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Ti -like theories and their symmetries w/ Brunole Floch

1) Introduction

Carton of RG
Holographic RG [MaGaugh, MM, Verinde]


Love: don't do $K$ !

- each order in PT new div. $\Rightarrow$ ls of predictability extra structure
- IR dof not appropriate for UV physics open problem
- in AdS/CFT turning on inelevant ops destroys UV AdS Dirichlet wall

Motivation

- TT leads to novel UV behavior [Dulorsky, Flauger, Garbenko]
- Tricritical 1 sing $\rightleftarrows$ ling $S$-mex: $S_{2 \rightarrow 2}(S)=\frac{i M^{2}-S}{i M^{2}+S}$ [Zamdodchiker] Expect that $\leftarrow$ is a TF-like def.
- Solving a large family of $T \bar{T}$-like theories gives UV discovery machine

2) Strategy
3) $\theta_{\lambda} \frac{d S}{d \lambda} \propto \int \theta_{\lambda}$
4) $\theta_{\lambda}(y) \equiv \lim _{x \rightarrow y} \varepsilon^{\mu \nu} \int_{\mu}^{(1)}(x) \int_{\nu}^{(2)}(y)+$ (tot. der.) [Smimnov, Zandodchikov] [ $T^{\top} \alpha \operatorname{det} T_{\mu \nu}=\frac{1}{2} \varepsilon^{\alpha \beta} \varepsilon^{\mu \nu} T_{\alpha \mu} T_{\beta \nu}$ belongs to this class.] sues on curved spaces.
5) Factorization for thy on $S_{L} \times \mathbb{R}$ :

$$
\left.\langle n| \sigma_{\lambda}(y)|n\rangle_{\lambda}=\varepsilon^{\mu \nu}\langle n| J_{\mu}^{(1)}\left|n X_{\lambda}\right| n\left|\int_{\nu}^{(2)}\right| n\right\rangle_{\lambda}
$$

4) $\left\langle J_{t}\right\rangle_{n}=\frac{Q_{n}}{L},\left\langle J_{x}\right\rangle_{n}=$ ?

Fa $T \bar{T}$ we are lucky, since $\left\langle T_{t x}\right\rangle_{n}=\left\langle T_{x+}\right\rangle_{n}=\frac{i P_{n}}{L},\left\langle T_{x+}\right\rangle_{n}=-d_{i} E_{n}$ Burgers eq. readily follows: $\partial_{\lambda} E_{n}=E_{n} \partial_{L} E_{n}+\frac{P_{n}^{2}}{L}$
3) Determining $\left\langle J_{x}\right\rangle_{n}$

- In the case of $J \bar{T}$ defamation can use holomorphy of $J$ to conclude $\left\langle J_{x}\right\rangle_{n}=-i\left\langle J_{t}\right\rangle_{n}[z \equiv x+i t]$
[Guica; Chaknaborty, Giveon, kutasov]
- In more general theories W/CFT seed [Le Flock, MM]
i) $\left.\begin{array}{rl}\frac{\partial}{\partial a} S & =\int i J_{x} \\ \frac{\partial}{\partial b} S & =\int i T_{t x}\end{array}\right\}$ keep $J_{t}, T_{x t}$ fixed

The defamations commute, $S(a, \bar{a}, b)$ can be determined
[Equivalent to coupling to const bind gauge field and metric.]
This approach doesn't work for KdV currents.
ii) Conjecture eq. around a generic point

$$
\begin{equation*}
\partial_{\lambda} S=\int \sigma_{\lambda}+O\left(b^{\#}, a^{\#}\right) \tag{*}
\end{equation*}
$$

based on classical scalar.
No dependence on $\lambda$ !
iii) Solve the enlarged system

Solution: $0=A E_{n}^{2}+B E_{n}+C$ (**)
(v) Checks:

- (*) works in $\sigma$-models, complex scalar w/ potential
- (*) true in quantum PT
- (**) matches AdS/CFT results (far $a, b=0$ ) [Giveon et al.,...]
v) Would be interesting to understand how other approaches fix $\left\langle J_{x}\right\rangle_{n}$.
Integrability: [Conte, Negro, Tateo]
Lightcone gauge string: [Frolor]
Path integral: [Agcilera - Damia et al., Tolley] Kevel from dual string [Hashimoto, Kutasor]
- We find that $\left\langle J_{x}^{(k \alpha V)}\right\rangle_{n}$ is fixed in terms of $p_{n}^{(k d V)}$ and $\partial_{L} P_{n}^{(K d V)} \quad[L e F l o c h, M M]$

4) Symmetries and flow of KdV charges What ssm's are preserved?
$\sigma_{\lambda}=\varepsilon^{\mu \nu} J_{\mu}^{(1)} J_{\nu}^{(2)}$ deformation preserves any $J_{\mu}^{(c)}$ for which $\left[Q^{(1,2)}, Q^{(c)}\right]=0[$ Le Foch, MM]
$\Rightarrow$ only def's $w /\left[Q^{(1)}, Q^{(2)}\right]=0$ make sense
Example: $K d V$ currents
CRT: $T_{s+1}=: T^{(s+1) / 2}:+\ldots, \bar{\delta} T_{s+1}=0$

$$
P_{S}=\frac{1}{2 \pi} \int d z T_{S+1}(z),\left[P_{ \pm 1}, P_{S}\right]=0
$$

Deformed theory: $0=\bar{\delta} T_{s+1}-\partial \theta_{s-1}$

$$
P_{S}=\frac{1}{2 \pi} \int\left(d z T_{S+1}(z)+d \bar{z} \Theta_{S-1}(\bar{z})\right)
$$

