_{23}) \end{pmatrix} \begin{pmatrix} \psi_2(r) \\ \psi_3(r) \end{pmatrix} = 0,$$

$$\boxed{^{7}\mathrm{Li} + ^{208}\mathrm{Pb}}$$

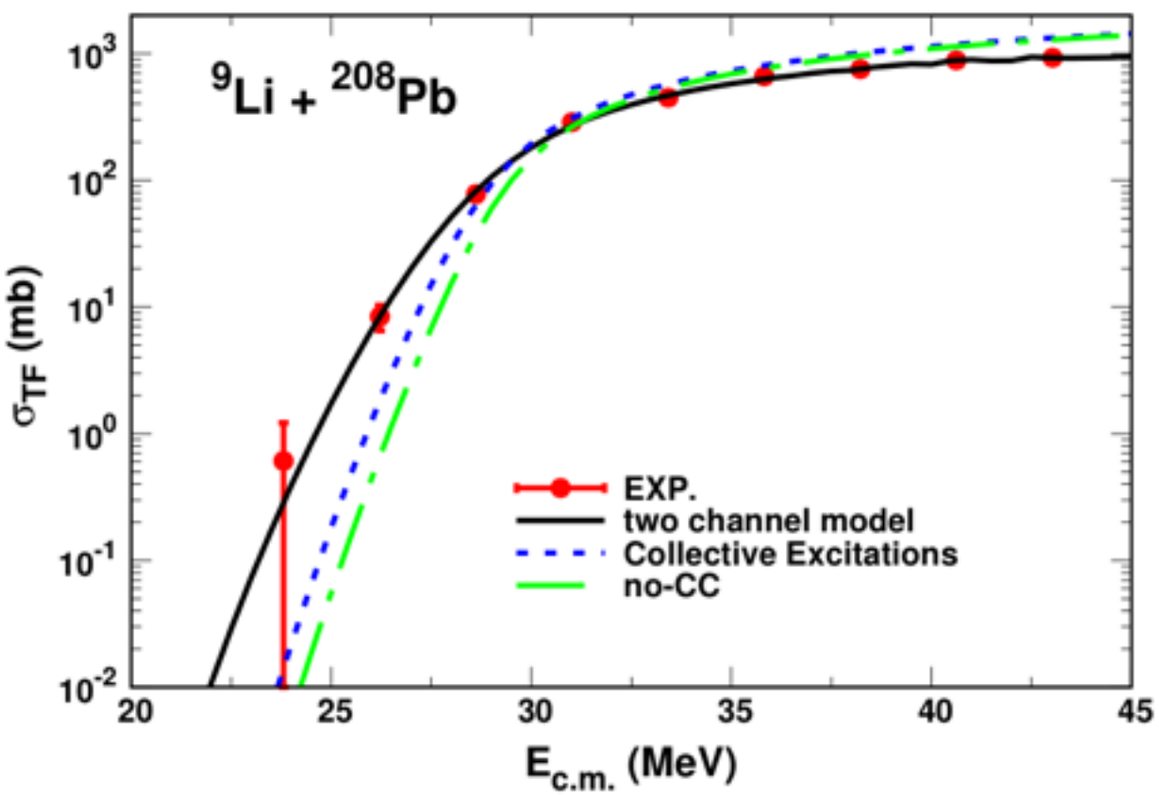
2-channel

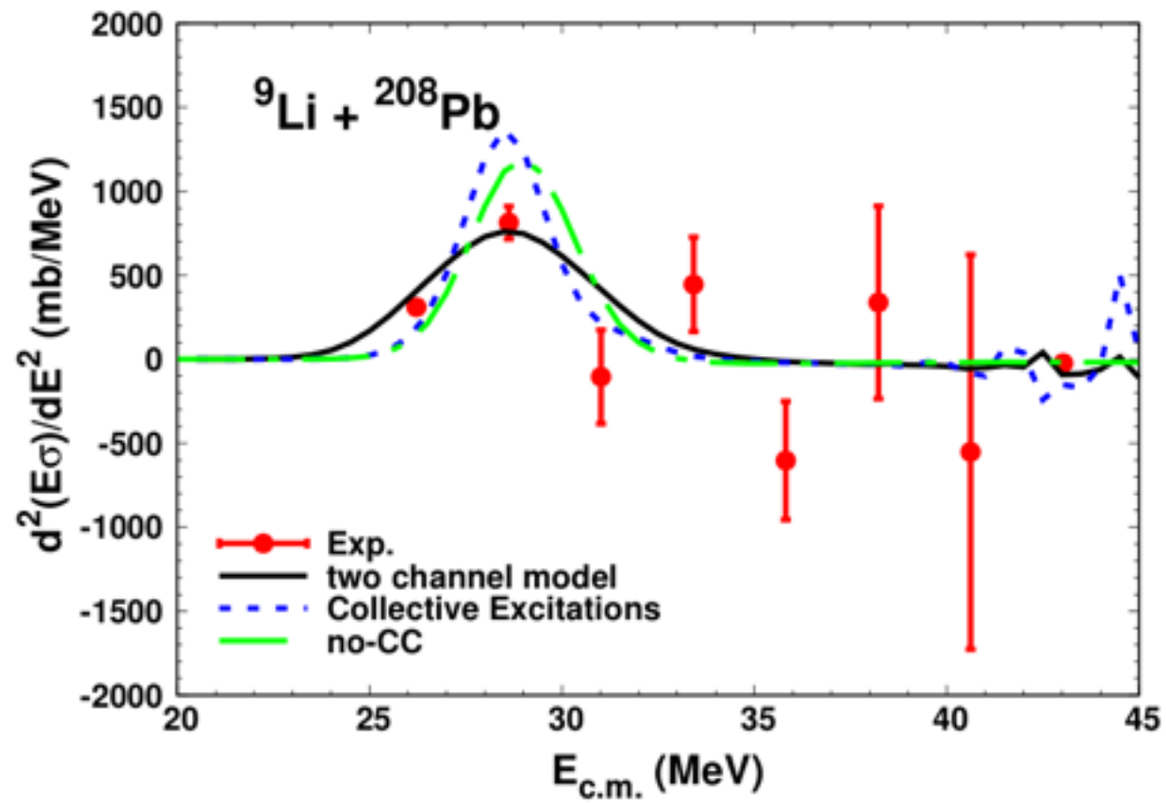
$^{9}Li + ^{208}Pb$ fusion reaction with inelastic channel



