Inflation, Primordial Black Holes, Gravitational Waves

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Motivations

- Slow-roll inflation for the first 7 e-foldings (61>N>54)

  Planck CMB + LSS + Sne + ...

- Binary black holes detected by aLIGO/VIRGO may be primordial black holes produced during inflation (N ~ 40) ?

  PBHs may be dark matter and aLIGO has seen dark matter!

Then…

- While we will learn more details about the slow-roll inflation (r, n_s, running n_s,..), there may be an opportunity to probe beyond the inflationary standard model
- Formation of PBHs, associated with gravitational waves in inflation

Primordial Black Holes

- Formed at high-density contrasts over a wide range of scales or masses in the radiation-dominated Universe
- There have been stringent astrophysical and cosmological constraints on M_{PBH}
- $10M_{\odot}$PBHs could be the binary BHs observed by aLIGO gravity-wave detectors

  Bird et al. 16., Clesse et al. 16, Sasaki et al. 16
- PBHs behave like cold dark matter

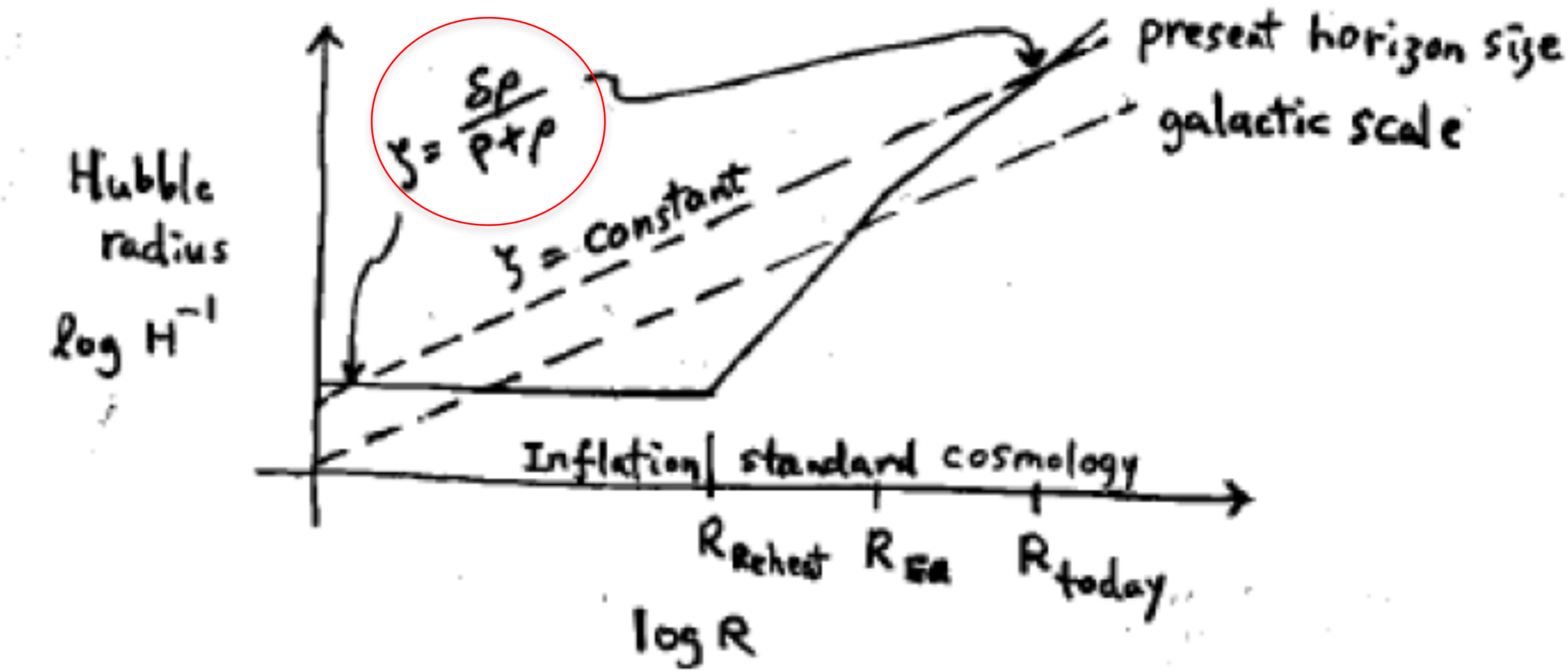
  García-Bellido, Linde, Wands 96
- They, although being of baryonic origin, do not participate in big-bang nucleosynthesis

PBH Formation

- spontaneously formed in phase transitions, e.g., 1st order electroweak, quark-hadron phase transition $M_{BH} \sim M_{\odot}$
- arisen from the collapse of horizon-sized large matter inhomogeneity seeded by quantum fluctuations during inflation that re-enter the horizon in the subsequent radiation-dominated Universe, with M_{BH} spans a wide range of mass

Production of PBHs in inflation

Evolution of cosmological density perturbation



$$M_{\rm BH} = \frac{4\pi M_p^2}{H} e^{2N} = 2.74 \times 10^{-38} e^{2N} \left(\frac{M_p}{H}\right) M_\odot$$

N is e-foldings before the end of inflation

PBH Production in Inflation

- Single-field slow-roll inflation models, matter density perturbation ($\delta\rho/\rho \sim 10^{-5}$) too small
- Modified inflation potential to achieve blue-tilted matter power spectra or running spectral indices, leading to large density perturbation at the end of inflation, but mostly $M_{PBH} \ll M_{\odot}$

  García-Bellido, Linde, Wands 96
- To boost M_{PBH}, hybrid inflation, double inflation, curvaton models by inflating small-scale density perturbation to the size of a stellar-mass to supermassive PBH.

  Kawasaki, Kohri, Yokoyama, Yanagida….

Interacting inflaton

Quantum environment Wu et al. JCAP 07

$$\mathcal{L} = \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi + \frac{1}{2} g^{\mu\nu} \partial_\mu \sigma \, \partial_\nu \sigma - V(\phi) - \frac{m_\sigma^2}{2} \sigma^2 - \frac{g^2}{2} \phi^2 \sigma^2$$

Trapped Inflation Kofman, Linde et al. 04, Green et al.09

Inflaton coupled to instantaneously massless fields

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) + \frac{1}{2} \sum_i (\partial_\mu \chi_i \partial^\mu \chi_i - g^2 (\phi - \phi_i)^2 \chi_i^2) + \cdots ,$$

Natural Inflation with axion-like couplings Sorbo, Kaloper,Peloso,..

$$S = \int d^4x \sqrt{-g} \left[\frac{M_p^2}{2} R - \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - V(\varphi) - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \frac{\alpha}{4f} \varphi \, \tilde{F}^{\mu\nu} \, F_{\mu\nu} \right]$$

Trapped axion Inflation

We consider a version of the trapped inflation driven by a pseudoscalar φ that couples to a $U(1)$ gauge field A_μ:

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}R - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - V(\varphi) - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{\alpha}{4f}\varphi\tilde{F}^{\mu\nu}F_{\mu\nu}\right], \quad (3)$$

$$\varphi = \phi(\eta) + \delta\varphi(\eta, \vec{x})$$

Under the temporal gauge, $A_\mu = (0, \vec{A})$, we decompose $\vec{A}(\eta, \vec{x})$ into its right and left circularly polarized Fourier modes, $A_\pm(\eta, \vec{k})$, whose equation of motion is then given by

