Complement on Thermodynamics of Taub-NUT-AdS spacetime

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Pin point

1. **Introduction to Taub-NUT-AdS spacetime**
2. **Non-charged TB-AdS (Analogy)**

Sch-AdS black hole	non-charged TB-AdS
Small black hole/ Negative heat capacity	Small TB-AdS/ Negative heat capacity
Large black hole/ Positive heat capacity	Large TB-AdS/ Positive heat capacity

3. **Dyonic TB-AdS (Differences)**

dyonic RN black hole	dyonic TB-AdS
Regularity condition on A is NOT essential for thermodynamics	Regularity condition on A is essential for thermodynamics
electric potential $\Phi_E^{(1)} = \Phi_E^{(2)}$	electric potential $\Phi_E^{(1)} \neq \Phi_E^{(2)}$
Q and P charges are independent	Q and P charges are NOT independent

4. **Zero temperature limit of TB-AdS**

Introduction to Taub-NUT(-AdS) spacetime

Solutions to Einstein EQ. with cosmological const.

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- **Taub-NUT-AdS spacetime**

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- **Taub-NUT-AdS spacetime**
- **Eguchi-Hanson-AdS spacetime**

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$$ds^2_{\mathrm{AdS}} = -\left(1 + \frac{r^2}{l^2}\right) dt^2 + \left(1 + \frac{r^2}{l^2}\right)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2\theta d\phi^2),$$

$$R_{abcd} = -\frac{1}{l^2}\left(g_{ac} g_{bd} - g_{ad} g_{bc}\right)$$

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- **Taub-NUT-AdS spacetime** : Asymptotically locally AdS spacetime

$$\lim_{r\to\infty} ds^2_{\text{TN-AdS}} \neq ds^2_{\mathrm{AdS}}, \qquad R_{abcd} \sim -\frac{1}{l^2}\left(g_{ac}g_{bd} - g_{ad}g_{bc}\right)$$

- **Eguchi-Hanson-AdS spacetime**

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- **Eguchi-Hanson-AdS spacetime**

$$\lim_{r\to\infty} ds^2_{\text{EH-AdS}} \neq ds^2_{\text{AdS}}, \qquad R_{abcd} \neq -\frac{1}{l^2}\left(g_{ac} g_{bd} - g_{ad} g_{bc}\right)$$

Misner string for $\kappa = 1$

$$e^{\hat{t}} = F(r)[dt + 4s\sin^2\left(\frac{\theta}{2}\right)d\phi], \qquad e^{\hat{r}} = F(r)^{-1}dr,$$
$$e^{\hat{\theta}} = (r^2+s^2)^{1/2}d\theta, \qquad e^{\hat{\phi}} = (r^2+s^2)^{1/2}\sin(\theta)d\phi$$

- string singularity : $(\nabla t)^2$ is regular as $\theta \to 0$, it diverges as $\theta \to \pi$

$$-(\nabla t)^2 = \frac{1}{F(r)^2} - \frac{(2s)^2}{r^2+s^2}\tan^2\left(\frac{\theta}{2}\right)$$

- On constant r hypersurface

$$e^{\hat{t}} = dt_N + 4s\sin^2\left(\frac{\theta}{2}\right)d\phi, \qquad (0 \le \theta < \pi)$$
$$e^{\hat{t}} = dt_S - 4s\cos^2\left(\frac{\theta}{2}\right)d\phi, \qquad (0 < \theta \le \pi)$$

- combine two patches $\to$ time coordinate becomes periodic

$$t_N \equiv t_N + 8\pi s$$

Euclidean Taub-NUT-AdS space

Euclidean Taub-NUT-AdS is written

$$ds_E^2 = F_E(r)^2 \left[d\tau + 4\chi \sin^2\left(\frac{\theta}{2}\right) d\phi \right]^2 + \frac{dr^2}{F_E(r)^2} + (r^2 - \chi^2) d\Omega_2^2,$$

$$F_E(r)^2 = \frac{l^2(r^2 - \chi^2)^2 + (\kappa - 4l^{-2}\chi^2)(r^2 + \chi^2) - 2Mr}{r^2 - \chi^2}$$

- Horizon is located at $F_E(r_+) = 0$
- **NUT** solution occurs when the fixed point set of ∂_τ is zero dimensional, e.g. $r_+ = \chi$ (denoted as **TN-AdS**)
- **Bolt** solution occurs when the fixed point set of ∂_τ is two dimensional, e.g. $r_+ \neq \chi$ (denoted as **TB-AdS**)

Thermodynamics of non-charged TB/TN-AdS space

non-charged TB/TN-AdS

The Euclidean action and metric

$$I_E = -\frac{1}{16\pi G_4}\int d^4x\sqrt{g}\left(R + \frac{6}{l^2}\right) - \frac{1}{8\pi G_4}\int d^3x\sqrt{h}K \quad (1)$$

$$I_{\text{ren.}} = I_E + \frac{1}{8\pi G_4}\int d^3x\sqrt{h}\left(\frac{2}{l} + \frac{l}{2}R_3\right), \quad (2)$$

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \frac{3}{l^2}g_{\mu\nu} = 0 \quad (3)$$

$$ds^2_{\text{E}} = f_E(r)(d\tau + 2\chi\lambda(\theta)d\phi)^2 + \frac{dr^2}{f_E(r)} + (r^2 - \chi^2)(d\theta^2 + Y(\theta)^2 d\phi^2), \quad (4)$$

$$f_E = \frac{l^{-2}(r^2-\chi^2)^2 + (\kappa - 4l^{-2}\chi^2)(r^2+\chi^2) - 2Mr}{r^2-\chi^2}.$$

$$\lambda(\theta) = \begin{cases} \cos\theta \\ -\theta \\ -\cosh\theta \end{cases}, \qquad Y(\theta) = \begin{cases} \sin\theta & \text{for } \kappa = 1 \\ 1 & \text{for } \kappa = 0 \\ \sinh\theta & \text{for } \kappa = -1, \end{cases}$$

Thermodynamics of non-charged TN-AdS

- Taub-"NUT"-AdS solution when $r_+ = \chi$ where $f_E(r+) = 0$
- metric function

$$f_n(r) = \frac{l^{-2}(r-\chi)^2(r+3\chi)\chi + \kappa(r-\chi)\chi}{\chi(r+\chi)}, \qquad M_n = \chi\kappa - \frac{4\chi^3}{l^2}$$

- Hawking temperature

$$T_n = \frac{1}{4\pi} f_n'(\chi) = \frac{\kappa}{8\pi\chi}$$

- Entropy and Energy

$$S = \left(\beta\frac{\partial}{\partial\beta} - 1\right) I_{\text{ren.}} = \frac{2\pi\omega\chi^2}{G}\left(\kappa - \frac{6\chi^2}{l^2}\right),$$

$$E = \partial_\beta I_{\text{ren.}} = \frac{\chi\omega}{2G}\left(\kappa - \frac{4\chi^2}{l^2}\right)$$

- First law and free energy

$$dE = TdS, \qquad F = E - TS$$

Thermodynamics of non-charged TB-AdS ($\kappa \neq 1$)

- Taub-Bolt-AdS solution when $r_+ \neq \chi$ where $f_E(r+) = 0$
- mass parameter

$$M_b = \frac{-6\chi^2 r_b^2 + r_b^4 - 3\chi^4 + \kappa\left(r_b^2 + \chi^2\right) l^2}{2l^2 r_b}.$$