A. M. Vinodkumar : Phys. Rev. C 80, 054609 (2009)

^{9}Li + ^{208}Pb fusion reaction with transfer channel





potentials

V_0	r_0	a_0	V_{02}	r_{02}	a_{02}
47.500	1.177	0.636	47.347	1.177	0.625

$^{9}\mathrm{Li} + ^{208}\mathrm{Pb}$ $\quad$ $^{7}\mathrm{Li} + ^{208}\mathrm{Pb}$

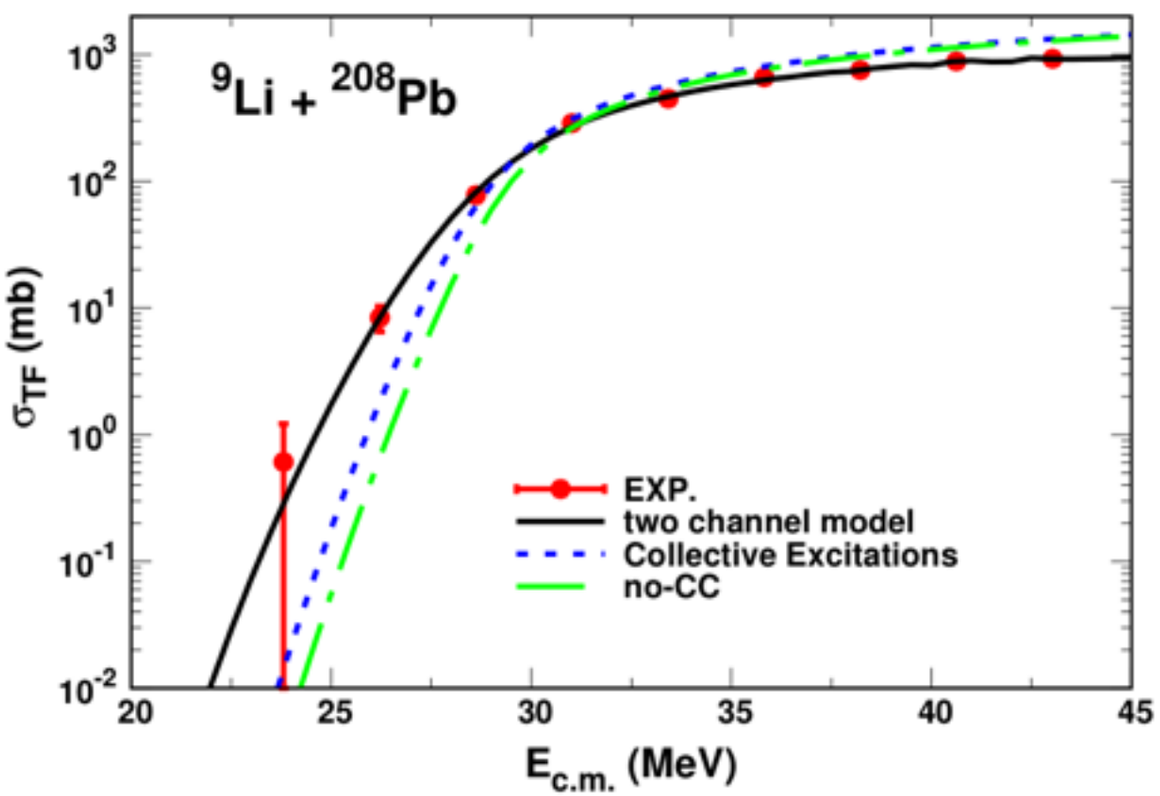
form factor

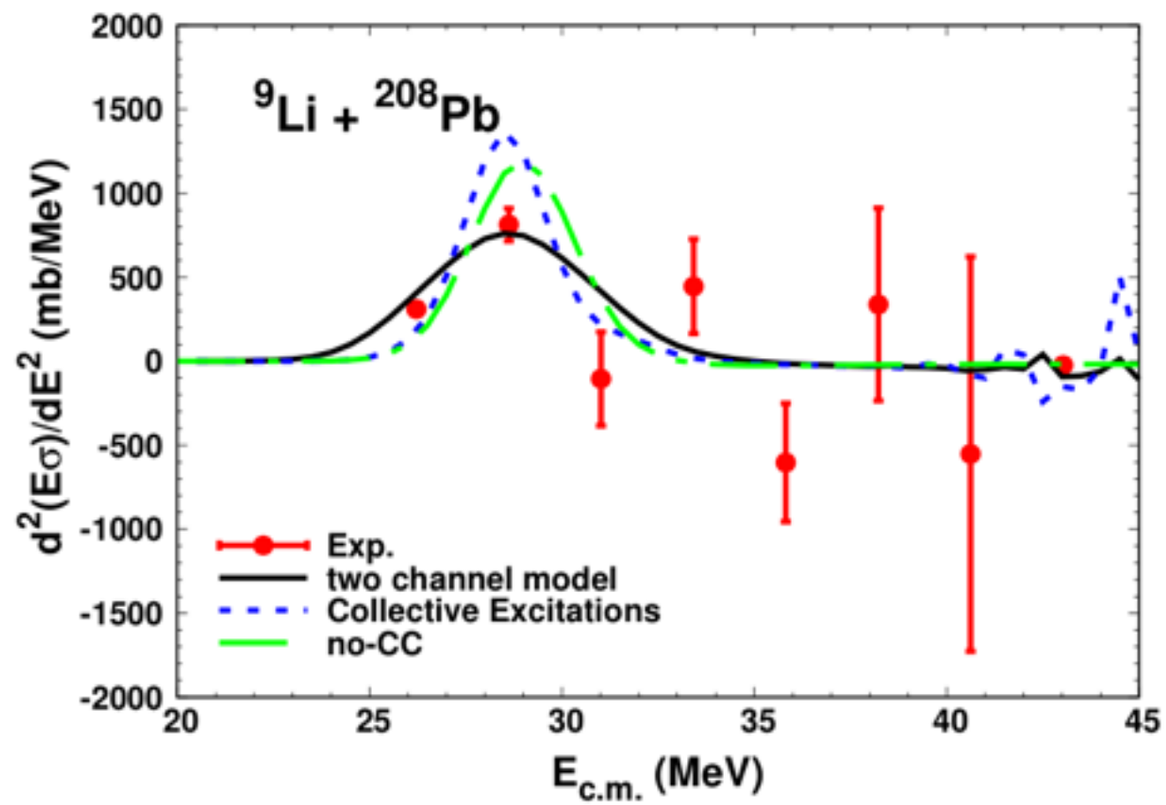
Q-value	F_t(MeVfm)	r_{coup}(fm)	a_{coup}(fm)
-3.204	51.367	1.357	0.264

