Cosmological consequences of quantum black holes

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BH thermodynamics, Hawking radiation, Bekenstein-Hawking entropy (Thermodynamics)

information loss paradox, firewall (quantum information, AdS/CFT)

- Novel pictures of BH such as
- fuzzball, quantized horizon area, membrane paradigm etc… (string theory, loop quantum gravity, fluid/gravity correspondence)

Quantum BH has been discussed/studied mainly from the fundamental physics

Cosmology?Quantum black holes \rightleftharpoons Astrophysics?Particle physics?

Astrophysics

GW/scalar emission (ringdown, superradiance, echoes) Particle physics Higgs metastability (seeded vacuum decay)

Quantum Black Hole

Cosmology

Inflation (initial condition, thermal nature of Schwarzschild-dS)

Footprints of quantum gravity in radiation from QBHs

N.O., N. Afshordi, S. Mukohyama (2020), in preparation
N.O., D. Tsuna, and N. Afshordi (2020), arXiv: 2004.06276
N.O., D. Tsuna, and N. Afshordi (2020), arXiv: 2001.11642
Q. Wang, N.O., and N. Afshordi (2019), arXiv: 1905.00446
N.O., Q. Wang, and N. Afshordi (2019), arXiv: 1905.00464
N.O. and N. Afshordi (2018), arXiv: 1807.10287

How can we observationally probe the quantum properties of BHs ?

How about Hawking radiation ?

$$M = M_{\odot}$$
 $T_H \sim 10^{-6} \text{ K} \ll T_{\text{CMB}} \simeq 2.7 \text{ K}$

How about the Planck size structure of space?

To reach the Planck scale with a particle accelerator, its size should be comparable to the solar system.







Ringdown GWs tell us about the horizon structure.

Gravitational waves from a binary black hole





GW150914 (the first detection of GWs by LIGO)

- consist of specific modes (Quaisi-Normal Modes)
- QNMs determined only by its mass and angular momentum

ringdown is useful to test a BH



Standard Ringdown

Frequency and damping rate are independent of initial conditions.



massless scalar field (spin-0 field) I=2 M=1/2



Quantum nature of spacetime could (and how) modify those universal properties?

Effects of the Lifshitz scaling on the BH ringing (model of microstructure of spacetime)

Quantum Theory of Gravity (spacetime)

Discretized (coarse-grained) spacetime with
a certain size?Bekenstein-Hawking entropy

Quantum Gravity → Lorentz breaking theory at high energies?

Lifshitz scaling

Anisotropy between space and time

$$x \to bx \qquad t \to b^z t$$

This idea was originally developed in condensed-matter systems. (Lifshitz scalar field in D+1 dim)

Applying this to quantum gravity theory, it becomes a renormalizable theory in a power-counting level. Horava (2009)

Afterwards, (projectable) Horava-Lifhshitz gravity was proven to be exactly renormalizable! Barvinsky et al. (2015)

Simplified Model

$$L = \int d^4x \sqrt{-g} \left[\mathcal{L}_{\rm EH} + \mathcal{L}_{\rm SG} + \mathcal{L}_{\rm GW} \right]$$

(background)

$$\mathcal{L}_{\rm EH} \equiv \frac{2}{\kappa^2} R$$

(preferred frame)

(perturbations) $\mathcal{L}_{\mathrm{GW}}\equiv\psi(\mathcal{F}(\Delta)+\Box)\psi$

$$\mathcal{F}(\Delta) \equiv -\Delta^3 / M_{\rm HL}^4 + \nu_4 \Delta^2 / M_{\rm HL}^2$$

Lifshitz scaling in gravity leads to...