$k/(aH) < 2|\xi|$

Spinoidal instability

$$\left[\frac{d^2}{d\eta^2} + k^2 \mp 2aHk\xi\right]A_\pm(\eta, k) = 0, \quad \xi \equiv \frac{\alpha}{2fH}\frac{d\phi}{dt}. \quad (5)$$

Background

$$\frac{d^2\phi}{dt^2} + 3H\frac{d\phi}{dt} + \frac{dV}{d\phi} = \frac{\alpha}{f}\langle \vec{E}\cdot\vec{B}\rangle,$$

$$3H^2 = \frac{1}{M_p^2}\left[\frac{1}{2}\left(\frac{d\phi}{dt}\right)^2 + V(\phi) + \frac{1}{2}\langle \vec{E}^2 + \vec{B}^2\rangle\right]$$

$$\boxed{\begin{aligned}\langle \vec{E}\cdot\vec{B}\rangle &\simeq -2.4\cdot 10^{-4}\frac{H^4}{\xi^4}\,\mathrm{e}^{2\pi\xi},\\ \left\langle\frac{\vec{E}^2+\vec{B}^2}{2}\right\rangle &\simeq 1.4\cdot 10^{-4}\frac{H^4}{\xi^3}\,\mathrm{e}^{2\pi\xi}.\end{aligned}}$$

$$\frac{1}{2}\langle \vec{E}^2 + \vec{B}^2\rangle = \int \frac{dk\,k^2}{4\pi^2 a^4}\sum_{\lambda=\pm}\left(\left|\frac{dA_\lambda}{d\eta}\right|^2 + k^2|A_\lambda|^2\right),$$

$$\langle \vec{E}\cdot\vec{B}\rangle = -\int \frac{dk\,k^3}{4\pi^2 a^4}\frac{d}{d\eta}\left(|A_+|^2 - |A_-|^2\right).$$

Perturbation

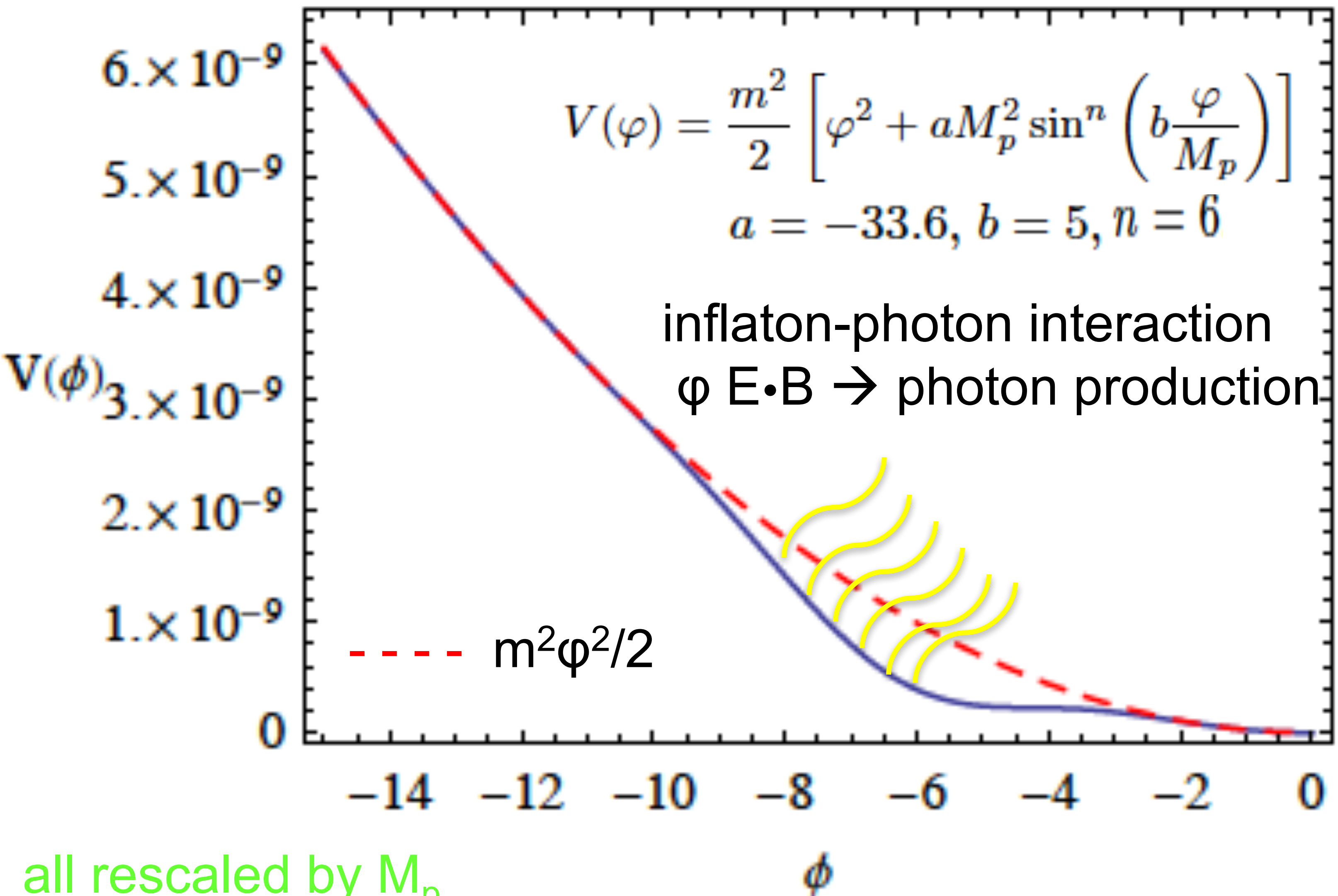
$$\left[\frac{\partial^2}{\partial t^2} + 3\beta H\frac{\partial}{\partial t} - \frac{\vec{\nabla}^2}{a^2} + \frac{d^2V}{d\phi^2}\right]\delta\varphi(t,\vec{x}) = \frac{\alpha}{f}\left(\vec{E}\cdot\vec{B} - \langle \vec{E}\cdot\vec{B}\rangle\right)$$

$$\delta\varphi = \frac{\alpha}{3\beta f H^2}\left(\vec{E}\cdot\vec{B} - \langle \vec{E}\cdot\vec{B}\rangle\right)$$