- Hawking temperature

$$T = \frac{1}{4\pi} f_{\mathrm{E}}'(\chi) = \frac{M_b\left(r_b^2 + \chi^2\right) + (9\chi^4 r_b - 2\chi^2 r_b^3 + r_b^5) l^{-2} - 2\kappa r_b \chi^2}{2\pi\left(\chi^2 - r_b^2\right)^2}.$$

- Entropy and Energy

$$S_{(\kappa\neq 1)} = \left(\beta \frac{\partial}{\partial \beta} - 1\right) I_{\mathrm{ren.}} = \frac{\pi\omega}{2G}(r_b^2 - \chi^2)$$

$$E_{(\kappa\neq 1)} = \partial_\beta I_{\mathrm{ren.}} = \frac{\omega}{4G}\left(\kappa + \frac{r_b^3 - 3r_b\chi^2}{l^2}\right)$$

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Thermodynamics of non-charged TB-AdS ($\kappa = 1$)

- Hawking temperature

$$T = \frac{1}{4\pi} f'_{\mathrm{E}}(\chi) = \frac{M_b\left(r_b^2 + \chi^2\right) + (9\chi^4 r_b - 2\chi^2 r_b^3 + r_b^5) l^{-2} - 2\kappa r_b \chi^2}{2\pi\left(\chi^2 - r_b^2\right)^2}$$

$$= \frac{1}{8\pi\chi}$$

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$$\rightarrow \quad r_b = \frac{1}{12\chi}\left(l^2 \pm \sqrt{l^4 + 144\chi^4 - 48 l^2 \chi^2 \kappa}\right), \quad \chi = \chi_{\mathrm{max}} = \frac{l}{2}\sqrt{\frac{1}{3}(2 - \sqrt{3})}$$

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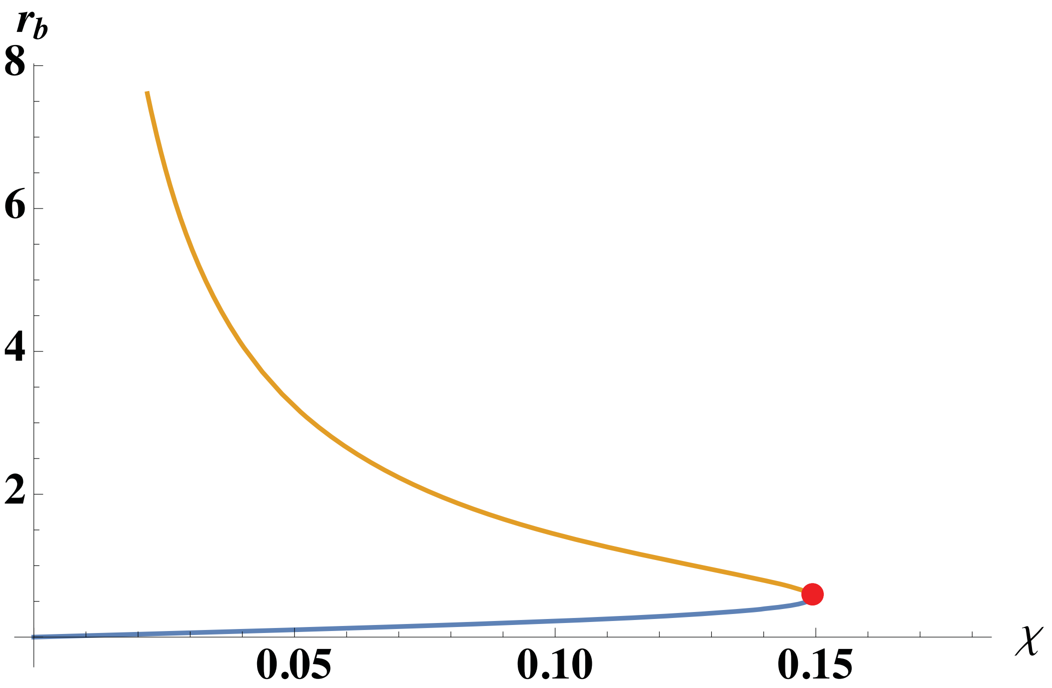

Thermodynamics of non-charged TB-AdS ($\kappa = 1$)

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$$S_{(\kappa=1)} = \left(\beta \frac{\partial}{\partial \beta} - 1\right) I_{\text{ren.}} = \frac{2\pi\omega\chi}{G}\left(M_b + \frac{r_b^3 - 3r_b\chi^2}{l^2}\right),$$

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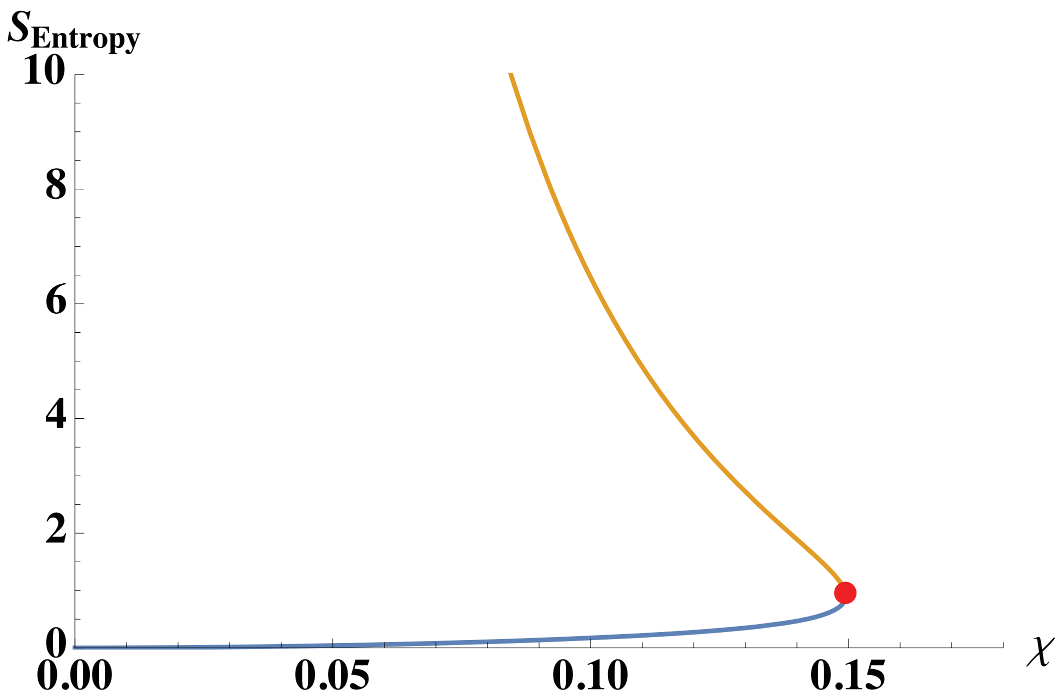

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- Heat capacity

$$C = T\frac{\partial S}{\partial T} = T\left(\frac{\partial T}{\partial M}\frac{\partial M}{\partial r_b}\frac{\partial r_b}{\partial S}\right)^{-1} = \frac{\pi\omega}{G}\frac{r_b(12\chi^3 - l^2 r_b)}{(-12 r_b \chi + l^2)}$$

