Unpolarized parton distribution functions of light vector mesons

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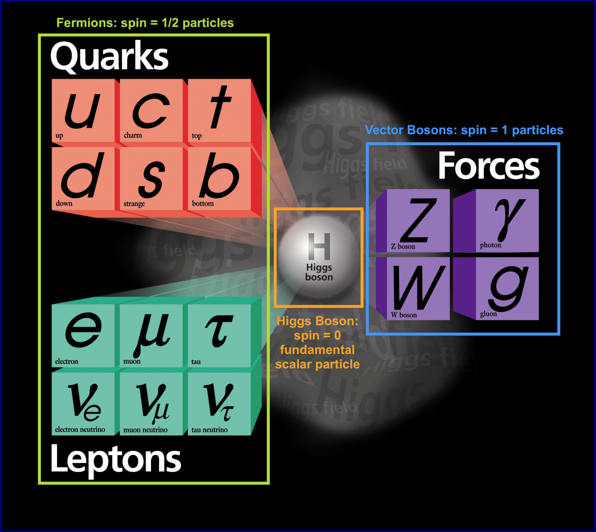
Building Block of Matter



Lagrangian of the Standard Model of Particle Physics

These three lines in the Standard Model are ultra-specific to the gluon, the boson that carries the strong force. Gluons come in the eight types, interact among themselves and have what's called a color charge [1]

$$-\frac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \frac{1}{4}g_s^2 f^{abc}f^{ade}g^b_\mu g^c_\nu g^d_\mu g^e_\nu + \frac{1}{2}ig_s^2(\bar{q}^\sigma_i \gamma^\mu q^\sigma_j)g^a_\mu + \bar{G}^a\partial^2 G^a + g_s f^{abc}\partial_\mu \bar{G}^a G^b g^c_\mu - \partial_\nu W^+_\mu \partial_\nu W^-_\mu - M^2 W^+_\mu W^-_\mu - \frac{1}{2}\partial_\nu Z^0_\mu \partial_\nu Z^0_\mu - \frac{1}{2c_w^2}M^2 Z^0_\mu Z^0_\mu - \frac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - \frac{1}{2}\partial_\mu H \partial_\mu H -$$

[1] http://www.symmetrymagazine.org/article/the-deconstructed-standard-model-equation

Lagrangian of the Standard Model of Particle Physics

Almost half of this equation is dedicated to explaining interactions between bosons, particularly W and Z bosons. Bosons are force-carrying particles, and there are four species of bosons that interact with other particles using three fundamental forces. Photons carry electromagnetism, gluons carry the strong force and W and Z bosons carry the weak force. The most recently discovered boson, the Higgs boson, is a bit different; its interactions appear in the next part of the equation [2]