$F_{2\to3}(r)$

M. Dasgupta : Phys. Rev. C 70, 024606 (2004)

$^{9}Li + ^{208}Pb$ fusion reaction with transfer channel

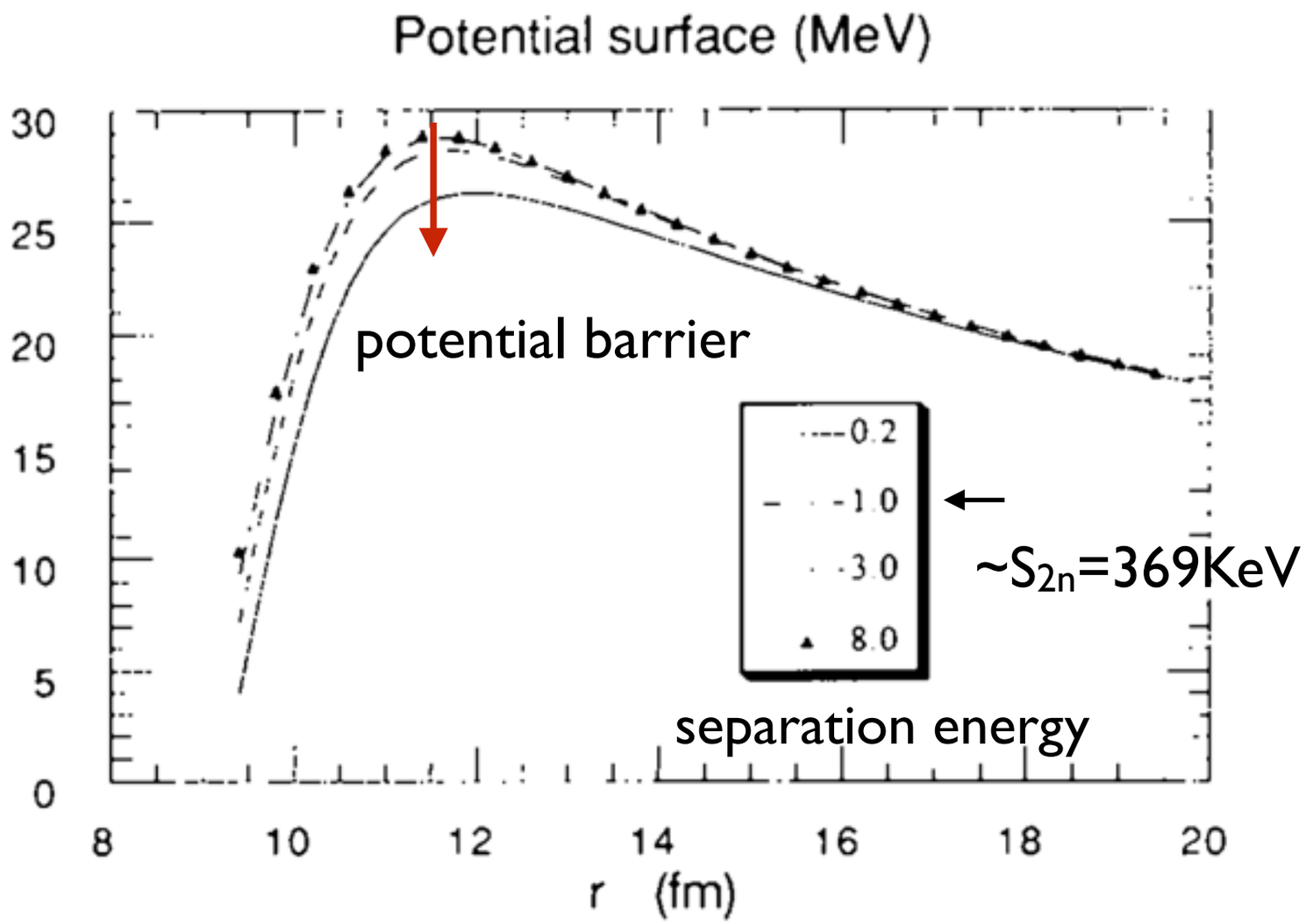




Channel	V_0 (MeV)	r_0 (fm)	a_0 (fm)	V_b (MeV)	R_b (fm)	$\hbar\Omega$ (MeV)
$^{9}Li + ^{208}Pb$ (channel 2)	47.500	1.177	0.636	29.1	11.5	4.07
$^{7}Li + ^{210}Pb$ (channel 3)	47.347	1.177	0.625	29.6	11.3	4.71

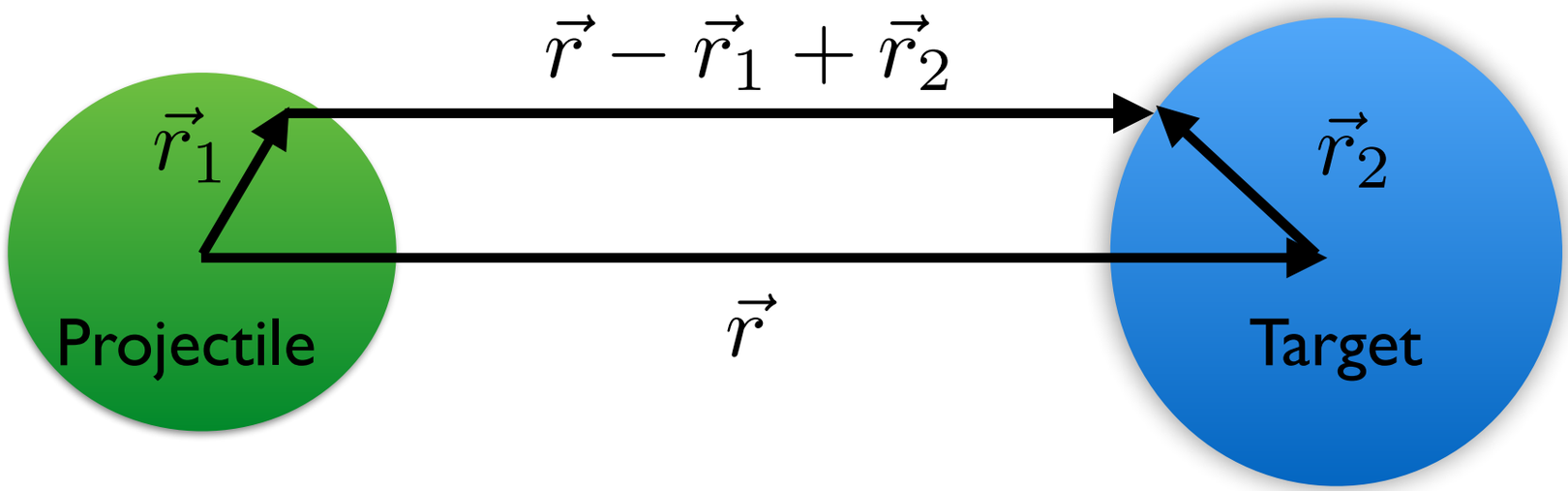
M. Dasgupta : Phys. Rev. C 70, 024606 (2004)

Separation energy dependence of the potential surface for $^{11}\mathrm{Li} +^{208}\mathrm{Pb}$