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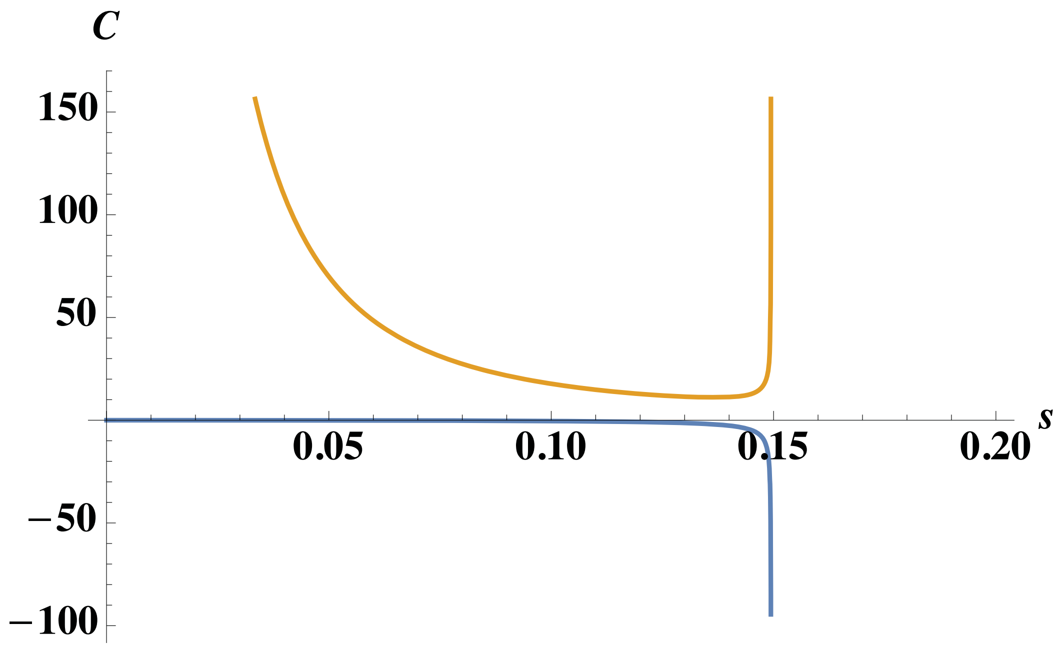

Thermodynamics of dyonic RN black hole

Dyonic RN black hole

$$I_{\mathrm{ren}} = \int_{\mathcal{M}} \sqrt{-g}\left(\frac{1}{\kappa^2}R - \frac{1}{4}F^2\right) + \frac{2}{\kappa^2}\int_{\partial\mathcal{M}} \sqrt{-h}(K - \hat{K}), \tag{5}$$

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_2^2, \qquad f(r_h) = 0, \tag{6}$$

$$f(r) = 1 - \frac{2M}{r} + \frac{q^2 + p^2}{4r^2}, \qquad F = \frac{1}{\kappa}\left(\frac{q}{r^2}dt \wedge dr + p\Omega_2\right). \tag{7}$$

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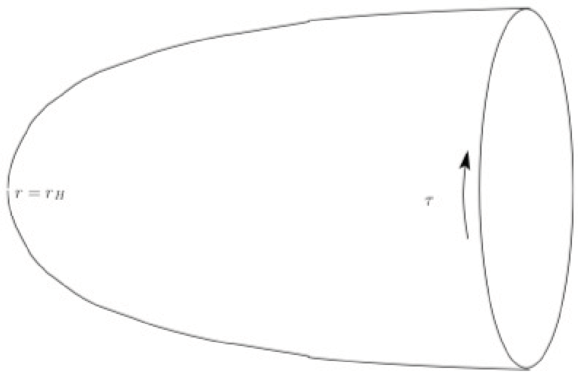

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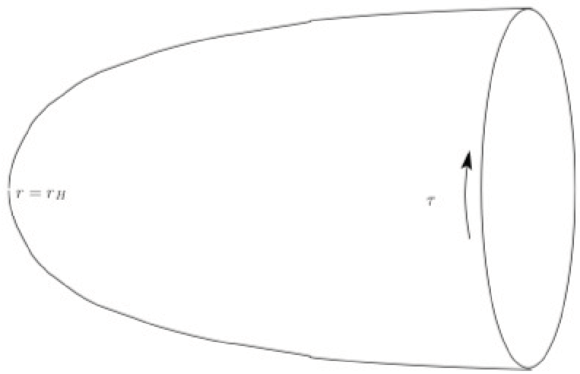

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- Regularity Condition on ds_E^2 : Time periodicity $\rightarrow$ Hawking Temperature

$$f(r)d\tau^2 + \frac{dr^2}{f(r)}\bigg|_{r\sim r_h+\epsilon} \sim \rho^2 d\psi^2 + d\rho^2 \quad (8)$$

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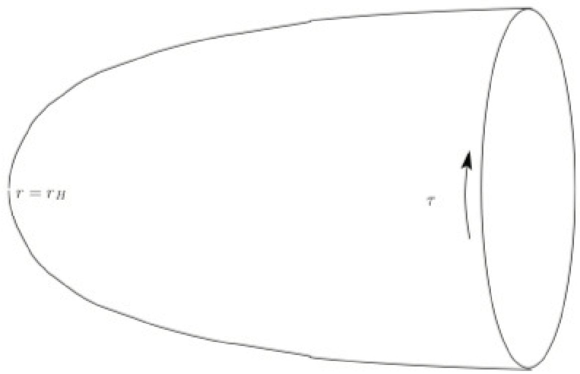

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- Regularity Condition on A ($F = dA$) : A should be regular at the horizon

$$A_t(r_h) = 0 \quad\rightarrow\quad A_t(r) = \frac{1}{\kappa}\left(\frac{q}{r} - \frac{q}{r_h}\right), \qquad A^2(r_h) = \frac{p^2}{\kappa^2 r_h^2}\cot^2(\theta) \tag{9}$$

Dyonic RN black hole

- Electric potential $\Phi_E^{(1)}$: conjugate variable of Q

$$I_{\mathrm{E,\ ren}} = \beta F, \qquad F = E - TS - \Phi_E^{(1)} Q \tag{10}$$

$$\frac{\partial I_{\mathrm{ren}}}{\partial \beta} = E - \Phi_E^{(1)} Q, \qquad \Phi_E^{(1)} = \frac{\partial E}{\partial Q}\bigg|_{r_b} = \frac{1}{\kappa}\frac{q}{r_h} \tag{11}$$

- Electric potential $\Phi_E^{(2)}$

$$\Phi_E^{(2)} = A_t(r_h) - A_t(\infty) = \frac{1}{\kappa}\frac{q}{r_h} \tag{12}$$