$$
\begin{aligned}
&-\tfrac12\partial_\nu g^a_\mu\partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \tfrac14 g_s^2 f^{abc}f^{ade} g^b_\mu g^c_\nu g^d_\mu g^e_\nu + \\
&\tfrac12 i g_s^2(\bar q_i^\sigma\gamma^\mu q_j^\sigma)g^a_\mu + \bar G^a\partial^2 G^a + g_s f^{abc}\partial_\mu \bar G^a G^b g^c_\mu - \partial_\nu W^+_\mu\partial_\nu W^-_\mu - \\
&M^2W^+_\mu W^-_\mu - \tfrac12\partial_\nu Z^0_\mu\partial_\nu Z^0_\mu - \tfrac{1}{2c_w^2}M^2 Z^0_\mu Z^0_\mu - \tfrac12\partial_\mu A_\nu\partial_\mu A_\nu - \tfrac12\partial_\mu H\partial_\mu H - \\
&\tfrac12 m_h^2H^2 - \partial_\mu\phi^+\partial_\mu\phi^- - M^2\phi^+\phi^- - \tfrac12\partial_\mu\phi^0\partial_\mu\phi^0 - \tfrac{1}{2c_w^2}M\phi^0\phi^0 - \beta_h[\tfrac{2M^2}{g^2} + \\
&\tfrac{2M}{g}H + \tfrac12(H^2+\phi^0\phi^0+2\phi^+\phi^-)] + \tfrac{2M^4}{g^2}\alpha_h - igc_w[\partial_\nu Z^0_\mu(W^+_\mu W^-_\nu - \\
&W^+_\nu W^-_\mu) - Z^0_\nu(W^+_\mu\partial_\nu W^-_\mu - W^-_\mu\partial_\nu W^+_\mu) + Z^0_\mu(W^+_\nu\partial_\nu W^-_\mu - \\
&W^-_\nu\partial_\nu W^+_\mu)] - igs_w[\partial_\nu A_\mu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu(W^+_\mu\partial_\nu W^-_\mu - \\
&W^-_\mu\partial_\nu W^+_\mu) + A_\mu(W^+_\nu\partial_\nu W^-_\mu - W^-_\nu\partial_\nu W^+_\mu)] - \tfrac12 g^2W^+_\mu W^-_\mu W^+_\nu W^-_\nu + \\
&\tfrac12 g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu + g^2c_w^2(Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) + \\
&g^2s_w^2(A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2s_wc_w[A_\mu Z^0_\nu(W^+_\mu W^-_\nu - \\
&W^+_\nu W^-_\mu) - 2A_\mu Z^0_\mu W^+_\nu W^-_\nu] - g\alpha[H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-] - \\
&\tfrac18 g^2\alpha_h[H^4+(\phi^0)^4+4(\phi^+\phi^-)^2+4(\phi^0)^2\phi^+\phi^-+4H^2\phi^+\phi^-+2(\phi^0)^2H^2] - \\
&gMW^+_\mu W^-_\mu H - \tfrac12 g\tfrac{M}{c_w^2}Z^0_\mu Z^0_\mu H - \tfrac12 ig[W^+_\mu(\phi^0\partial_\mu\phi^- - \phi^-\partial_\mu\phi^0) - \\
&W^-_\mu(\phi^0\partial_\mu\phi^+ - \phi^+\partial_\mu\phi^0)] + \tfrac12 g[W^+_\mu(H\partial_\mu\phi^- - \phi^-\partial_\mu H) - W^-_\mu(H\partial_\mu\phi^+ - \\
&\phi^+\partial_\mu H)] + \tfrac12 g\tfrac{1}{c_w}(Z^0_\mu(H\partial_\mu\phi^0 - \phi^0\partial_\mu H) - ig\tfrac{s_w^2}{c_w}MZ^0_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) + \\
&igs_wMA_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) - ig\tfrac{1-2c_w^2}{2c_w}Z^0_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) + \\
&igs_wA_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) - \tfrac14 g^2W^+_\mu W^-_\mu[H^2+(\phi^0)^2+2\phi^+\phi^-] - \\
&\tfrac14 g^2\tfrac{1}{c_w^2}Z^0_\mu Z^0_\mu[H^2+(\phi^0)^2+2(2s_w^2-1)^2\phi^+\phi^-] - \tfrac12 g^2\tfrac{s_w^2}{c_w}Z^0_\mu\phi^0(W^+_\mu\phi^- + \\
&W^-_\mu\phi^+) - \tfrac12 ig^2\tfrac{s_w}{c_w}Z^0_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) + \tfrac12 g^2s_wA_\mu\phi^0(W^+_\mu\phi^- + \\
&W^-_\mu\phi^+) + \tfrac12 ig^2s_wA_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) - g^2\tfrac{s_w}{c_w}(2c_w^2-1)Z^0_\mu A_\mu\phi^+\phi^- - \\
&g^1s_w^2A_\mu A_\mu\phi^+\phi^- - \bar e^\lambda(\gamma\partial+m_e^\lambda)e^\lambda - \bar\nu^\lambda\gamma\partial\nu^\lambda - \bar u_j^\lambda(\gamma\partial+m_u^\lambda)u_j^\lambda - \\
&\bar d_j^\lambda(\gamma\partial+m_d^\lambda)d_j^\lambda + igs_wA_\mu[-(\bar e^\lambda\gamma^\mu e^\lambda)+\tfrac23(\bar u_j^\lambda\gamma^\mu u_j^\lambda)-\tfrac13(\bar d_j^\lambda\gamma^\mu d_j^\lambda)] +
\end{aligned}
$$

[2] http://www.symmetrymagazine.org/article/the-deconstructed-standard-model-equation