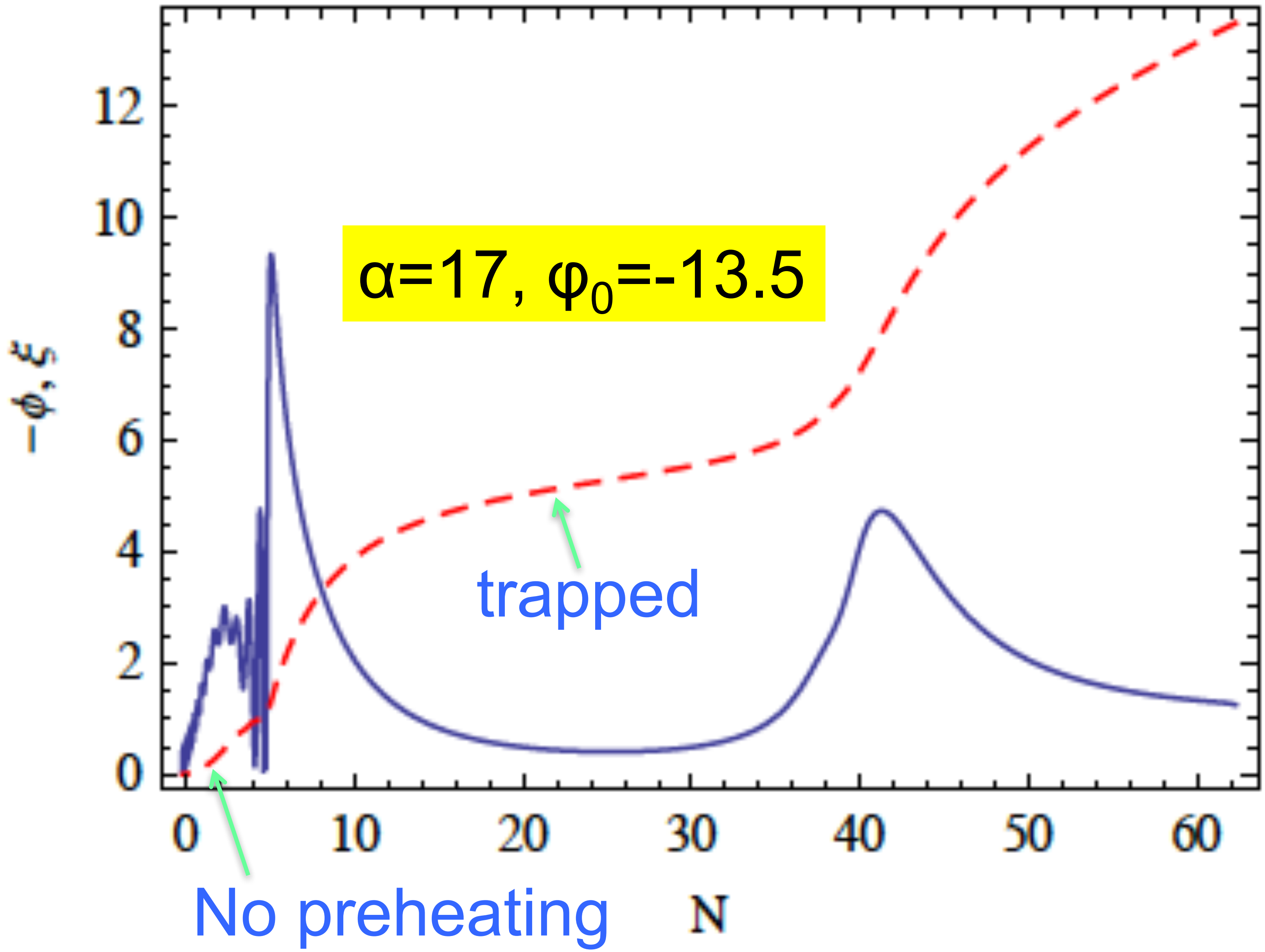
$$\boxed{\beta \equiv 1 - 2\pi\xi\frac{\alpha}{f}\frac{\langle \vec{E}\cdot\vec{B}\rangle}{3H(d\phi/dt)}}$$

$$\Delta_\zeta^2(k) = \langle \zeta(x)^2\rangle = \frac{H^2\langle\delta\varphi^2\rangle}{(d\phi/dt)^2} = \left[\frac{\alpha\langle \vec{E}\cdot\vec{B}\rangle}{3\beta f H(d\phi/dt)}\right]^2$$