N. Takigawa : Phys. lett. B 265, 23 (1991)

Folding Potential assumption for ^{11}Li + ^{208}Pb

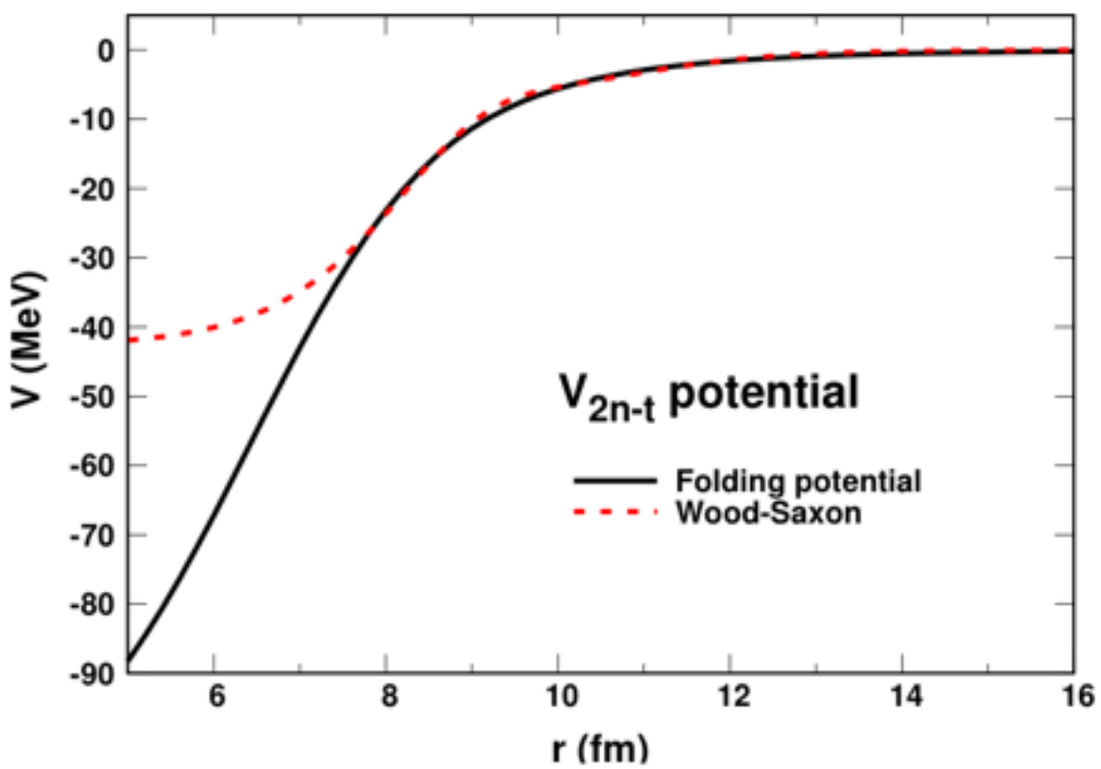


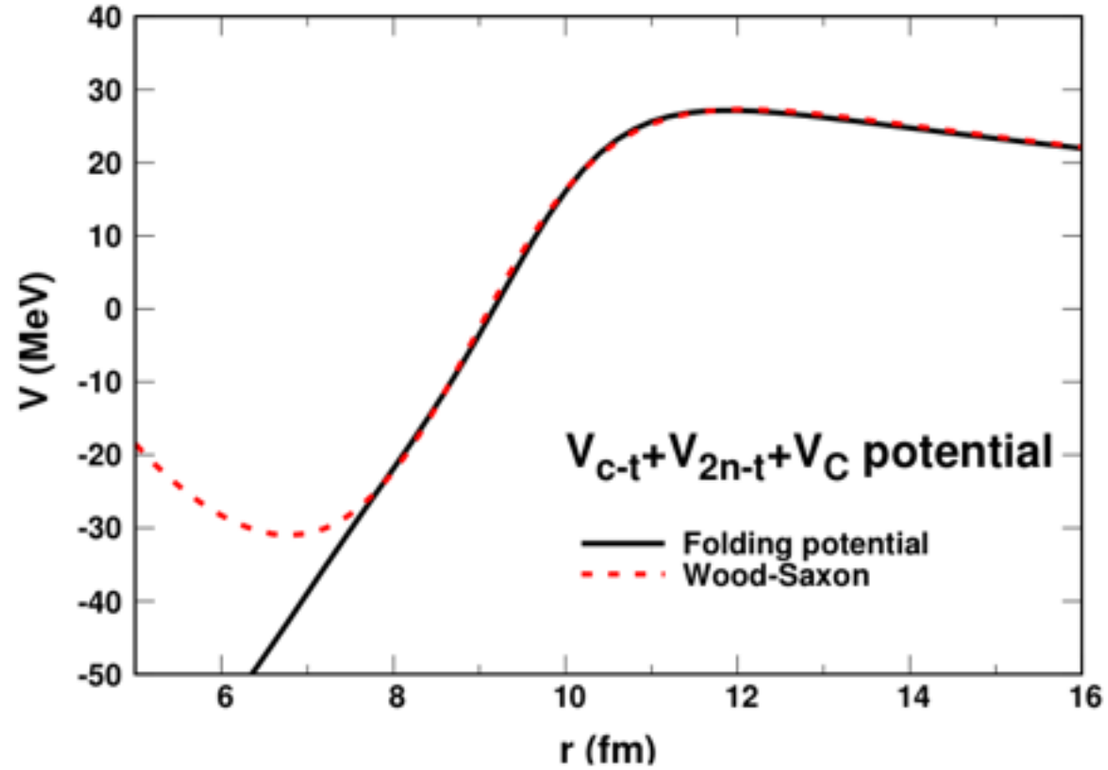
$$V(r) = \int d\mathbf{r}_1 \int d\mathbf{r}_2 \rho_p(r_1)\rho_t(r_2) V_{NN}\left(r_{12} = |\mathbf{r} - \mathbf{r}_1 + \mathbf{r}_2|\right)$$