- Modification to GW dispersion relation → superluminal propagation
- Scalar-graviton (khronon field) \rightarrow preferred frame







Preferred frame

Solve the khronon field equation



• khronon field equation Blas et al. (2011)

$$\frac{\partial_{\xi}^{2}U}{U} - c_{\chi}^{2}\frac{\partial_{\xi}^{2}V}{V} + \frac{2c_{\chi}^{2}}{\xi^{2}} = 0$$
$$U^{2} - V^{2} = 1 - \xi \qquad \xi \equiv \frac{1}{r} \quad U \equiv u_{t} \quad V \equiv u^{2}$$

Background spacetime: Schwarzschild solution

$$ds^2 = f(r)dt^2 - rac{dr^2}{f(r)} - r^2 d\Omega_2^2$$

 $f(r) \equiv 1 - rac{r_s}{r}$
Set $r_s = 1$

• Analytic solutions $U = 1 - \frac{1}{2r} \qquad (c_{\chi} \to 0)$

$$U = 1 - \frac{1}{r} + \frac{27}{16^2} \frac{1}{r^4} \qquad (c_{\chi} \to \infty)$$

Wave equation

$$\left[-\frac{\Delta^3}{M_{\rm HL}^4} + \nu_4 \frac{\Delta^2}{M_{\rm HL}^2} + \Box\right] \psi(\tau, r, \theta, \phi) = 0$$

Laplacian defined on constant $\mathcal{T} \rightarrow \Delta = \partial_{r^*}^2 + \frac{2U}{r} \partial_{r^*} - \frac{l(l+1)}{r^2} \quad dr^* \equiv \frac{dr}{U(r)}$

$$M_{\rm HL} \qquad B \equiv \frac{1}{M_{\rm HL}^4 r_s^4}$$
$$\nu_4 \qquad A \equiv \frac{\nu_4}{M_{\rm HL}^2 r_s^2}$$

 $\omega^2 = k^2 + Ak^4 + Bk^6 \qquad r \to \infty$

Results -ringdown-

$$\omega^2 = k^2 + Ak^4 + Bk^6$$



Results - reflectivity-

A = 0.1 B = 0.01 $c_{\chi} \to \overline{\infty}$



Why superradiance? -intuitive interpretation-





Prohibited mass extraction

Superluminal propagation → mass extraction can be possible

Why superradiance? -What causes the SR?-

 $A = \overline{0.1}$ $B = \overline{0.01}$ $c_{\chi} \to \infty$



Observational constraints



arXiv: 2010.14529 (LIGO and Virgo collaboration)

Lifshitz scaling effects could be dominant for asteroid-mass BHs

 $M_{\rm BH} \lesssim 10^{-8} M_{\odot}$



Important issues

- Superradiance of PBHs -> footprints on the stochastic GWs?
 - Inomata et al. arXiv:2003.10455
- Application to the D->4 Einstein-Gauss-Bonnet gravity Aoki et al. arXiv:2005.03859
- Complicated wave equation from the general Horava-Lifshitz gravity
- Modification to the background should be also taken into account when extending to more general cases.
- The generalized 2nd law of thermodynamics would be broken due to the Lifshitz scaling.

What if the interior does not exist?



Firewall proposal / BH complementarity state there is no interior for any observer / distant observer



membrane paradigm

According to an infalling observer,

information causally disappears.

K. Thorne+ (1986)

According to a distant observer,

information is dissipated due to the viscosity.



Thermality of Horizon Padmanabhan (2019)

(see also Hartle & Hawking (1976), Damour & Ruffini (1976)) Path integral approach Horizon tunneling approach

Feynman propagator

$$\sum_{\text{paths}} \exp\left[-im\ell(x_1, x_2)\right] = G(x_1, x_2) \xrightarrow{\text{Killing vector}} G(x_1, x_2)$$

$$\xi = \partial/\partial\tau \xrightarrow{t}$$

Amplitude of propagation with $E=\omega$ from x1 to x2

$$A(\omega) = \int d\tau G(\tau) e^{-i\omega\tau}$$

case 1 $A(\omega) = A(-\omega)$

case 2
$$|A(\omega)|^2/|A(-\omega)|^2 = \exp\left[-\omega/T(a)\right]$$
 $T(a) \equiv \frac{a}{2\pi}$
Boltzmann factor

reflectivity of QBHs = Boltzmann factor?

reflection at the apparent horizon leads to GW echoes



Tentative evidence of GW echoes



1e39 4.2 σ Fewer than 4 similar peaks in 3 days data Time series of -X(t, fpeak) 6 GW echoes signal found by Abedi and Afshordi following a 5 NS collapse to BH 3σ $-1 \times X(t, f_{peak})$ [strain]⁻² 4 2σ Independent determination of BH collapse, from EM 3 emission by Gill et al., 2019 2 1 0 0.25 0.50 0.75 1.00 1.25 1.50 1.75 0.00 2.00 t - t_{merger} [sec]

Abedi and Afshordi (2018)

Abedi, Afshordi, NO, Wang (2020)

Echoes from a quantum BH



NO, Wang, Afshordi (2019)

Superradiance

Superradiance may cause the instability of ringdown GWs.