- $\Phi_E^{(1)}$ and $\Phi_E^{(2)}$ are agreed : $\Phi_E^{(1)} = \Phi_E^{(2)} = \Phi_E$
- First law

$$dE = TdS + \Phi_e dQ + \Phi_m dP \tag{13}$$

where

$$Q_m = \int *F, \qquad Q_m = \int F \tag{14}$$

Thermodynamics of charged TB/TN-AdS space

Charged TB/TN-AdS spacetime

$$I_E = -\frac{1}{16\pi G_4}\int d^4x\sqrt{g}\left(R + \frac{6}{l^2} - F^{\mu\nu}F_{\mu\nu}\right) - \frac{1}{8\pi G_4}\int d^3x\sqrt{h}K \quad (15)$$

$$I_{\text{ren.}} = I_E + \frac{1}{8\pi G_4}\int d^3x\sqrt{h}\left(\frac{2}{l} + \frac{l}{2}R_3\right), \quad (16)$$

$$ds_{\text{E}}^2 = f_E(r)(d\tau + 2\chi\lambda(\theta)d\phi)^2 + \frac{dr^2}{f_E(r)} + (r^2 - \chi^2)(d\theta^2 + Y(\theta)^2 d\phi^2), \quad (17)$$

$$f_E = \frac{l^{-2}(r^2-\chi^2)^2 + (\kappa - 4l^{-2}\chi^2)(r^2+\chi^2) - 2Mr + P^2 + Q^2}{r^2-\chi^2}, \quad (18)$$

$$A_{\text{E}} = \frac{1}{2\chi}h_E(r)\left(d\tau + 2\chi\lambda(\theta)d\phi\right), \qquad h_{\text{E}} = \frac{2iQ\chi r - P(r^2+\chi^2)}{r^2-\chi^2} \quad (19)$$

- TB-AdS solution when $r = r_b \neq \chi$, $f_E(r_b) = 0$
- TN-AdS solution when $r = r_n = \chi$, $f_E(r_n) = 0$

Electric and magnetic charge

- Electric charge

$$Q_e[\xi_t] \equiv \frac{1}{\omega}\int_{\partial\Sigma_\kappa} *F = \lim_{r\to\infty}\frac{Q\left(r^2 - s^2\right) - 2Prs}{r^2 + s^2} = Q, \tag{20}$$

- Magnetic charge

$$Q_m[\xi_\phi] \equiv \frac{1}{\omega}\int_{\partial\Sigma_\kappa} F = \lim_{r\to\infty}\frac{P\left(r^2 - s^2\right) + 2Qrs}{r^2 + s^2} = P \tag{21}$$

Regularity conditions on dyonic TB-AdS

- Regularity condition on ds_E^2 at $r = r_b$

$$T = \frac{1}{4\pi} f_E'(r_b) = \frac{l^2(\kappa r_b - M_b) + 2r_b\left(r_b^2 - 3\chi^2\right)}{2\pi l^2\left(r_b^2 - \chi^2\right)} \tag{22}$$

- Regularity condition on A at $r = r_b$

$$A_{\rm E} = \frac{1}{2\chi} h_E(r)\Big(d\tau + 2\chi\lambda(\theta)d\phi\Big), \quad h_{\rm E} = \frac{2iQ\chi r - P(r^2 + \chi^2)}{r^2 - \chi^2}, \tag{23}$$

$$A_{{\rm E},\tau}(r_b) = 0 \quad \rightarrow \quad h_E(r_b) = 0 \quad \rightarrow \quad P = \frac{2iQ\chi r_b}{r_b^2 + \chi^2} \tag{24}$$

Electric and magnetic Potential of dyonic TB-AdS

- Electric $\Phi_E^{(1)}$ and magnetic $\Phi_M^{(1)}$ potential : conjugate variable of Q and P

$$I_{\mathrm{E,\ ren}} = \beta F, \qquad F = E - TS - \Phi_E^{(1)} Q - \Phi_M^{(1)} P \tag{25}$$

$$\frac{\partial I_{\mathrm{ren}}}{\partial \beta} = E - \Phi_E^{(1)} Q - \Phi_M^{(1)} P, \tag{26}$$

$$\Phi_E^{(1)} = \left.\frac{\partial E}{\partial Q}\right|_{r_b, P}, \qquad \Phi_M^{(1)} = \left.\frac{\partial E}{\partial P}\right|_{r_b, Q} \tag{27}$$

- Electric $\Phi_E^{(2)}$ and magnetic $\Phi_M^{(2)}$ potential

$$\Phi_E^{(2)} = A_\tau(r_h) - A_\tau(\infty), \qquad \Phi_M^{(2)} = \Phi_M^{(2)}(r_h) - \Phi_M^{(2)}(\infty) \tag{28}$$

where

$$\Phi_M^{(2)}(r) = \int^r dr' B(r'), \qquad B(r) = \frac{1}{\sqrt{g}} \epsilon^{tr\theta\phi} F_{\theta\phi} \tag{29}$$

- $\Phi_E^{(1)} \neq \Phi_E^{(2)}$ and $\Phi_M^{(1)} \neq \Phi_M^{(2)}$
- Once imposing the regularity condition, $\Phi_E^{(1)} = \Phi_E^{(2)}$

Thermodynamics of dyonic TB-AdS ($\kappa \neq 1$)

- temperature and entropy

$$T = \frac{1}{4\pi} f_E'(r_b) = \frac{l^2(\kappa r_b - M_b) + 2r_b\left(r_b^2 - 3\chi^2\right)}{2\pi l^2\left(r_b^2 - \chi^2\right)},$$

$$S \equiv \left(\beta \frac{\partial}{\partial\beta} - 1\right) I_{\mathrm{E}} = \frac{\pi\omega\left(r_b^2 - \chi^2\right)}{2G},$$

- by using thermodynamic relations

$$\Phi_{\mathrm{E}}^{(1)} = \frac{\omega}{4\pi G}\frac{Q\left(r_b^2 + \chi^2\right)r_b + 2iPr_b^2\chi}{\left(r_b^2 - \chi^2\right)^2}, \qquad \Phi_{\mathrm{M}}^{(1)} = \frac{\omega}{4\pi G}\frac{2iQr_b^2\chi - P\left(r_b^2 + \chi^2\right)r_b}{\left(r_b^2 - \chi^2\right)^2}$$

$$E^{(1)} = \frac{r_b\omega}{8\pi G}\left(\kappa + \frac{r_b^2 - 3\chi^2}{l^2} + \frac{\left(Q^2 - P^2\right)\left(r_b^2 + \chi^2\right) + 4iPQr_b\chi}{\left(r_b^2 - \chi^2\right)^2}\right),$$

- by using conventional method

$$\Phi_{\mathrm{E}}^{(2)} = \frac{\omega}{4\pi G}\frac{Qr_b + iP\chi}{\left(r_b^2 - \chi^2\right)}, \qquad \Phi_{\mathrm{M}}^{(2)} = \frac{\omega}{4\pi G}\frac{iQ\chi - Pr_b}{\left(r_b^2 - \chi^2\right)}.$$

$$E^{(2)} = \frac{\omega}{8\pi G}\left(\kappa r_b + \frac{r_b^3 - 3r_b\chi^2}{l^2} + \frac{\left(P^2 r_b - 2iPQ\chi\right)\left(r_b^2 + \chi^2\right) + Q^2 r_b\left(r_b^2 - 3\chi^2\right)}{\left(r_b^2 - \chi^2\right)^2}\right).$$

Thermodynamics of dyonic TB-AdS ($\kappa \neq 1$)

- imposing the regularity condition

$$E^{(1)} = E^{(2)} = E = \frac{\omega}{8\pi G}\left(\kappa r_b + \frac{r_b\left(r_b^2 - 3\chi^2\right)}{l^2} + \frac{Q^2 r_b}{r_b^2 + \chi^2}\right), \quad (30)$$

$$\Phi_{\mathrm{E}}^{(1)} = \Phi_{\mathrm{E}}^{(2)} = \Phi_{\mathrm{E}} = \frac{\omega}{4\pi G}\frac{Q r_b}{\left(r_b^2 + \chi^2\right)}, \quad (31)$$

$$\Phi_{\mathrm{M}}^{(1)} = 0 \neq \Phi_{\mathrm{M}}^{(2)} = -\frac{\omega}{4\pi G}\frac{iQ\chi}{\left(r_b^2 + \chi^2\right)} \quad (32)$$

- thermodynamic relations are satisfied

$$dE = TdS + \Phi_{\mathrm{E}} dQ, \quad (33)$$

$$F = E - TS - \Phi_{\mathrm{E}} Q \quad (34)$$

upon using $\Phi_{\mathrm{M}}^{(1)} = \Phi_{\mathrm{M}} = 0$.