$$\sim \quad = \int d\mathbf{r}_1 \int d\mathbf{r}_2 \left(\rho_c(r) + \rho_{2n}(r)\right) \rho_t(r_2) V_{NN}\left(r_{12}\right)$$

$$= V_{ct}(r) + V_{2nt}(r),$$

Folding Potential assumption for $^{11}Li + ^{208}Pb$





$$V(r) = \int d\mathbf{r}_1 \int d\mathbf{r}_2 \left(\rho_c(r) + \rho_{2n}(r)\right) \rho_t(r_2) V_{NN}(r_{12})$$
$$= V_{ct}(r) + V_{2nt}(r), \sim V_{WS}^{ct}(\vec{R}) + V_{WS}^{2nt}(\vec{R})$$

Modification of coupled channel equation

$$\begin{pmatrix} K+V_1-E & F_{1\to2}(r) & 0 \\ F_{1\to2}(r) & K+V_2-(E-Q_2) & F_{2\to3}(r) \\ 0 & F_{2\to3}(r) & K+V_2-(E-Q_2-Q_{23}) \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} = 0$$

$^{11}\mathrm{Li}+^{208}\mathrm{Pb}$			$^{9}\mathrm{Li}+^{210}\mathrm{Pb}$			$^{7}\mathrm{Li}+^{212}\mathrm{Pb}$		
Ch 1			Ch 2			Ch 3		
V_0	r_0	a_0	V_{02}	r_{02}	a_{02}	V_{03}	r_{03}	a_{03}
90.309	1.090	0.852	47.304	1.178	0.636	47.298	1.177	0.626
WS by folding potential			AW potential			AW potential		