H. Nakano+ (2017)

Prog. Theor. Exp. Phys. 2017, 071E01 (10 pages) DOI: 10.1093/ptep/ptx093

Letter

PTEP

Black hole ringdown echoes and howls

Hiroyuki Nakano^{1,2,*}, Norichika Sago³, Hideyuki Tagoshi⁴, and Takahiro Tanaka^{2,5}

The super-radiant amplification looks dangerous. There are extensive works on this problem (see, e.g., Refs. [24–26], and Ref. [27] for a review). The latest analysis [28] shows that the time scale can be larger than the age of the Universe if the location of the reflection boundary is sufficiently far from the horizon. The above means that if BHs have a complete reflecting boundary at a distance of the order of the Planck length from the horizon, all astrophysical BHs become non-rotating, i.e., Schwarzschild BHs. If we observe GW howls due to the super-radiant amplification, it means that only Schwarzschild BHs can exist in our universe.

We should confirm NO instability !



Consistency with the tentative detection of GW echoes

 $\operatorname{Re}[2M\omega]$

73 Hz 100

50

f/Hz

200

NO, D. Tsuna, N. Afshordi (2020)

n=1

20

0.4

0.3

 $\mathrm{Im}[-2M\omega]_{\sim}$

0.0

0.1

1040

1039

1038

1037

1036

1035

10

Xff

0.2

$[\mathrm{strain}]^{-2}$ $X(t, f, \delta t)$ $+ 3\sigma$ $\sim 6 \times 10^{39}$ 4:92:11:24.2σ 2:3 $mized(X(0.5s \le t - t_{max}))$ $_{out} \leq 1.5s, f, -10msec \leq \delta t \leq 10msec)$ 100 50 150 200 $72~Hz_{\rm Frequency~[Hz]}$

J. Abedi and N. Afshordi (2018)

Figure 11: Amplitude-frequency representations peak of $t_{\text{peak}}=1.0$ sec after the merger. In this analysis the detectors are shifted within ± 10 ms and the X(t, f) is minimized over ($t_{\text{peak}}=0.5, t_{\text{peak}}+0.5$) sec data range after the merger. Here the minimum peak again is the one with 72 Hz frequency, which happens at 2.62 ms time shift between Hanford and Livingston detectors, consistent with main event (see A). Resonance frequencies are at $\frac{2}{9}: \frac{1}{4}: \frac{4}{9}: \frac{1}{2}: \frac{2}{3}: 1: 2$ of 72 Hz.

Importance of overtones of QNMs

a/M = 0.5

n=4

0.2

 $\operatorname{Re}[2M\omega]$

n=3_{n=2}

0.3

n=1

0.4

 $\operatorname{Im}[-2M\omega]$

0.0

0.1

Giesler et al. (2019)

Black hole ringdown: the importance of overtones

Matthew Giesler,^{1,*} Maximiliano Isi,^{2,3,†} Mark A. Scheel,¹ and Saul A. Teukolsk

¹TAPIR, Walter Burke Institute for Theoretical Physics, California Institute of Technology, Pasadena, CA 91125, USA

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³LIGO Laboratory, California Institute of Technology, Pasadena, California 91125, USA

⁴Cornell Center for Astrophysics and Planetary Science, Cornell University, Ithaca, New York 14853, USA (Detect. March 21, 2010)

(Dated: March 21, 2019)

It is possible to infer the mass and spin of the remnant black hole from binary black hole mergers by comparing the ringdown gravitational wave signal to results from studies of perturbed Kerr spacetimes. Typically these studies are based on the fundamental quasinormal mode of the dominant $\ell = m = 2$ harmonic. By modeling the ringdown of accurate numerical relativity simulations, we find that the fundamental mode alone is insufficient to recover the true underlying mass and spin, unless the analysis is started very late in the ringdown. Including higher overtones associated with this $\ell = m = 2$ harmonic resolves this issue, and provides an unbiased estimate of the true remnant parameters. Further, including overtones allows for the modeling of the ringdown signal for all times beyond the peak strain amplitude, indicating that the linear quasinormal regime starts much sooner than previously expected. A model for the ringdown beginning at the peak strain amplitude can exploit the higher signal-to-noise ratio in detectors, reducing uncertainties in the extracted remnant quantities. Tests of the no-hair theorem should consider incorporating overtones in the analysis.