Thermodynamics of dyonic TB-AdS ($\kappa = 1$)

- Thermodynamic quantities

$$T = \frac{1}{4\pi} f'_E(r_b) = \frac{1}{8\pi\chi}, \quad S = \frac{\pi\omega}{2G}\left[r_b^2 + \chi^2 - \frac{24\chi^3 r_b}{l^2} + \frac{8Q^2\chi^3 r_b}{\left(r_b^2+\chi^2\right)^2}\right]$$

- by using thermodynamic relations

$$\Phi_{\mathrm{E}}^{(1)} = \frac{\omega}{4\pi G}\frac{r_b\left(2iP\chi r_b + Q(r_b^2+\chi^2)\right)}{\left(\chi^2 - r_b^2\right)^2}, \qquad \Phi_{\mathrm{M}}^{(1)} = -\frac{\omega}{4\pi G}\frac{r_b\left(P(r_b^2+\chi^2) - 2iQ\chi r_b\right)}{\left(\chi^2 - r_b^2\right)^2}.$$

$$E^{(1)} = \frac{\omega}{8\pi G}\left[M_b + \frac{r_b^2+\chi^2}{4\chi} - \frac{3\chi^2 r_b + r_b^3}{l^2} + \frac{\left(Q^2 - P^2\right)\left(-3\chi^4 r_b - 6\chi^2 r_b^3 + r_b^5\right)}{\left(r_b^2-\chi^2\right)^3} + \frac{16iPQ\chi^3 r_b^2}{\left(\chi^2 - r_b^2\right)^3}\right],$$

- by using conventional method

$$\Phi_{\mathrm{E}}^{(2)} = \frac{\omega}{4\pi G}\frac{(Qr_b + iP\chi)}{\left(r_b^2 - \chi^2\right)}, \quad \Phi_{\mathrm{M}}^{(2)} = \frac{\omega}{4\pi G}\frac{(-Pr_b + iQ\chi)}{\left(r_b^2-\chi^2\right)}$$

$$E^{(2)} = \frac{\omega}{8\pi G}\left[M_b + \frac{r_b^2+\chi^2}{4\chi} - \frac{r_b\left(r_b^2+3\chi^2\right)}{l^2} + \frac{1}{(r_b^2-\chi^2)^3}\Big(P^2 r_b(r_b^4 + 6r_b^2\chi^2 + \chi^4) + 2iPQ\chi(-3r_b^4 - 6r_b^2\chi^2 + \chi^4) + Q^2 r_b(r_b^4 - 10r_b^2\chi^2 + \chi^4)\Big)\right].$$

Thermodynamics of dyonic TB-AdS ($\kappa = 1$)

- imposing the regularity condition

$$E = \frac{\omega}{8\pi G}\left[M_b + \frac{r_b^2+\chi^2}{4\chi} - \frac{r_b\left(r_b^2+3\chi^2\right)}{l^2} + \frac{Q^2\left(3\chi^2 r_b + r_b^3\right)}{\left(r_b^2+\chi^2\right)^2}\right], \tag{35}$$

$$S = \frac{\omega}{4G}\left[r_b^2+\chi^2 - \frac{24\chi^3 r_b}{l^2} + \frac{8Q^2\chi^3 r_b}{\left(r_b^2+\chi^2\right)^2}\right], \tag{36}$$

$$\Phi_{\mathrm{E}} = \frac{\omega}{4\pi G}\frac{Q r_b}{\left(r_b^2+\chi^2\right)} \tag{37}$$

- thermodynamic relations are satisfied

$$dE = TdS + \Phi_{\mathrm{E}} dQ \tag{38}$$

$$F = E - TS - \Phi_{\mathrm{E}} Q \tag{39}$$

- Heat Capacity ($P = -2i\chi g$)

$$C_{P,Q} \equiv -\beta\frac{\partial S}{\partial \beta} = \frac{\omega l^2 r_b \chi^2}{2Ga_1}\left[\frac{\left(r_b^2-\chi^2\right)^2\left(l^2 r_b - 12\chi^3\right)}{l^2\chi^2} + \frac{4\chi\left(Q^2-P^2\right)\left(9r_b^4+14r_b^2\chi^2+\chi^4\right)+16iPQr_b\left(r_b^4+8r_b^2\chi^2+3\chi^4\right)}{\left(r_b^2-\chi^2\right)^2}\right]$$

Thermodynamics of charged TB-AdS ($\kappa = 1$)