Lagrangian of the Standard Model of Particle Physics

This part of the equation describes how elementary matter particles interact with the weak force. According to this formulation, matter particles come in three generations, each with different masses. The weak force helps massive matter particles decay into less massive matter particles. This section also includes basic interactions with the Higgs field, from which some elementary particles receive their mass [3]

$$W_\mu^-\phi^+) + \frac{1}{2} i g^2 s_w A_\mu H (W_\mu^+\phi^- - W_\mu^-\phi^+) - g^2 \frac{s_w}{c_w}(2c_w^2-1) Z_\mu^0 A_\mu \phi^+\phi^- -$$

$$g^1 s_w^2 A_\mu A_\mu \phi^+\phi^- - \bar{e}^\lambda(\gamma\partial + m_e^\lambda)e^\lambda - \bar{\nu}^\lambda\gamma\partial\nu^\lambda - \bar{u}_j^\lambda(\gamma\partial+m_u^\lambda)u_j^\lambda -$$

$$\bar{d}_j^\lambda(\gamma\partial + m_d^\lambda)d_j^\lambda + igs_w A_\mu[-(\bar{e}^\lambda\gamma^\mu e^\lambda) + \tfrac{2}{3}(\bar{u}_j^\lambda\gamma^\mu u_j^\lambda) - \tfrac{1}{3}(\bar{d}_j^\lambda\gamma^\mu d_j^\lambda)] +$$

$$\frac{ig}{4c_w} Z_\mu^0[(\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{e}^\lambda\gamma^\mu(4s_w^2 - 1 - \gamma^5)e^\lambda) + (\bar{u}_j^\lambda\gamma^\mu(\tfrac{4}{3}s_w^2 -$$

$$1 - \gamma^5)u_j^\lambda) + (\bar{d}_j^\lambda\gamma^\mu(1 - \tfrac{8}{3}s_w^2 - \gamma^5)d_j^\lambda)] + \frac{ig}{2\sqrt{2}} W_\mu^+[(\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)e^\lambda) +$$

$$(\bar{u}_j^\lambda\gamma^\mu(1+\gamma^5)C_{\lambda\kappa}d_j^\kappa)] + \frac{ig}{2\sqrt{2}} W_\mu^-[(\bar{e}^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{d}_j^\kappa C_{\lambda\kappa}^\dagger\gamma^\mu(1+$$

$$\gamma^5)u_j^\lambda)] + \frac{ig}{2\sqrt{2}}\frac{m_e^\lambda}{M}[-\phi^+(\bar{\nu}^\lambda(1-\gamma^5)e^\lambda) + \phi^-(\bar{e}^\lambda(1+\gamma^5)\nu^\lambda)] -$$

$$\frac{g}{2}\frac{m_e^\lambda}{M}[H(\bar{e}^\lambda e^\lambda) + i\phi^0(\bar{e}^\lambda\gamma^5 e^\lambda)] + \frac{ig}{2M\sqrt{2}}\phi^+[-m_d^\kappa(\bar{u}_j^\lambda C_{\lambda\kappa}(1-\gamma^5)d_j^\kappa) +$$

[3] http://www.symmetrymagazine.org/article/the-deconstructed-standard-model-equation

Lagrangian of the Standard Model of Particle Physics

This part of the equation describes how matter particles interact with Higgs ghosts, virtual artifacts from the Higgs field [4]

$$(\bar{u}_j^\lambda \gamma^\mu (1+\gamma^5) C_{\lambda\kappa} d_j^\kappa)] + \frac{ig}{2\sqrt{2}} W_\mu^- [(\bar{e}^\lambda \gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{d}_j^\kappa C^\dagger_{\lambda\kappa}\gamma^\mu(1+$$

$$\gamma^5)u_j^\lambda)] + \frac{ig}{2\sqrt{2}}\frac{m_e^\lambda}{M}[-\phi^+(\bar\nu^\lambda(1-\gamma^5)e^\lambda) + \phi^-(\bar e^\lambda(1+\gamma^5)\nu^\lambda)] -$$

$$\frac{g}{2}\frac{m_e^\lambda}{M}[H(\bar e^\lambda e^\lambda) + i\phi^0(\bar e^\lambda\gamma^5 e^\lambda)] + \frac{ig}{2M\sqrt{2}}\phi^+[-m_d^\kappa(\bar u_j^\lambda C_{\lambda\kappa}(1-\gamma^5)d_j^\kappa) +$$

$$m_u^\lambda(\bar u_j^\lambda C_{\lambda\kappa}(1+\gamma^5)d_j^\kappa] + \frac{ig}{2M\sqrt{2}}\phi^-[m_d^\lambda(\bar d_j^\lambda C^\dagger_{\lambda\kappa}(1+\gamma^5)u_j^\kappa) - m_u^\kappa(\bar d_j^\lambda C^\dagger_{\lambda\kappa}(1-$$