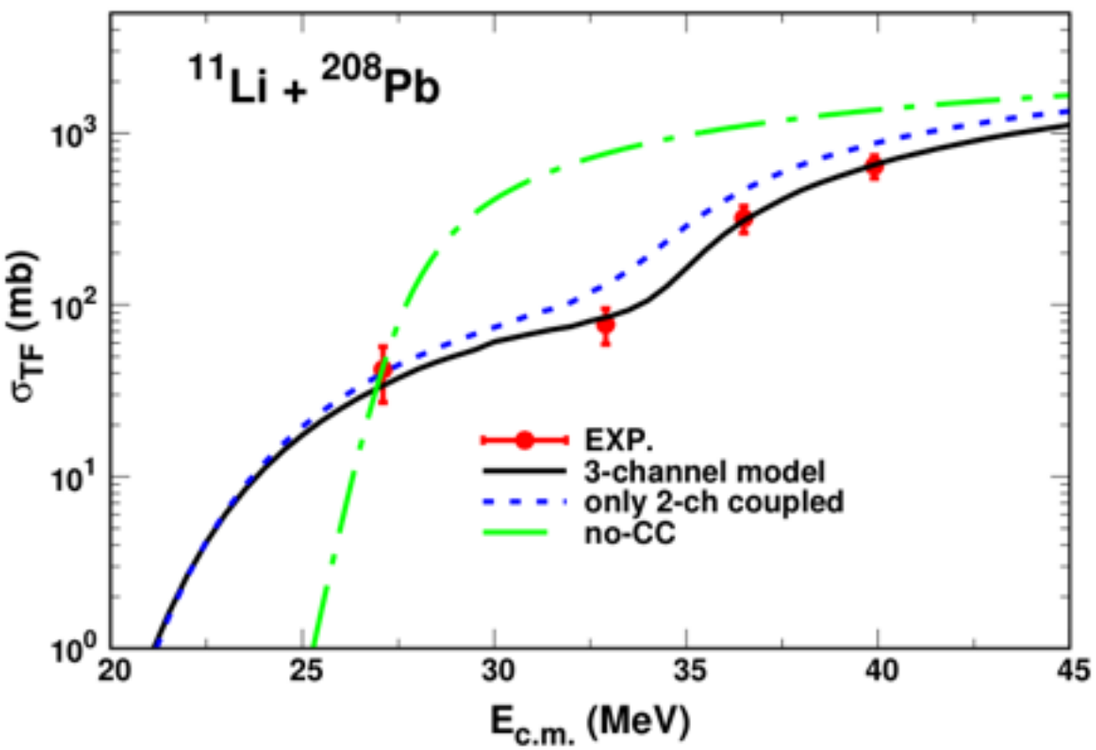
$F_{2\to3}(r)$ form factor

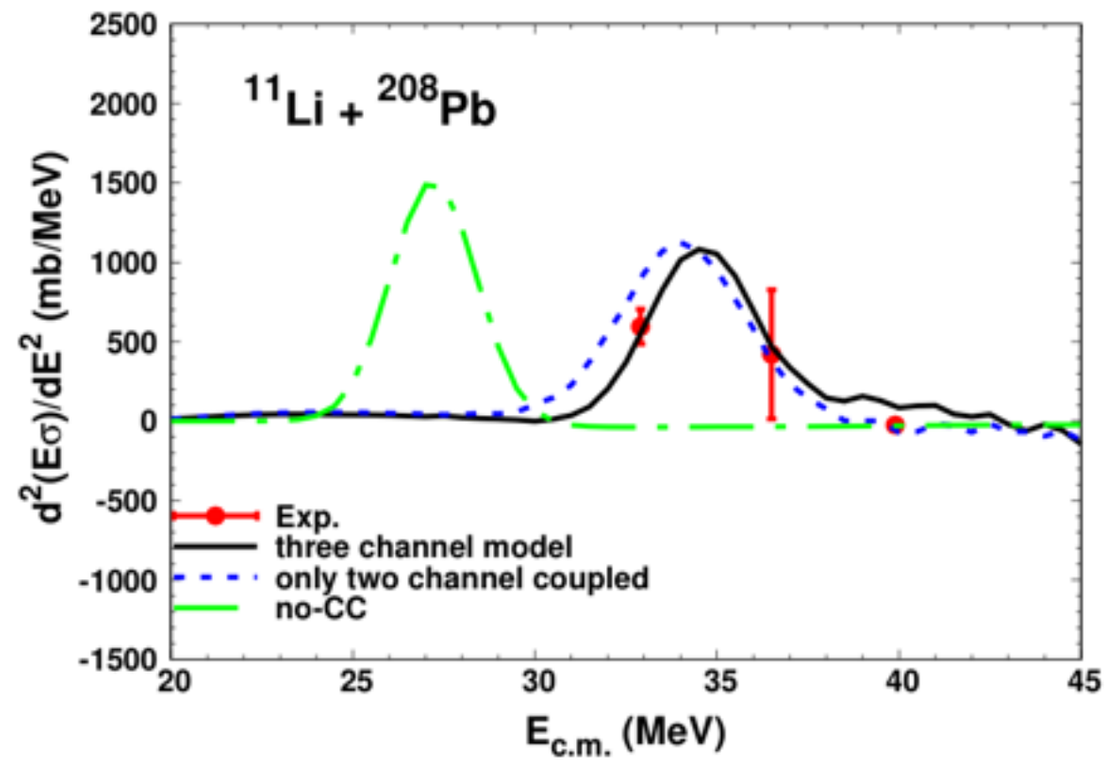
Q-value	F_t(MeVfm)	r_{coup}(fm)	a_{coup}(fm)
-3.204	51.367	1.357	0.264

$F_{1\to2}(r)$ form factor is remained

4 fitting parameters