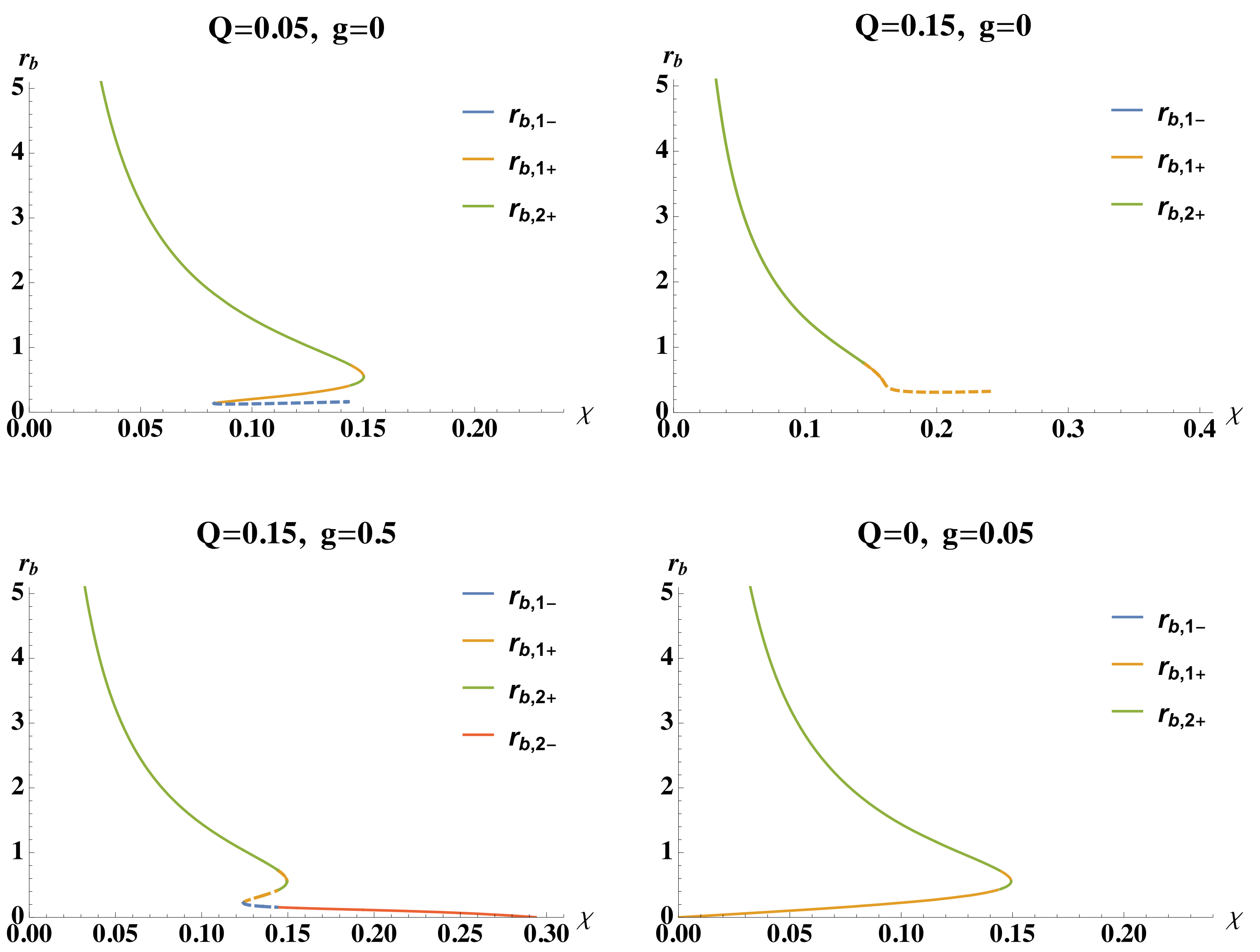

Figure: The horizon radius of TB-AdS for $\kappa = 1$ vs χ for fixed values of Q and g with $G = l = 1$.

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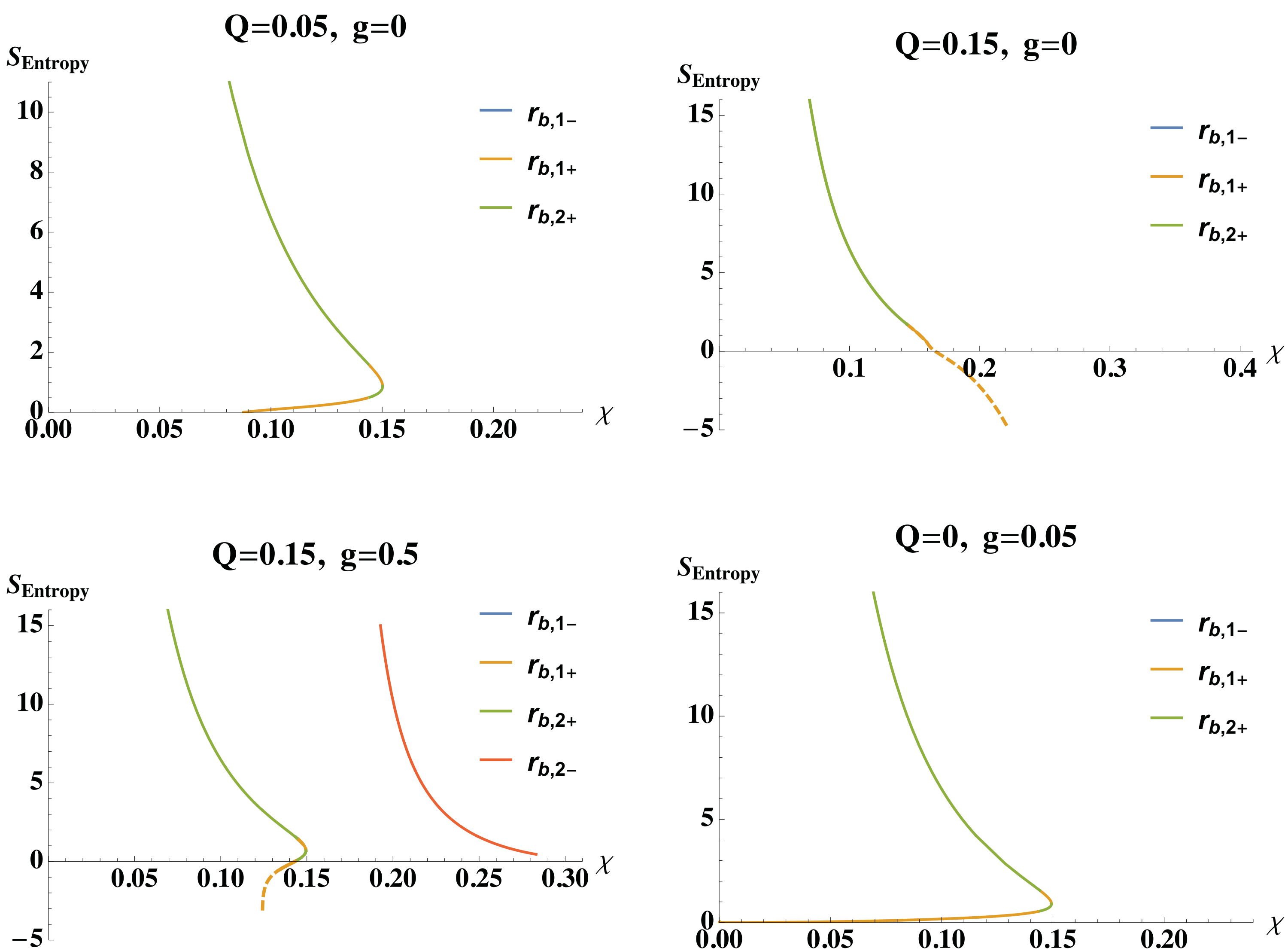

Figure: Entropy of TB-AdS for $\kappa = 1$ vs χ for fixed values of Q and g with $G = l = 1$

Thermodynamics of charged TB-AdS ($\kappa = 1$)