e.g. Trapped axion inflation with a steep potential Cheng, Lee, Ng 16

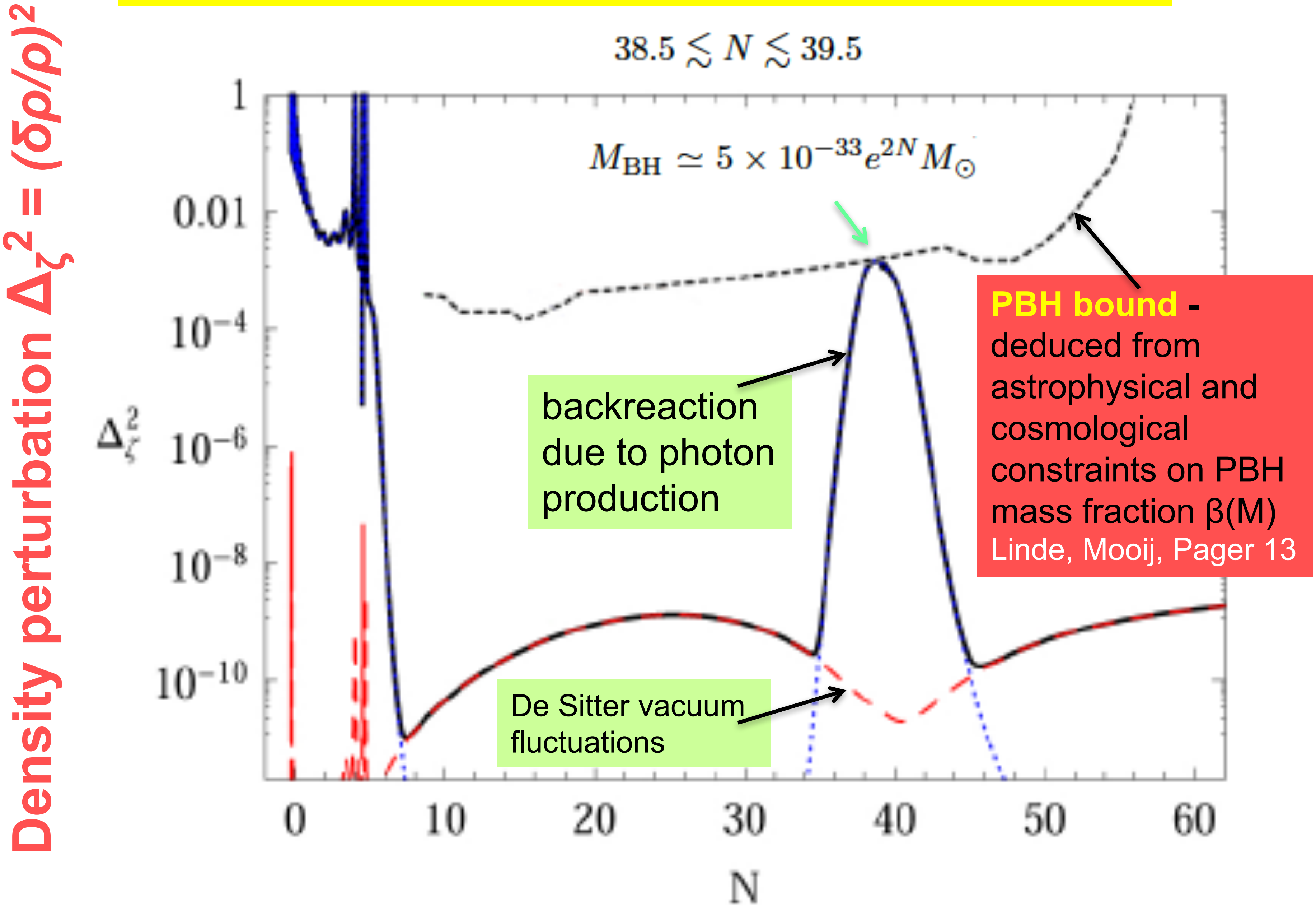


all rescaled by M_p



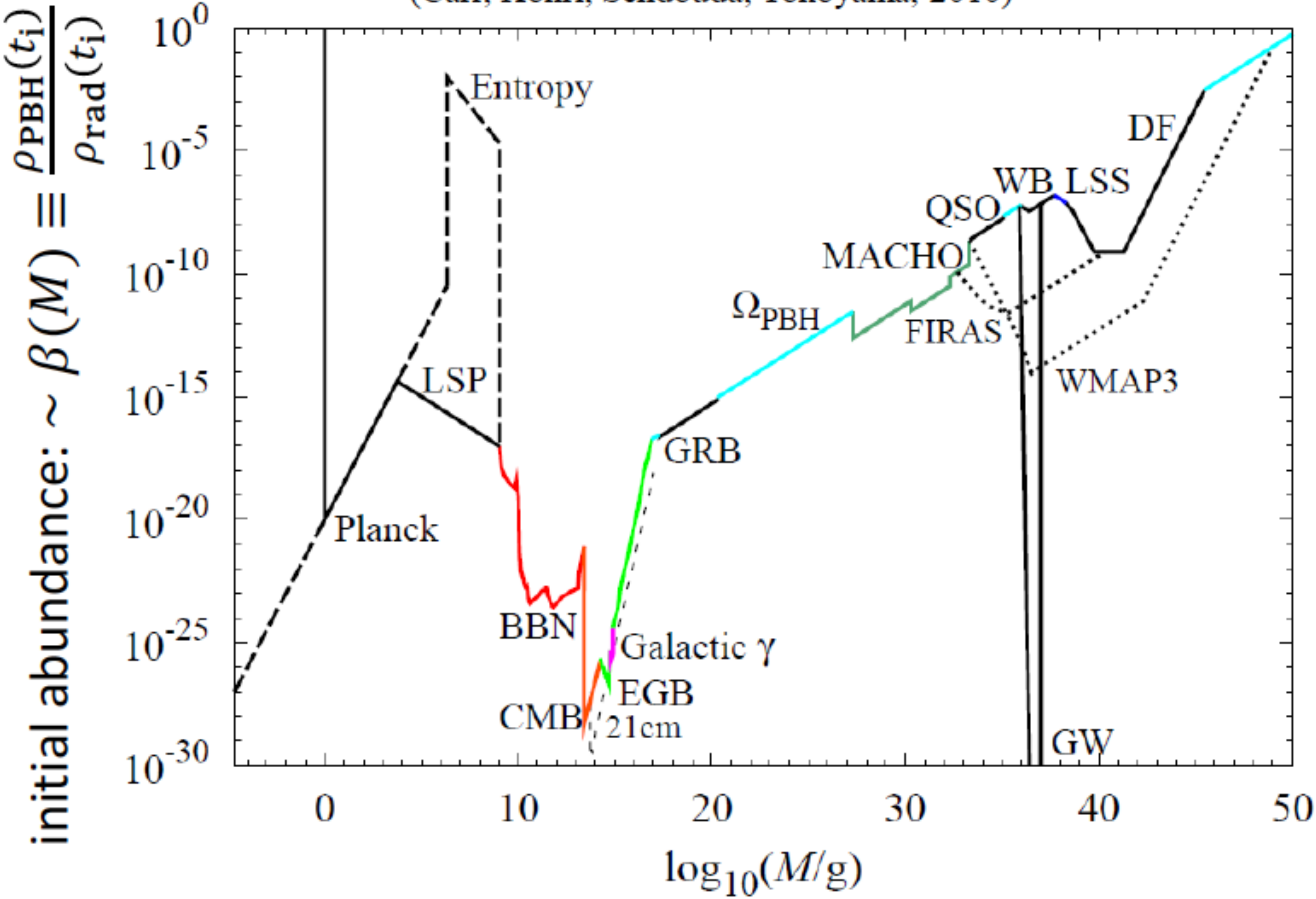
Note that inflation starts at N=61

Production of 10-100$M_\odot$ PBHs



Observational constraint on PBH abundance

(Carr, Kohri, Sendouda, Yokoyama, 2010)



PBH Associated Gravitational Waves in Inflation

$$\left[\frac{\partial^2}{\partial\eta^2} + \frac{2}{a}\frac{da}{d\eta}\frac{\partial}{\partial\eta} - \vec{\nabla}^2\right] h_{ij} = 0$$

Free gravitational wave equation

De Sitter vacuum fluctuations during inflation lead to almost scale-invariant primordial gravitational waves

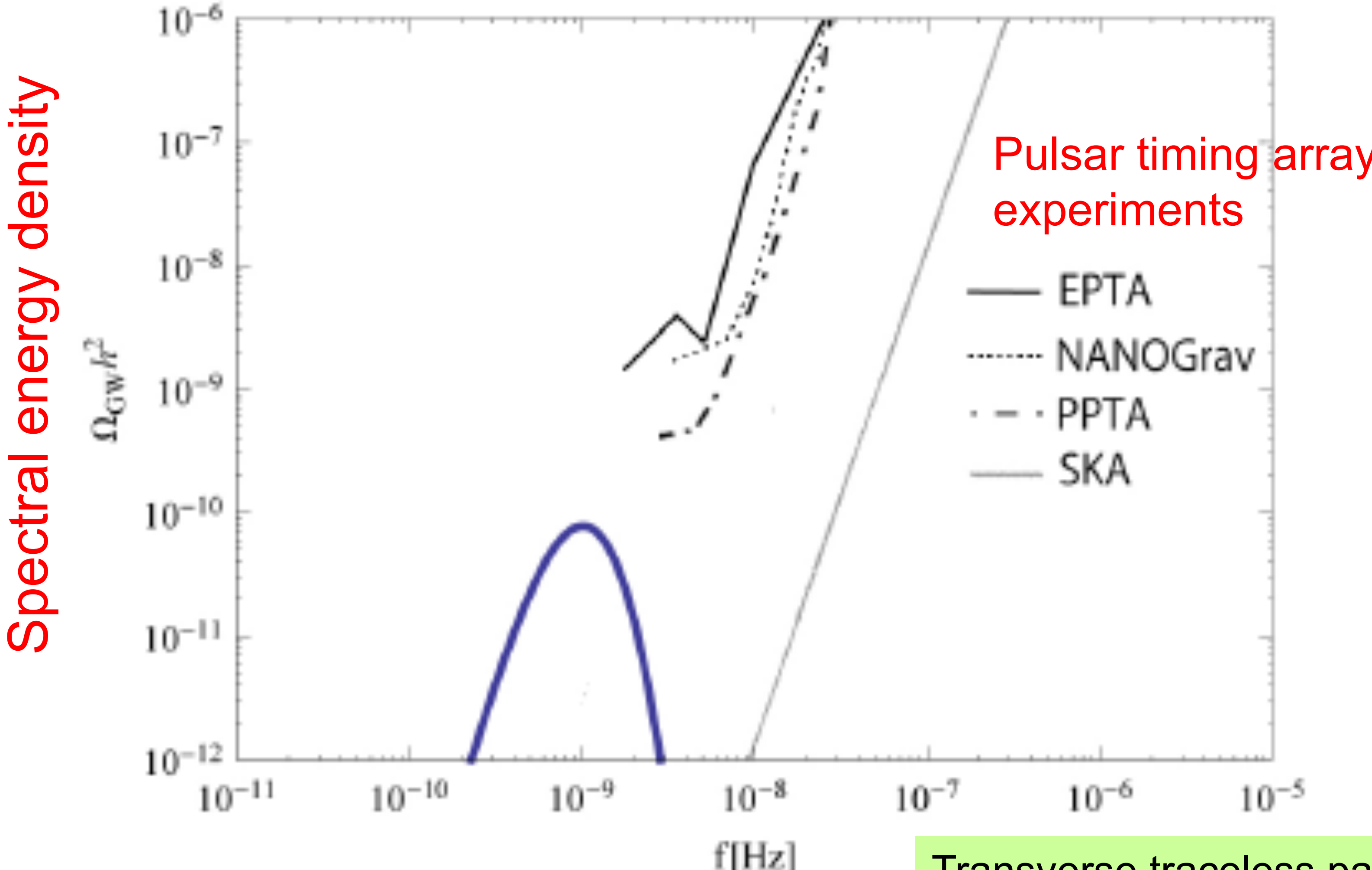
$\Delta_\zeta^2 = <\zeta\zeta> = (\delta\rho/\rho)^2 \sim 2\text{x}10^{-9}$