^{11}Li + ^{208}Pb fusion reaction with transfer contribution





form factor

channel potential

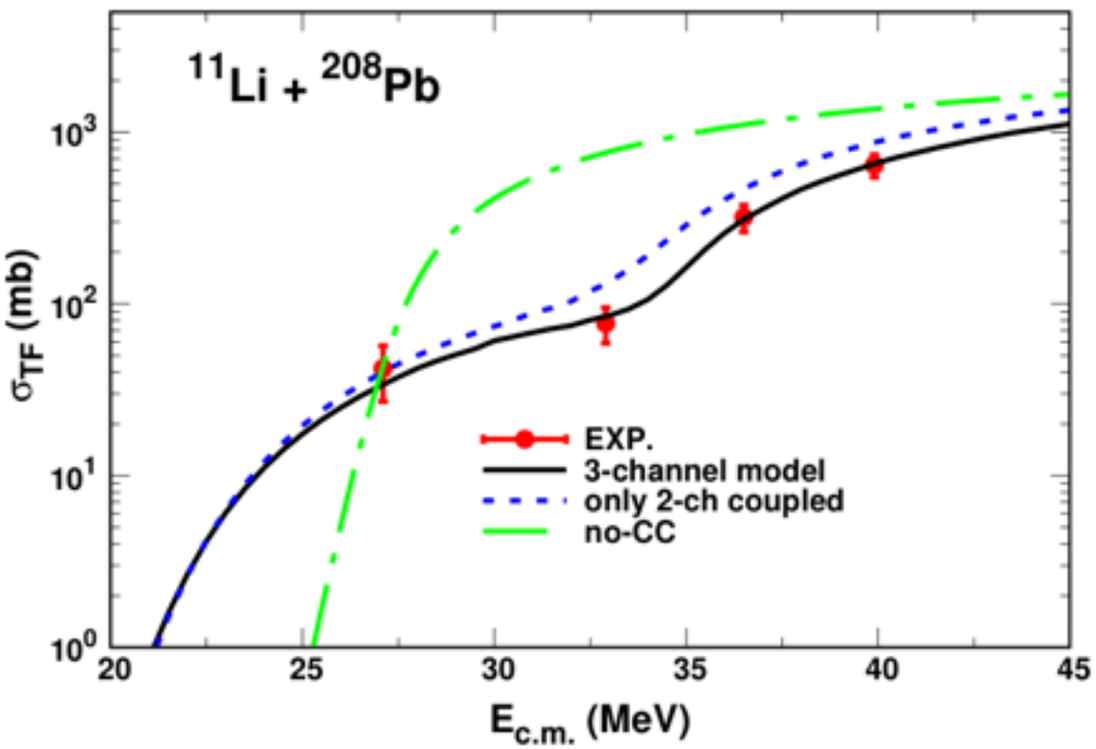
V_0	r_0	a_0	V_{02}	r_{02}	a_{02}	V_{03}	r_{03}	a_{03}
90.309	1.090	0.852	47.304	1.178	0.636	47.298	1.177	0.626