Third-generation GW detectors



Figure 40. Spectra of ringdown and echo phases in the Boltzmann reflectivity model with $\bar{a} = 0.1$, $\epsilon_{\rm rd} = 2.4 \times 10^{-6}$, $M = 2.4 M_{\odot}$, $\theta = 90^{\circ}$, and $D_o = 1$ Mpc. Here we also assume $\gamma = 10^{-10}$, $T_{\rm H}/T_{\rm QH} = 1$ (left) and $T_{\rm H}/T_{\rm QH} = 1.37 \times 10^{-6}$ (right).


GW ringdown from a failed supernova



Can we test echo emission with the third-generation GW detector if a failed supernova happens within ~ 10 Mpc?

NO, D. Tsuna, N. Afshordi (2020)



Summary

- Lifshitz scaling could lead to the superradiance.
- It may leave some footprints, for example, on the background GW signal originating from the reheating process that induces stochastic GW.
- Thermality of horizon could lead to the Boltzmann reflectivity that can be consistent with the tentative detection although no conclusive evidence exists.
- Third-generation GW detectors may be able to test the echo emission with their enough precisions.
- Failed supernova is another candidate to test echo GW in addition to compact binary coalescences if it happens within ~ 10 Mpc with an optimistic situation.

Astrophysics GW/scalar emission (ringdown, superradiance, echoes) Particle physics Higgs metastability (seeded vacuum decay)

Quantum Black Hole

Cosmology Inflation (initial condition, thermal nature of DS)

Hawking-Moss transition with black holes

R. Gregory, I. G. Moss, NO, S. Patrick (2020) arXiv: 2007.11428

R. Gregory, I. G. Moss, NO, (2020) arXiv: 2003.04927



Initial condition problem for inflation

- Numerical calculation of inhomogeneous spaces (inhomogeneous matter + positive energy density): linear and non-linear computations
- Linear computations → slow-roll trajectory for large field inflation is a local attractor A. Albrecht+ (1985) D.S. Goldwirth (1991) R.Easther+ (2014)
 H. A. Feldman+ (1989) R.H.Brandenberger+ (1989)
- beyond linear theory → numerical relativity (non-linear fluctuations)
- 1-dim simulations: inflation is likely in the case of large field inflation H. Kurki-Suonio + (1987) P. Laguna + (1991) H. Kurki-Suonio + (1993)
- Black holes form at the pre-inflation epoch \rightarrow BHs diluted due to expansion

W. East + (2015)

Simulation of inhomogeneous space at a pre-inflation

W. East + (2015)



$$k \lesssim H_0$$

$$\frac{4\pi}{3}Gk^{-3}\rho \gtrsim k^{-1}/2 \implies k \lesssim H$$

mass of the over-density Schwarzschild radius



What if the vacuum state is metastable? (e.g. Higgs metastability and/or landscape picture)





Figure 1: Left: SM RG evolution of the gauge couplings $g_1 = \sqrt{5/3}g'$, $g_2 = g$, $g_3 = g_s$, of the top and bottom Yukawa couplings (y_t, y_b) , and of the Higgs quartic coupling λ . All couplings are defined in the $\overline{\text{MS}}$ scheme. The thickness indicates the $\pm 1\sigma$ uncertainty. Right: RG evolution of λ varying M_t , M_h and α_s by $\pm 3\sigma$.



Vacuum Phase Transition



$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} R + \int d^4x \sqrt{-g} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right]$$

Computation of decay rate -Euclidean path integral-

Based on the Euclidean path integral and semi-classical approximation,





CDL vs HM

CDL bounce solution exists when $|V^{\prime\prime}|/H^2>4$

is satisfied at the top of potential barrier.

Otherwise, only the HM bounce is allowed.