$$\left[\frac{\partial^2}{\partial\eta^2} + \frac{2}{a}\frac{da}{d\eta}\frac{\partial}{\partial\eta} - \vec{\nabla}^2\right] h_{ij} = GT^{\mu\nu}$$

Sources due to transverse traceless part of second-order density perturbation $\zeta\zeta$

$\Delta_\zeta^2 = <\zeta\zeta> = (\delta\rho/\rho)^2 \sim 10^{-3}$

Associated Gravitational Waves in Trapped Inflation

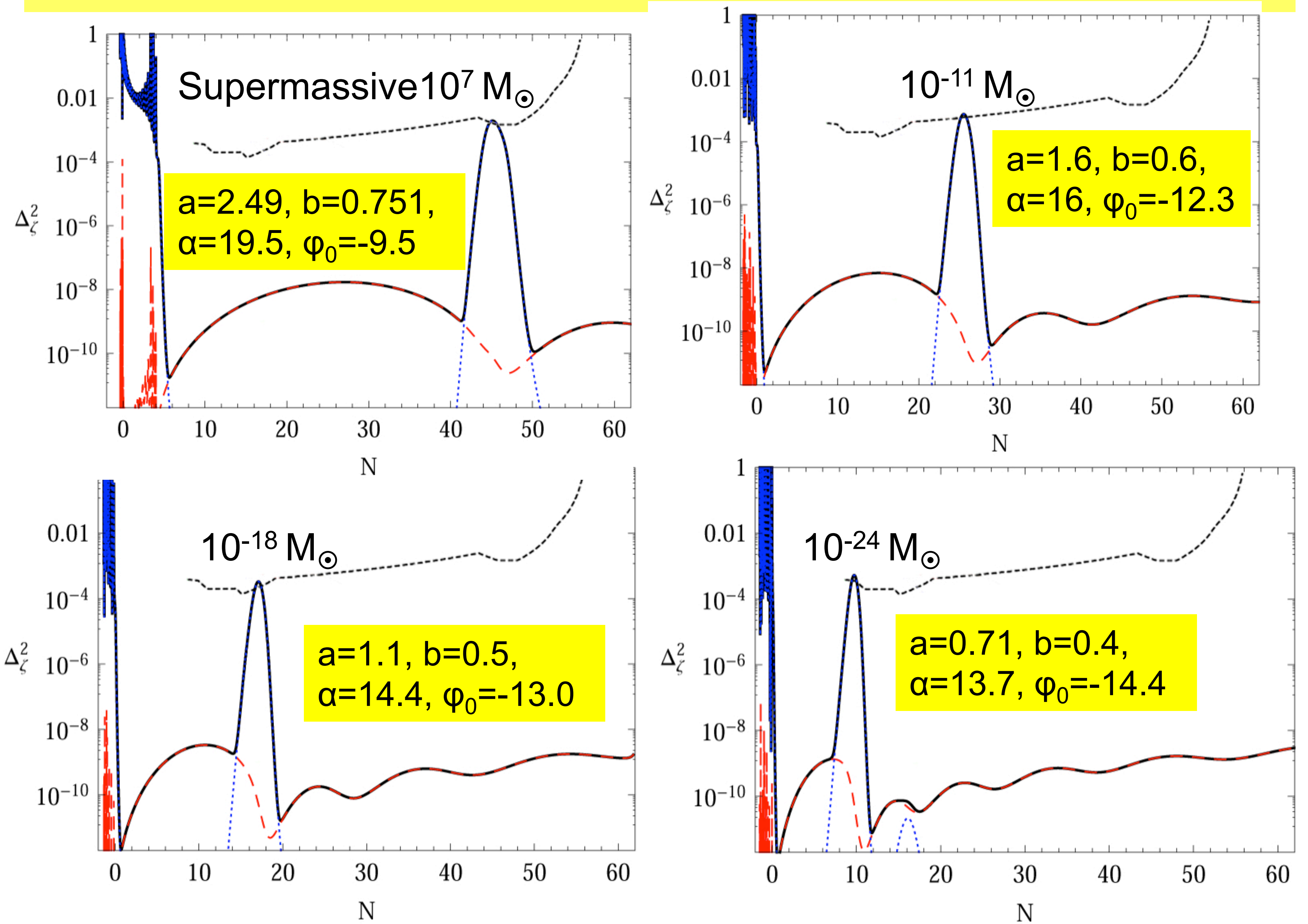


$$\left[\frac{\partial^2}{\partial\eta^2}+\frac{2}{a}\frac{da}{d\eta}\frac{\partial}{\partial\eta}-\vec{\nabla}^2\right]h_{ij}=\frac{2a^2}{M_p^2}(-E_iE_j-B_iB_j)^{TT}$$

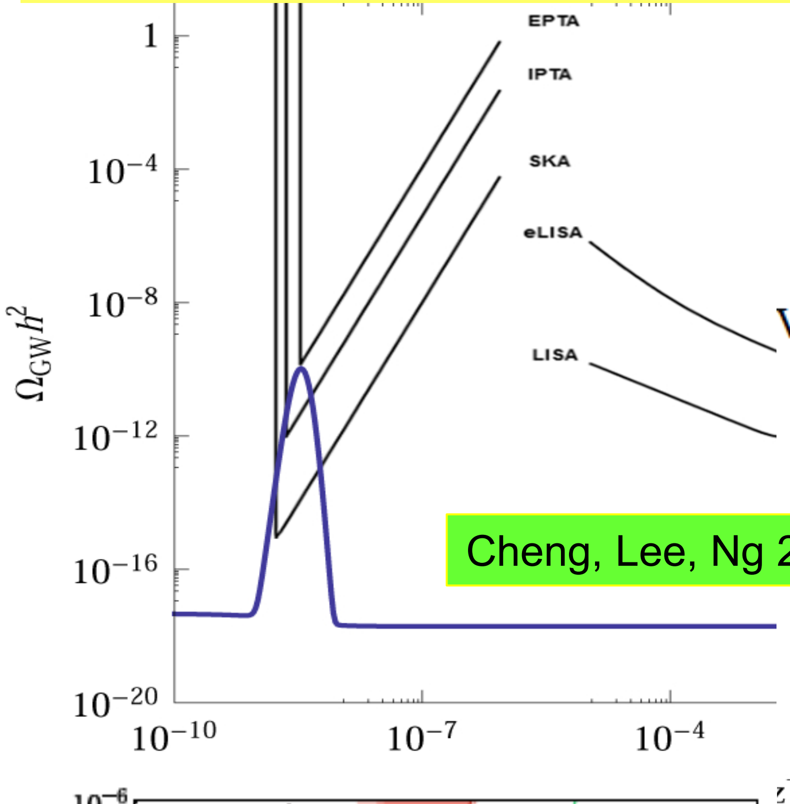
Transverse traceless part of photon energy-momentum