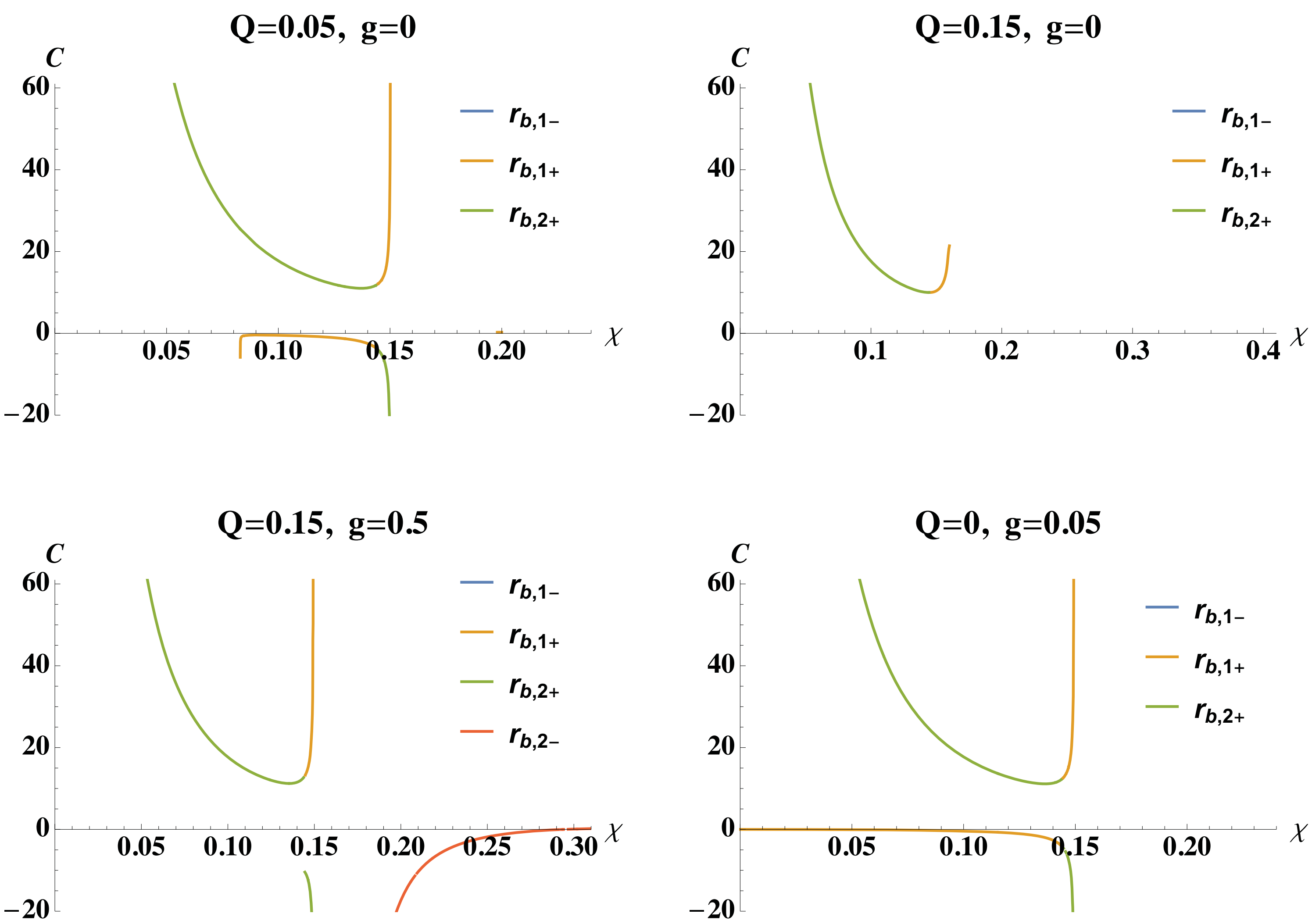

Figure: Heat capacity for TB-AdS for $\kappa = 1$ vs χ for fixed Q and g with $G = l = 1$

Thermodynamics of charged TN-AdS ($\kappa = 1$)

- Taub-"NUT"-AdS solution when $r_+ = \chi$ where $f_E(r+) = 0$

$$f_n(r) = \frac{l^{-2}(r-\chi)^2(r+3\chi)\chi + \kappa(r-\chi)\chi - (P^2+Q^2)}{\chi(r+\chi)}, \tag{40}$$

$$l^{-2}(r_+-\chi)^2(r_++3\chi)\chi + \kappa(r_+-\chi)\chi - P^2 - Q^2 = 0 \tag{41}$$

$$\to P^2 + Q^2 = 0 \;\; (P = iQ) \;\; \text{is required}$$

- Hawking temperature

$$T_n = \frac{1}{4\pi} f_n'(\chi) = \frac{\kappa}{8\pi\chi}$$

- entropy, energy, and gauge field potential ($P = 2iv\chi \to Q = 2v\chi$)

$$S = \left(\beta\frac{\partial}{\partial\beta} - 1\right) I_{\text{ren.}} = \frac{2\pi\omega\chi^2}{G}\left(1 + 2v^2 - \frac{6\chi^2}{l^2}\right),$$

$$E = \partial_\beta I_{\text{ren.}} = \frac{\omega\chi}{2G}\left(1 + 2v^2 - \frac{4\chi^2}{l^2}\right),$$

$$\Phi = \frac{i\omega}{2G}\frac{1}{2\chi}\Big(h_{\text{E}}(\chi) - h_{\text{E}}(\infty)\Big) = \frac{\omega}{2G}v$$

- First law and free energy

$$dE = T_n dS + \Phi dQ, \qquad F = E - T_n S - \Phi Q.$$

Thermodynamics of charged TN-AdS ($\kappa \neq 1$)

For $\kappa = 0$ case,

- zero Hawking temperature
- energy and gauge potential

$$E = \frac{\omega}{2G}\left(\frac{Q^2}{4\chi} - \frac{\chi^3}{l^2}\right), \qquad \Phi = \frac{Q\omega}{4G\chi} \tag{42}$$

- First law and free energy

$$dE = \Phi dQ, \qquad F = E - \Phi Q. \tag{43}$$

For $\kappa = -1$ case, $f_n(r)$ becomes negative near $r_+ = \chi$ and so we exclude this case.

"Thermodynamics" of Extremal TB-AdS

Near Horizon Geometry of the extremal TB-AdS

The extremal limit

$$T_{\mathrm{H}} \to 0.$$

At the zero temperature limit, the metric is factorized as follows

$$f_{\mathrm{E}} = \frac{1}{r^2 l^2}(r + r_0 + \alpha)(r + r_0 - \alpha)(r - r_0)^2$$

where

$$r_0 = \sqrt{\chi^2 + \frac{1}{6} l \left(\sqrt{\kappa^2 l^2 + 12 \left(Q^2 + P^2\right)} - \kappa l \right)},$$

$$\alpha = \sqrt{4\chi^2 - \frac{1}{3} l \left(\sqrt{\kappa^2 l^2 + 12 \left(Q^2 + P^2\right)} + 2\kappa l \right)}.$$

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$$\begin{aligned} ds^2_{\rm ext} \sim & \frac{\tilde{r}^2}{l^2} \frac{(4r_0^2 - \alpha^2)}{r_0^2 - \chi^2} d\tilde{\tau}^2 + \frac{l^2}{\tilde{r}^2} \frac{r_0^2 - \chi^2}{(4r_0^2 - \alpha^2)} d\tilde{r}^2 + (r_0^2 - \chi^2)(d\theta^2 + Y(\theta)^2 d\phi^2), \\ \sim & AdS_2 \times S^2 \ (H^2 \text{ or } R^2) \end{aligned}$$

"Thermodynamics" of Extremal TB-AdS

- $\kappa \neq 1$ case : We can take the zero temperature limit.
- $\kappa = 1$ case : We imposed the following condition at finite temperature

$$T_H = \frac{1}{2\pi} f'(r_b) = \frac{1}{8\pi\chi} \tag{44}$$

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[Conclusion2.] **zero temperature limit for $\kappa = 1$ case without $\Delta\tau = 4\chi$**

"Thermodynamics" of Extremal TB-AdS

The mass parameter and charges are related as

$$M = \frac{\left(2\kappa l^2 - 12\chi^2 + l\Delta\right)\sqrt{6\chi^2 - l^2\kappa + l\Delta}}{3\sqrt{6}l^2},$$

$$\Delta = \sqrt{\kappa^2 l^2 + 12(Q^2 + P^2)}$$

The free energy and energy are

$$F_{ext} = -\frac{\omega r_b}{2G}\left(\frac{\kappa\chi^2}{r_b^2 - \chi^2} + \frac{r_b^2 + 3\chi^2}{l^2}\right),$$

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The first law and free energy are satisfied

$$F_{ext} = E_{ext} - \Phi_{\rm E} Q|_{ext},$$

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"Thermodynamics" of Extremal TB-AdS for $\kappa = 1$