Intermediate solutions between them?

Oscillating bounce solutions



A. Rajantie + (2017)

CDL, HM, and oscillating bounce



oscillating bounces with many oscillations → HM bounce

Static oscillating bounce in Schwarzschild-de Sitter space

static and spherical metric

$$ds^{2} = f(r)e^{2\delta(r)}d\tau^{2} + \frac{dr^{2}}{f(r)} + r^{2}(d\theta^{2} + \sin^{2}\theta d\varphi^{2})$$
$$f = 1 - \frac{2G\mu(r)}{f(r)}$$

r

E.O.M. of a scalar field and Einstein equations

$$f\phi'' + f'\phi' + \frac{2}{r}f\phi' + \delta'f\phi' - V_{\phi} = 0$$

$$\mu' = 4\pi r^2 \left(\frac{1}{2}f\phi'^2 + V\right)$$

 $\delta' = 4\pi G r \phi'^2$

RESULTS



R. Gregory, I. G. Moss, NO (2020)

~ Hawking-Moss bounce with a BH?



BHHM bounce

HM bounce around a BH (BHHM bounce)

Transition between two SdS spacetimes



HM vs BHHM

$$\Gamma_{\rm HM} \sim e^{-\Delta S_{\rm E}} = e^{\Delta A_{\rm c}/4G} \simeq e^{-\Delta E/T_{\rm dS}}$$

increment of the internal energy \rightarrow exponential suppression

In our case, there exists not only vacuum energy but also a seed BH. \rightarrow vacuum can consume the energy of BH to go to the potential top!

Internal energy inside the cosmological horizon is conserved

 $\prod_{\substack{\text{arXiv: 2007.11428}}} \frac{\text{Gregory, Moss, NO, Patrick (2020)}}{\text{arXiv: 2007.11428}}$

$$\Gamma_{\rm BHHM} \sim e^{-\Delta S_{\rm E}} = e^{\frac{(\Delta A_c + \Delta A_{BH})}{4G}} = e^{\frac{\Delta A_{BH}}{4G}}$$





Summary

- An oscillating bounce with a seed BH, which may be an intermediate solution between the CDL and HM bounce, was investigated.
- The oscillating bounce is well consistent with the HM bounce with a BH (i.e. field configuration and the values of Euclidean action).
- The initial condition of inflation may be very inhomogeneous and accommodated many PBHs.
- Mini BHs could catalyze the Hawking-Moss transition.
- The number density of pre-inflation BHs should be constrained in order for the present Universe to exist, provided that there was the vacuum metastability at the early Universe. -> shed light on the initial condition problem for inflation??

Astrophysics GW/scalar emission (ringdown, superradiance, echoes) Particle physics Higgs metastability (seeded vacuum decay)

Quantum Black Hole

Cosmology Inflation (initial condition, thermal nature of DS) Vacuum decay catalyzed by black holes or compact objects

T. Hayashi, K. Kamada, NO, J. Yokoyama (2020) arXiv: 2005.12808 NO, (2020) arXiv: 2002.11175 NO, K. Ueda, M. Yamaguchi (2019) arXiv: 1909.01378 NO, M. Yamada, M. Yamaguchi (2018) arXiv: 1808.01382

Situation where the Bekenstein-Hawking entropy affects a cosmological phenomenon



$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} R + \int d^4x \sqrt{-g} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right]$$

Why are the vacuum phase transitions important?

For example...

Cosmology Open inflation

Gott (1982) Bucher, Goldhaber, Turok (1995)

Particle Physics Higgs Metastability

ATLAS collaboration (2012) Sher (1989), Arnold (1989)



Johnson+ (2016)



A Higgs particle



A Higgs particle with ~125 GeV has been found!!



Figure 1: Left: SM RG evolution of the gauge couplings $g_1 = \sqrt{5/3}g'$, $g_2 = g$, $g_3 = g_s$, of the top and bottom Yukawa couplings (y_t, y_b) , and of the Higgs quartic coupling λ . All couplings are defined in the $\overline{\text{MS}}$ scheme. The thickness indicates the $\pm 1\sigma$ uncertainty. Right: RG evolution of λ varying M_t , M_h and α_s by $\pm 3\sigma$.