<E·E> ~ <E·B> ~ ζ

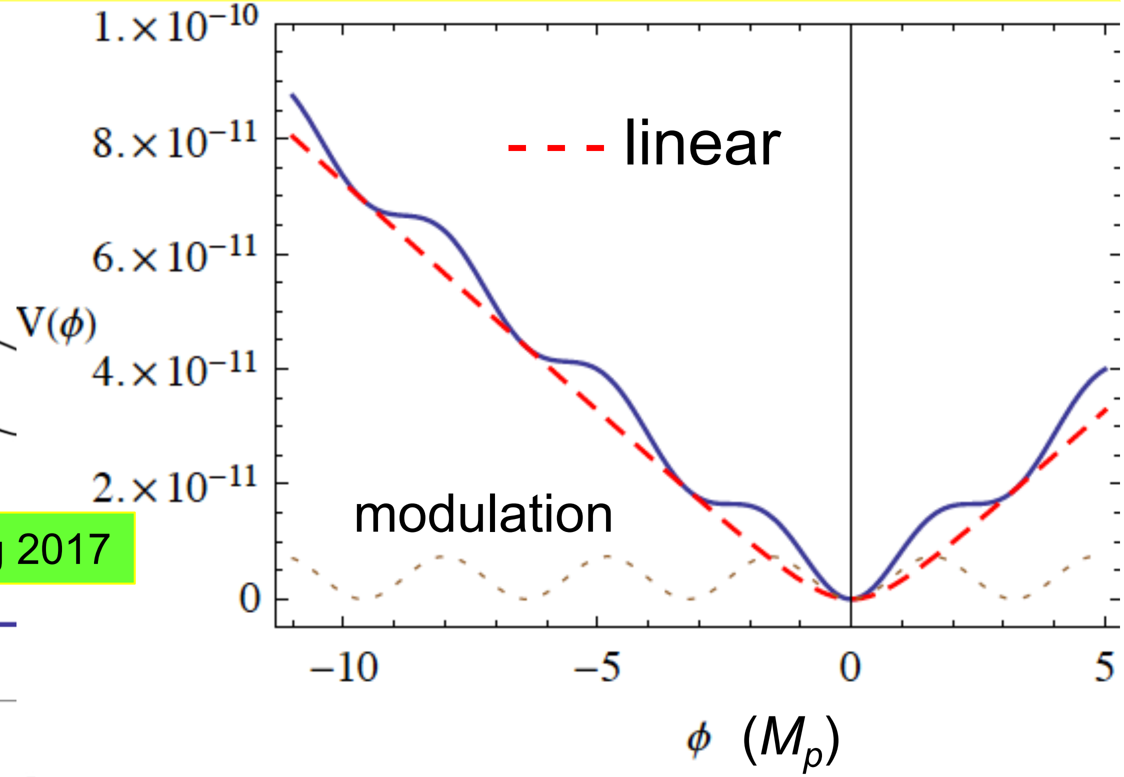
Producing PBHs with a wide range of masses