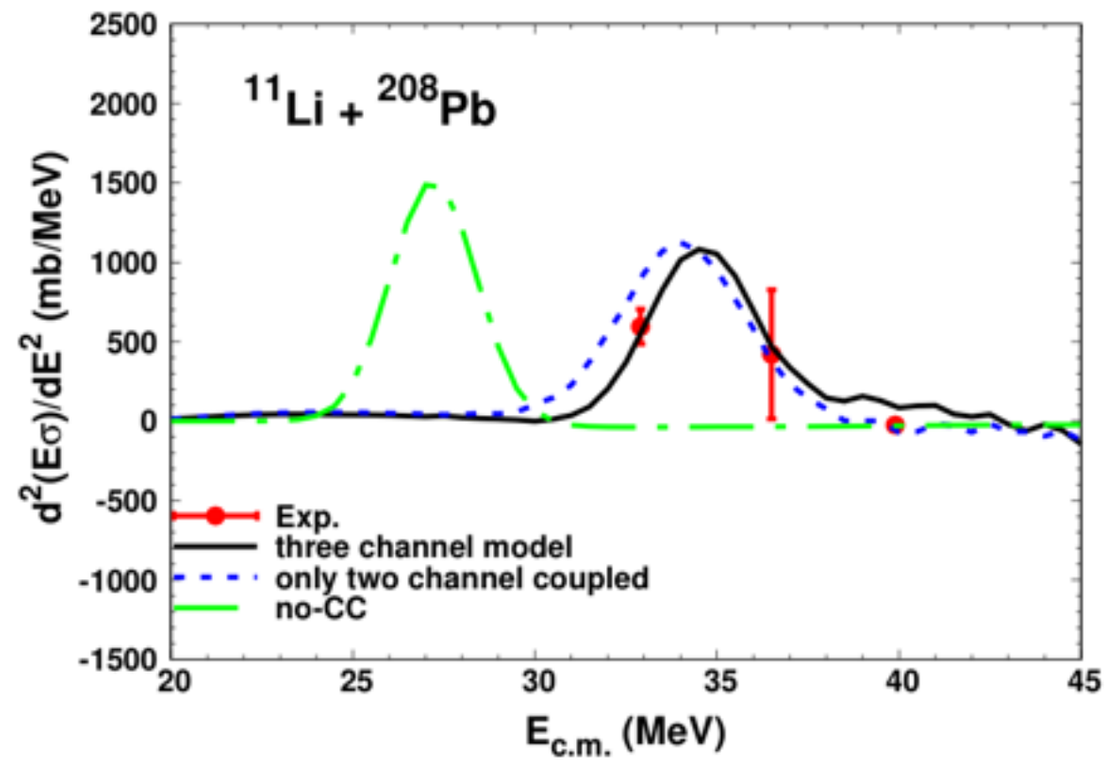
$^{11}\mathrm{Li} + ^{208}\mathrm{Pb}$ $^{9}\mathrm{Li} + ^{210}\mathrm{Pb}$ $^{7}\mathrm{Li} + ^{212}\mathrm{Pb}$

fitting parameters

Channel	Q (MeV)	F_t (MeV fm)	r_{coup} (fm)	a_{coup} (fm)
$1 \to 2$	+8.346	40.227	1.666	0.857
$2 \to 3$	−3.204	51.367	1.357	0.264

^{11}Li + ^{208}Pb fusion reaction with transfer contribution





Channel	V_0 (MeV)	r_0 (fm)	a_0 (fm)	V_b (MeV)	R_b (fm)	$\hbar\Omega$ (MeV)
^{11}Li + ^{208}Pb (channel 1)	90.309	1.090	0.852	27.3	12.1	3.04
^{9}Li + ^{210}Pb (channel 2)	47.304	1.178	0.636	29.0	11.5	4.06
^{7}Li + ^{212}Pb (channel 3)	47.298	1.177	0.626	29.6	11.29	4.70

Summary

- We estimated total fusion cross section of $^{11}Li + ^{208}Pb$ with coupled-channel method.
- We could describe explanation of experimental data. with general 2-channel coupled channel method.
- For reducing parameters and explicit treatment of unstable nuclei, we employed general 3-channel coupled channel method.
- We need additional discussion for describing total fusion cross section of $^{11}Li+^{208}Pb$ case about the contribution of each channel.
- We would try to apply similar system, i. e. $^{6}He+^{209}Bi$ case as treated $^{11}Li+^{208}Pb$ case although it is described with general way.