Degrassi et al. (2013)

The Higgs self-coupling can be negative at high energies.

$$ds^{2} = -fdt^{2} + \frac{dr^{2}}{f} + r^{2}d\Omega^{2}$$

$$f(r) = \begin{cases} 1 & \cdots \text{ Minkowski} \\ 1 + H^{2}r^{2} \cdots \text{ AdS} \end{cases}$$
Israel junction condition
$$\frac{\sqrt{1 + H^{2}R^{2} + \dot{R}^{2}}}{R} - \frac{\sqrt{1 + \dot{R}^{2}}}{R} = -4\pi G\sigma$$
energy density of wall
$$R = \gamma^{-1}\cosh\gamma\tau \cdots \text{ Lorentzian}$$

$$R = \gamma^{-1}\cos\gamma\tau_{E} \cdots \text{ Euclidean} \qquad \gamma = \frac{|H^{2} - (4\pi G\sigma)^{2}|}{8\pi G\sigma}$$

Evaluating the vacuum decay rate

(4-volume of the observable universe) × $\Gamma \sim \exp[540 - S_{\rm E}[\phi_f]] \ll 1$

However, this estimation is based on the homogeneity of the Universe.

In our universe, there are black holes, neutron stars, and so on.

How do "cosmic impurities" (such as black holes, neutron stars, monopoles) affect the cosmological phase transitions?

Enhancement of decay rate!!!

Black holes as bubble nucleation cites

Hiscock (1987) & Gregory et al. (2014)



$$\Gamma \sim e^{-B}$$







accretion of the surrounding radiation -> non-zero spin of PBHs

(this effect is very efficient for small PBHs) Dong+ (2016)

Thermal effects?

$$T_{\rm H} = \frac{1}{8\pi GM}$$

$$M \sim 10^6 M_{Pl} \implies T_{\rm H} \sim 10^{12} {\rm GeV}$$

Our Universe is rescued? * in the context of reheating process, c.f. Espinosa, Racco, Riotto (2018)




Can any spinning BHs catalyze vacuum decay?

NO.

There is the upper mass limit for BH catalysts. M_C

Interior curvature

 $M_C = M_C(a, H, \Sigma)$



More massive BHs are allowed to have the bounce solution for spinning case.

Small-mass naked singularities censored by the Higgs field?

NO, arXiv: 2002.11175 [CQG 37 (2020) 07LT01] $\Gamma \sim e^{-B} = e^{-B_{\text{wall}} + \Delta S}$ **Bekenstein-Hawking entropy** rotating vacuum bubble wall over-spinning naked singularity spinning BH $\Delta S = 0$ $\Delta S \gg 1$ $L_{\rm tot} = L_{\rm sin}$ $L_{\rm tot} = L_{\rm sin} + L_{\rm bubble}$ $L_{\rm sin} > L_{\rm max}$ $L_{\rm sin} < L_{\rm max}$ BH naked singularity

Vacuum decay catalyzed by a horizonless compact object



$$R\sim 2GM$$
 & $R\sim rac{2\Sigma}{H^2-\Sigma^2}$

Lifetime of vacuum can be shorter than the cosmic age

 $\Sigma \equiv 4\pi G\sigma$



The Bekenstein-Hawking entropy decreases

Transition rate is suppressed



NO, M. Yamada, M. Masahide (2018)

Horizonless object could catalyze vacuum decay more efficiently!!

Does thermal effect prevent the catalyst effect??

Kohri & Matsui (2018) Mukaida & Yamada (2018)

There are arguments that the catalyst effect would be suppressed by the Hawking radiation, which is inversely proportional to the BH mass.

However, there is no rigorous calculations and results to show the argument.



Result: hawking radiation slightly lower transition rate - Why can we ignore the thermal corrections of BH? transition rate against BH mass



For small BH $M_{BH} < 10^7 m_{pl}$, Higgs instability is still problematic, even taking Hawking radiation into account.

Summary

- spinning BHs can be catalysts for vacuum decay
- near extremal BHs (oppositely) stabilize a false vacuum
- upper mass limit for BH catalyst increases for a > 0
- BH catalyzing effect can be a cosmic censorship mechanism
- Hawking thermal corrections to the Higgs effective potential may be negligible.