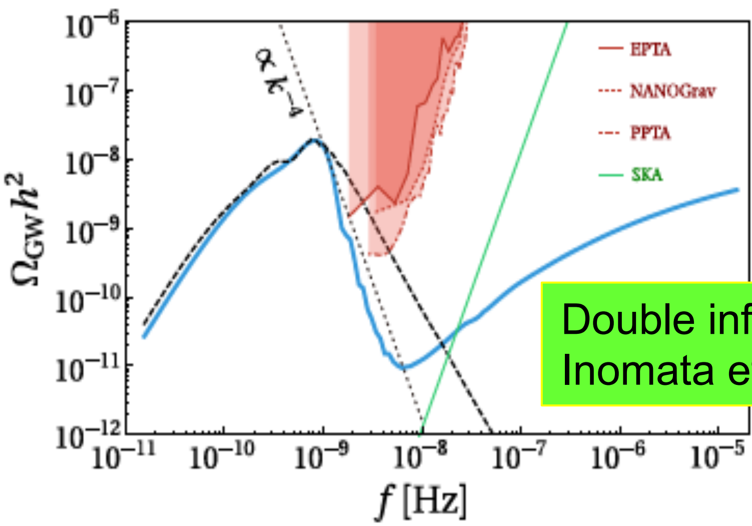


GWs associated with $10M_\odot$ PBHs in modulated axion inflation



Cheng, Lee, Ng 2017



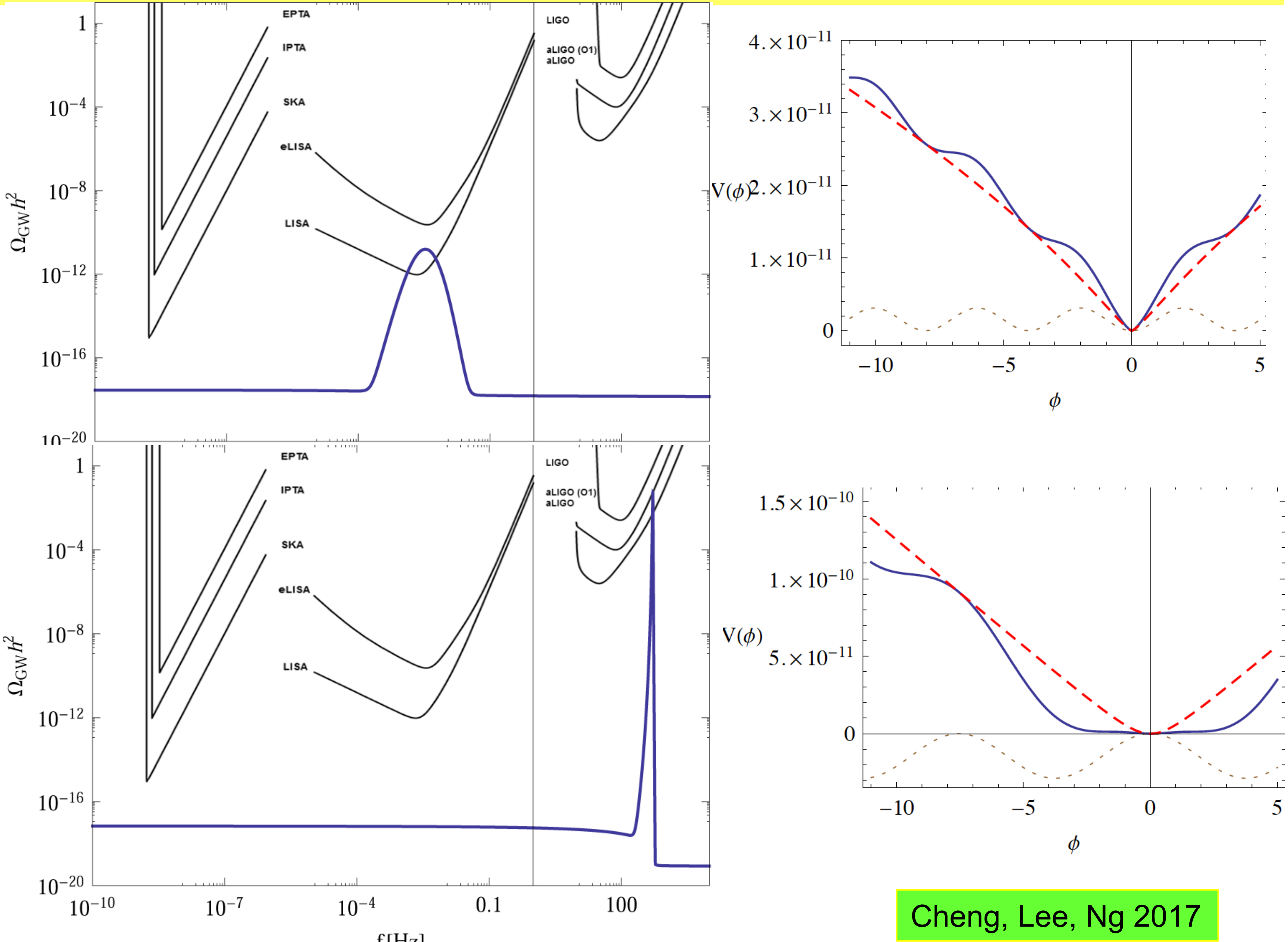


Double inflation
Inomata et al. 16

$$V(\varphi) = M_p\mu^3\sqrt{1+\left(\frac{\varphi}{M_p}\right)^2} - M_p\mu^3 + \Lambda^4\left[1+\cos\left(\frac{\varphi}{f}+\pi\right)\right]$$

M_p reduced Planck mass
f axion decay constant

GWs associated with smaller PBHs in modulated axion inflation



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Higgs Inflation

Bezrukov+Shaposhnikov

SM Higgs potential

Jordan frame

$$S_J = \int d^4x\sqrt{-g}\left\{ -\frac{M^2+\xi h^2}{2}R + \frac{\partial_\mu h \partial^\mu h}{2} - \frac{\lambda}{4}\left(h^2 - v^2\right)^2 \right\}$$

Conformal transformation

$$\hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}\ , \quad \Omega^2 = 1 + \frac{\xi h^2}{M_P^2}$$

Einstein frame

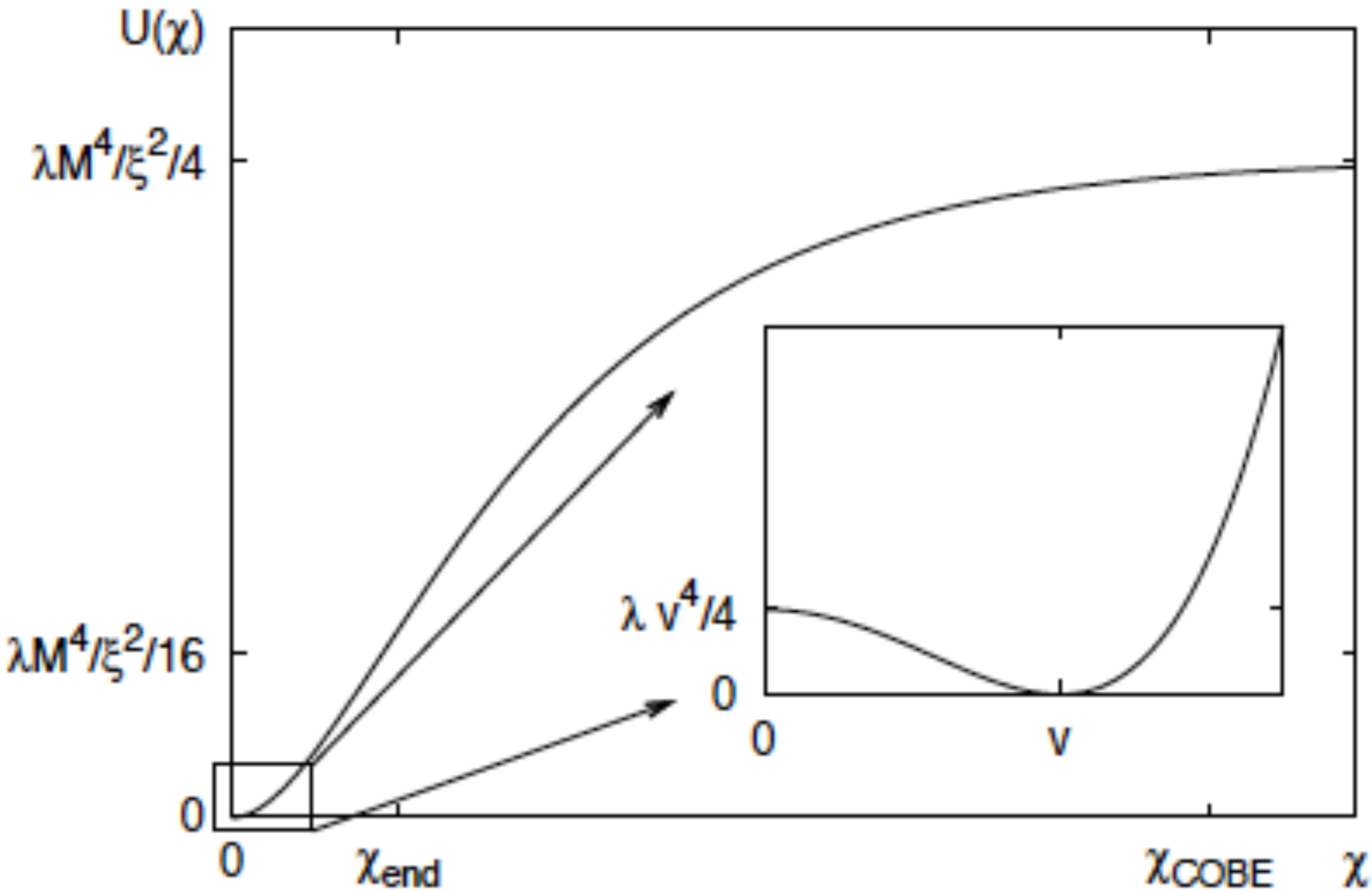
$$S_E = \int d^4x\sqrt{-\hat{g}}\left\{ -\frac{M_P^2}{2}\hat{R} + \frac{\partial_\mu \chi \partial^\mu \chi}{2} - U(\chi) \right\}$$