$\left[P_{\sigma}, P_{s}\right]=0$ equivalent to $\left[P_{\sigma}, d z T_{s+1}+d \bar{z} \theta_{s-1}\right]$ exact 1-farm [Sminno, Zandoddikou]
$\left[P_{\sigma}, T_{s+1}\right]=-i \partial A_{\sigma}^{s} \quad$ (similarly for $\theta_{s-1}$ )
Some properties: $A_{1}^{S}=T_{S+1}, A_{-1}^{S}=-\theta_{s-1} \quad$ (trivial)

$$
A_{s}^{\prime}=s T_{s+1}, A_{s}^{-1}=s \theta_{s-1} \quad(\text { hard })
$$

New factraizing composite op's: $\chi_{\sigma_{1} \sigma_{2}}^{s_{1} s_{2}} \equiv\left(A_{\left[\sigma_{1}\right.}^{s_{1}} A_{\sigma_{2}}^{s_{2}}\right)_{\text {reg }}$ $\left\langle\chi_{\sigma_{1} \sigma_{2}}^{s_{1} s_{2}}\right\rangle_{n}=\left\langle A_{\left[\sigma_{1}\right\rangle_{n}}^{s_{1}}\left\langle A_{\left.\sigma_{2}\right]}^{s_{2}}\right\rangle_{n}\right.$
$T_{s+1} \bar{T}_{s+1}$ is $\chi_{-1,1}^{-s, s}$ (hence $T \bar{T}$ is $\chi_{-1,1}^{-1,1}$ )
Can show that under $x_{-1,1}^{t u}$ defamation:

$$
\delta P_{S}=-\frac{1}{2 \pi^{2}} \int d x\left(\chi_{s, 1}^{t u}(x)+\chi_{-1, s}^{t u}(x)\right)+\text { ambiguities }
$$

To solve a $x_{-1,1}^{t u}$-deformed thy, have to follow $P_{S}$, not just $P_{ \pm 1}$ $\partial_{\lambda}\left\langle P_{S}\right\rangle_{n}=-\frac{1}{2 \pi}\left(\left\langle P_{t}\right\rangle_{n}\left\langle A_{S}^{u}\right\rangle_{n}-\left\langle P_{u}\right\rangle_{n}\left\langle A_{S}^{t}\right\rangle_{n}\right)$
To proceed we need Lorentz inv. + only $A_{ \pm 1}^{S} \& A_{S}^{ \pm 1}$
$\Rightarrow$ consider TT
Still need key step: $\delta_{L}\left\langle P_{S}\right\rangle_{n}=-\frac{S}{2 \pi}\left(\left\langle T_{S+1}\right\rangle_{n}-\left\langle\theta_{S-1}\right\rangle_{n}\right)$
Other linear comb. is $\left\langle P_{s}\right\rangle_{n} / L$
$\left.\partial_{\lambda}\left\langle P_{S}\right\rangle_{n}=E_{n} \partial_{L}\left\langle P_{s}\right\rangle_{n}+P_{n} \frac{S\left\langle P_{S}\right\rangle}{L} \right\rvert\,[$ also Conte, Negro, Tateo]

$$
\partial_{\lambda} E_{n}=E_{n} \partial_{L} E_{n}+\frac{P_{n}^{2}}{L}
$$

Burgers variables: $\lambda=-t, E_{n}=u, L=x$

$$
\begin{aligned}
& \partial_{t} u+u \partial_{x} u=-\frac{p^{2}}{x^{3}} \\
& \partial_{+} P_{S}+u \partial_{x} P_{S}=-\frac{S p}{x^{2}} P_{S} \quad \text { passive scalan }
\end{aligned}
$$

Another option is to ustrict to $P_{n}=0$, do $T_{u+1} \bar{T}$ deformation:

$$
\partial_{\lambda}\left\langle P_{s}\right\rangle_{n}=-\left\langle P_{u}\right\rangle_{n} \partial_{L}\left\langle P_{s}\right\rangle_{n}
$$

Density of states super-Hagedorn: $S(E) \sim \exp \left[\sqrt{\# \lambda} E^{(u+1) / 2}\right]$
5) Outlook: Coupling theories w/ TF [Ge, Le Flock, MM wip]

- Start w/ $N$ decoupled theories, have $T^{(i)}, P_{S}^{(i)}$

$$
\left[\begin{array}{c}
p_{s}^{(i)}, p_{t}^{(j)}
\end{array}\right]=0
$$

- From the above can deform by $\varepsilon^{\mu \nu} T_{\alpha \mu}^{(i)} T_{\beta \nu}^{(j)}$ Eng. $\quad O_{\lambda} \equiv \varepsilon^{\alpha \beta} \varepsilon^{\mu \nu}\left(\sum_{i} T_{\alpha \mu}^{(i)}\right)\left(\sum_{j} T_{\beta \nu}^{(j)}\right)$ double trace

$$
\theta_{\lambda} \equiv \varepsilon^{\alpha \beta} \varepsilon^{\mu \nu} \sum_{i}\left(T_{\alpha \mu}^{(i)} T_{\beta V}^{(i)}\right) \quad \text { single trace }
$$

[Giveon, Itzhaki, Kutcosou]
$-\partial^{\mu} T_{\alpha \mu}^{(i)}=0$ for $\lambda>0$, but $T_{\alpha \mu}^{(i)} \neq T_{\mu \alpha}^{(i)}$, no corresponding rotation chang rotation charge

- Q1: Can we solve this family?
- Q2: Can this unify double trace \& single trace $T \bar{T}$ ?
- Q 3: Geometric interpretation of $p(i)$ ?
- Q 4: Holographic dual? [Aharony, Clark, March]