Inspired by the idea[1] that

- $\omega \equiv \frac{1}{T}$ where ω is the conjugate variable to the charges $Q + J$
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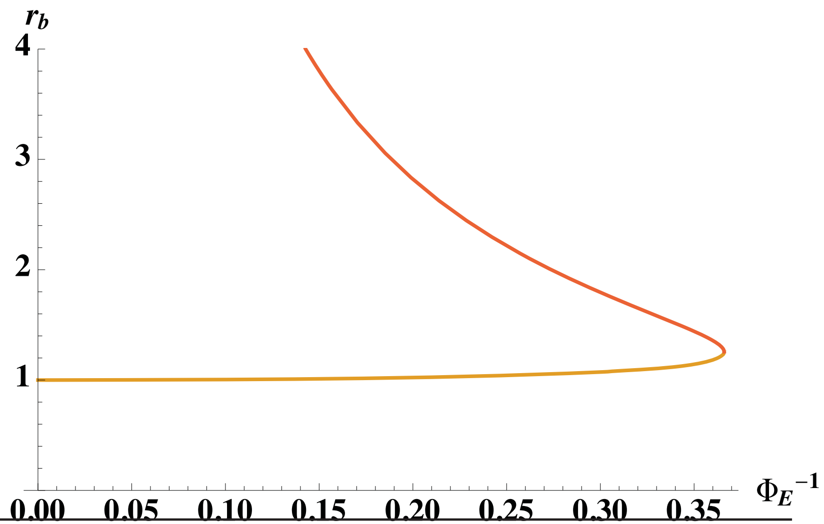

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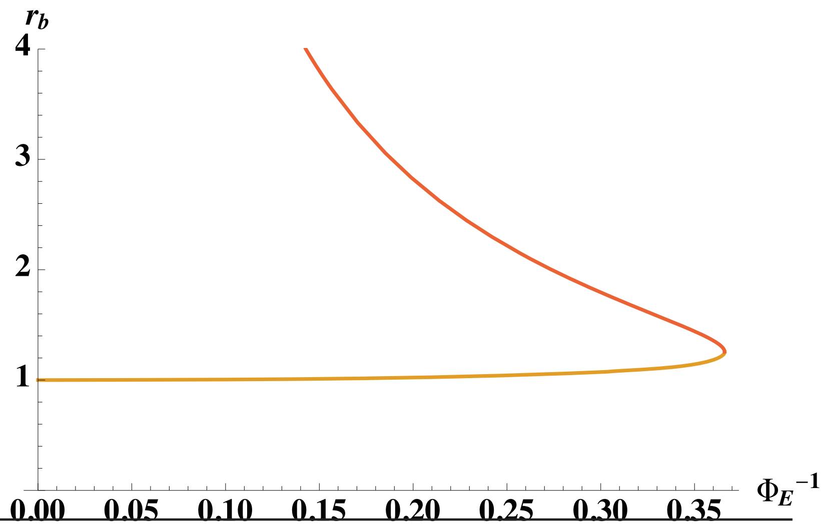

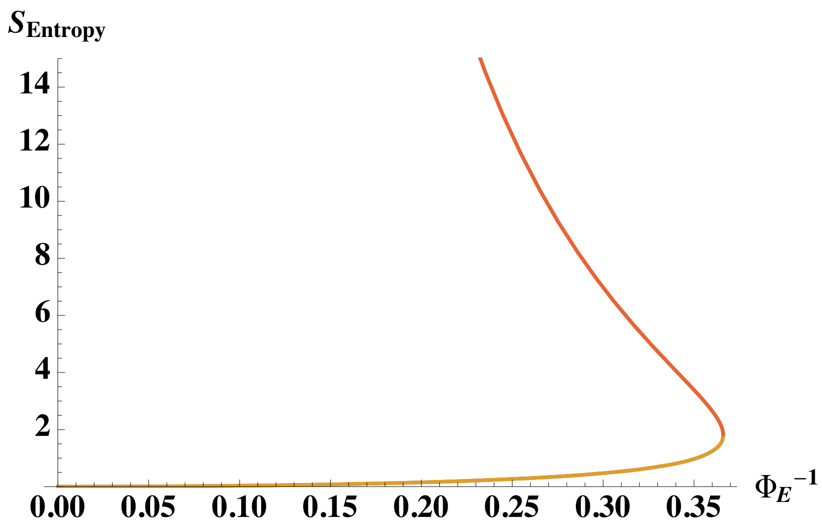

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Now let us define the heat capacity as follows

$$C \equiv T \frac{\partial S}{\partial T} \quad \rightarrow \quad C \equiv \Phi_{\mathrm{E}} \frac{\partial S}{\partial \Phi_{\mathrm{E}}}$$

then

$$C = \Phi_{\mathrm{E}} \frac{\partial S}{\partial \Phi_{\mathrm{E}}} = \frac{\pi \omega r_b^2 \left(r_b^2 - \chi^2\right) \left(3 r_b^2 + \kappa l^2 - 3\chi^2\right)}{G \left(-6\chi^2 r_b^2 + 3 r_b^4 - \kappa l^2 \chi^2 + 3\chi^4\right)}$$

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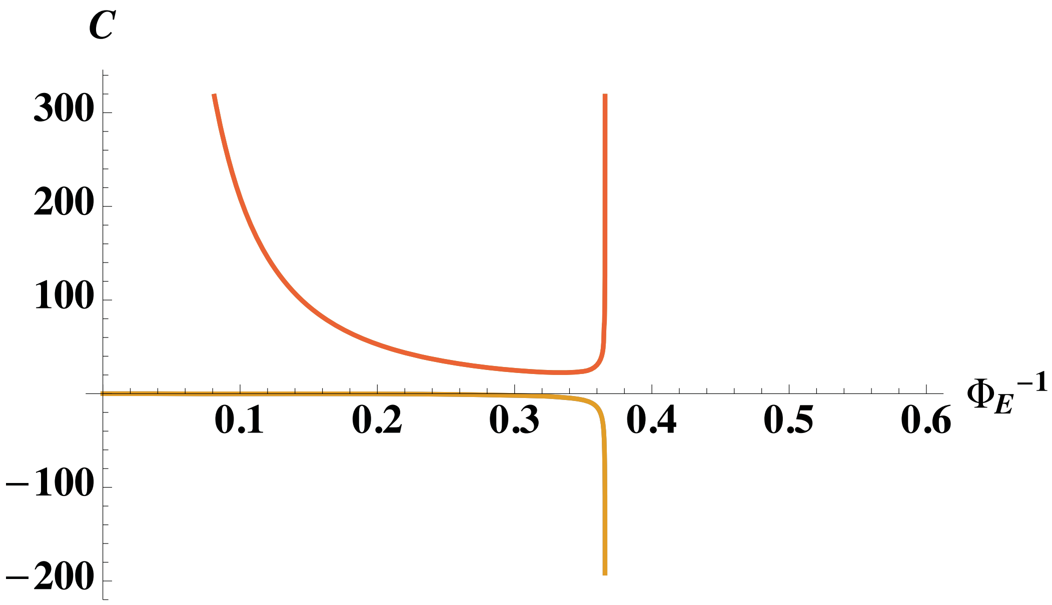

Summary and Future works

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- The regularity condition which comes from $A_t(r_+) = 0$ is necessary to satisfy the first law and free energy.
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- (Recently, there have been studied about thermodynamics of Lorentzian Taub-NUT-spacetimes.)

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