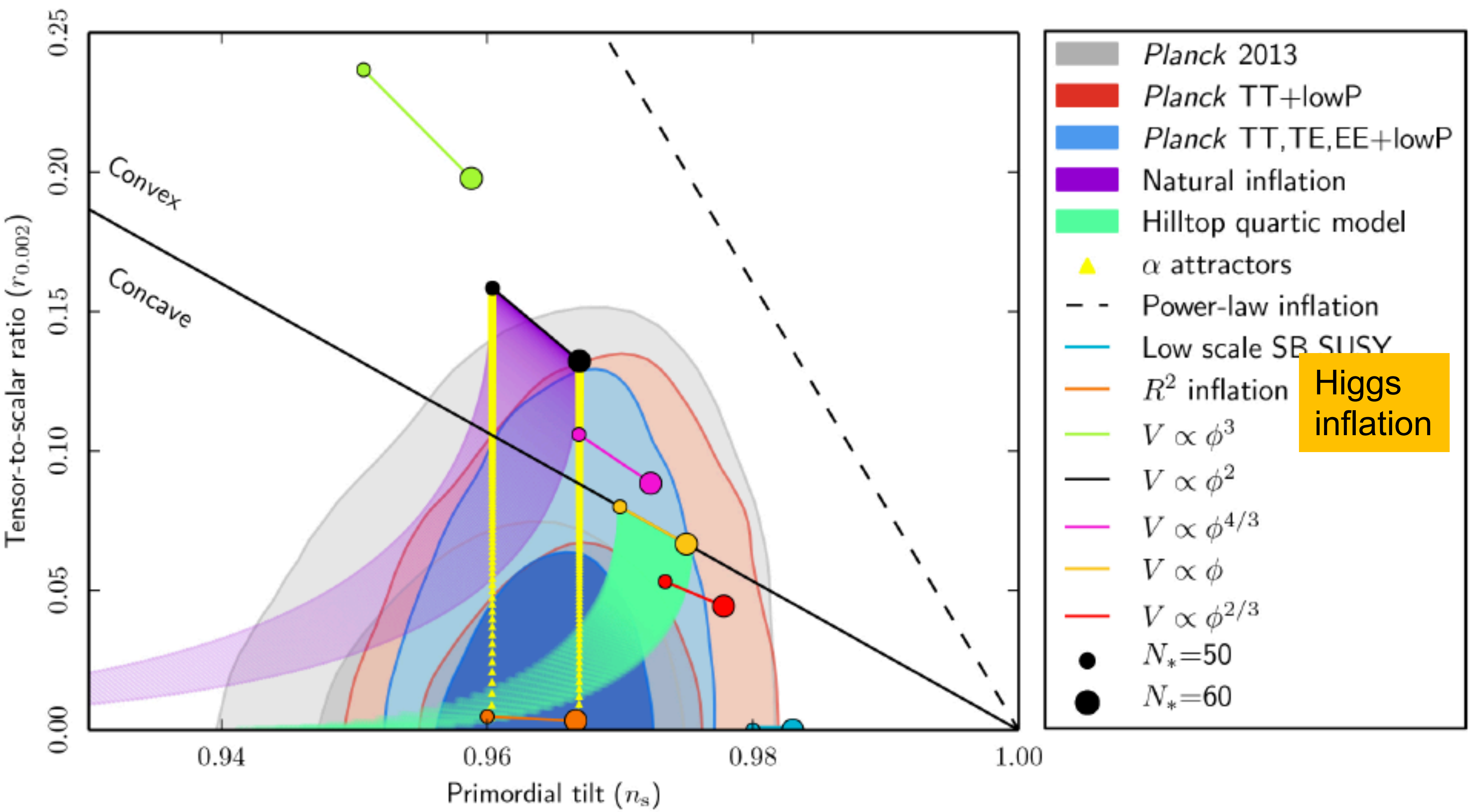
$$\frac{d\chi}{dh} = \sqrt{\frac{\Omega^2 + 6\xi^2 h^2/M_P^2}{\Omega^4}}$$

Canonical scalar field χ as inflaton

$$U(\chi) = \frac{1}{\Omega(\chi)^4}\frac{\lambda}{4}\left(h(\chi)^2 - v^2\right)^2$$

Inflation potential


U(χ)
λM⁴/ξ²/4
λM⁴/ξ²/16
0
0
χend
χCOBE
χ
λ v⁴/4
0
0
v



$\xi \sim 10^4 m_H/v$ to give $\Delta_\zeta^2 \sim 2x10^{-9}$

Higgs + photon

magnetogenesis

$$S_E = \int d^4x\sqrt{-\hat{g}}\left\{ -\frac{M_P^2}{2}\hat{R} + \frac{\partial_\mu\chi\partial^\mu\chi}{2} - U(\chi) - 1/4\, I(\chi)\, F^{\mu\nu}F_{\mu\nu} \right\}$$

$$\frac{\partial^2\vec{A}}{\partial\tau^2} - \vec{\nabla}^2\vec{A} + \frac{I'}{I}\frac{\partial\vec{A}}{\partial\tau} = 0$$

$$\vec{E} = -\frac{1}{a^2}\frac{\partial\vec{A}}{\partial\tau}, \qquad \vec{B} = \frac{1}{a^2}\vec{\nabla}\times\vec{A}.$$

$$\vec{A}(\tau,\vec{x}) = \sum_{\lambda=\pm}\int\frac{d^3k}{(2\pi)^{3/2}}\left[\vec{\epsilon}_\lambda(\vec{k})a_\lambda(\vec{k})A_\lambda(\tau,\vec{k})e^{i\vec{k}\cdot\vec{x}} + \text{h.c.}\right]$$

$$\left[\frac{\partial^2}{\partial\tau^2} + \frac{I'}{I}\frac{\partial}{\partial\tau} + k^2\right]A_\pm(\tau,k) = 0$$

$$\left[a_\lambda(\vec{k}), a^\dagger_{\lambda'}(\vec{k}')\right] = \delta_{\lambda\lambda'}\delta(\vec{k}-\vec{k}')$$

Background

$$\ddot{\phi} + (3H+\Gamma)\dot{\phi} + \frac{dV}{d\phi} = \frac{1}{2}\frac{dI}{d\phi}\langle\vec{E}^2 - \vec{B}^2\rangle$$

$\chi = \phi + \delta\phi$

$$3H^3 = \frac{1}{M_p^2}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi) + \rho_{EB} + \rho_\gamma\right]$$

Γ perturbative reheating

$$\dot{\rho}_\gamma + 4H\rho_\gamma = \Gamma\dot{\phi}^2$$

$$\rho_{\rm EM} \equiv \frac{I}{2}\langle\vec{E}^2 + \vec{B}^2\rangle = \frac{I}{4\pi^2 a^4}\int dk\, k^2\sum_{\lambda=\pm}\left(|A'_\lambda|^2 + k^2|A_\lambda|^2\right)$$

Conclusion

- The binary BHs observed by aLIGO/VIRGO gravity-wave detectors may be primordial BHs
- PBHs could be dark matter
- aLIGO saw indirect signals (GWs) of DM !?
- How to distinguish PBHs from astrophysical BHs?

  PBHs have less spins – need more data and statistics

- Production of PBHs with a wide range of masses in trapped inflation models

- From a toy trapped inflation model to axion monodromy inflation with modulations

- PBHs need axion decay constant $f \sim M_{Planck}$

  Oscillations in CMB from modulations with
  axion decay constant $f \ll M_{Planck}$ Flauger et al. 2010, Choi+Kim 2016

- Associated primordial gravitational waves in the frequency ranges of PTAs, LISA, aLIGO/VIRGO

  polarized (+ mode / - mode) GWs in axion inflation