$$\gamma^5)u_j^\kappa] - \frac{g}{2}\frac{m_u^\lambda}{M}H(\bar u_j^\lambda u_j^\lambda) - \frac{g}{2}\frac{m_d^\lambda}{M}H(\bar d_j^\lambda d_j^\lambda) + \frac{ig}{2}\frac{m_u^\lambda}{M}\phi^0(\bar u_j^\lambda\gamma^5 u_j^\lambda) -$$

$$\frac{ig}{2}\frac{m_d^\lambda}{M}\phi^0(\bar d_j^\lambda\gamma^5 d_j^\lambda) + \bar X^+(\partial^2 - M^2)X^+ + \bar X^-(\partial^2-M^2)X^- + \bar X^0(\partial^2 -$$

$$\frac{M^2}{c_w^2})X^0 + \bar Y\partial^2 Y + igc_w W_\mu^+(\partial_\mu \bar X^0 X^- - \partial_\mu \bar X^+ X^0) + igs_w W_\mu^+(\partial_\mu \bar Y X^- -$$

[4] http://www.symmetrymagazine.org/article/the-deconstructed-standard-model-equation

Lagrangian of the Standard Model of Particle Physics

This last part of the equation includes more ghosts. These ones are called Faddeev-Popov ghosts, and they cancel out redundancies that occur in interactions through the weak force [5]

$$
\gamma^5)u_j^\kappa] - \frac{g}{2}\frac{m_u^\lambda}{M}H(\bar{u}_j^\lambda u_j^\lambda) - \frac{g}{2}\frac{m_d^\lambda}{M}H(\bar{d}_j^\lambda d_j^\lambda) + \frac{ig}{2}\frac{m_u^\lambda}{M}\phi^0(\bar{u}_j^\lambda\gamma^5 u_j^\lambda) -
$$

$$
\frac{ig}{2}\frac{m_d^\lambda}{M}\phi^0(\bar{d}_j^\lambda\gamma^5 d_j^\lambda) + \bar{X}^+(\partial^2 - M^2)X^+ + \bar{X}^-(\partial^2 - M^2)X^- + \bar{X}^0(\partial^2 -
$$

$$
\frac{M^2}{c_w^2})X^0 + \bar{Y}\partial^2 Y + igc_w W_\mu^+(\partial_\mu\bar{X}^0X^- - \partial_\mu\bar{X}^+X^0) + igs_w W_\mu^+(\partial_\mu\bar{Y}X^- -
$$

$$
\partial_\mu\bar{X}^+Y) + igc_w W_\mu^-(\partial_\mu\bar{X}^-X^0 - \partial_\mu\bar{X}^0X^+) + igs_w W_\mu^-(\partial_\mu\bar{X}^-Y -
$$

$$
\partial_\mu\bar{Y}X^+) + igc_w Z_\mu^0(\partial_\mu\bar{X}^+X^+ - \partial_\mu\bar{X}^-X^-) + igs_w A_\mu(\partial_\mu\bar{X}^+X^+ -
$$

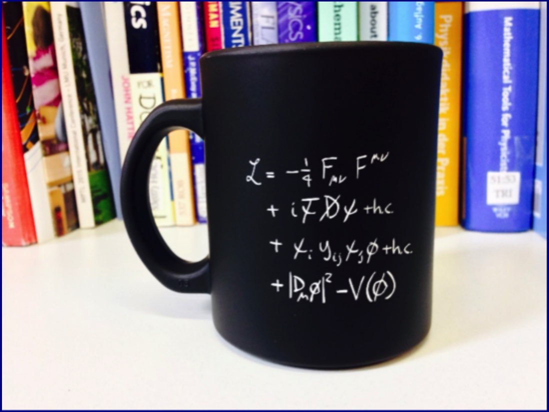
$$
\partial_\mu\bar{X}^-X^-) - \frac{1}{2}gM[\bar{X}^+X^+H + \bar{X}^-X^-H + \frac{1}{c_w^2}\bar{X}^0X^0H] +
$$

$$
\frac{1-2c_w^2}{2c_w}igM[\bar{X}^+X^0\phi^+ - \bar{X}^-X^0\phi^-] + \frac{1}{2c_w}igM[\bar{X}^0X^-\phi^+ - \bar{X}^0X^+\phi^-] +
$$

$$
igMs_w[\bar{X}^0X^-\phi^+ - \bar{X}^0X^+\phi^-] + \frac{1}{2}igM[\bar{X}^+X^+\phi^0 - \bar{X}^-X^-\phi^0]
$$

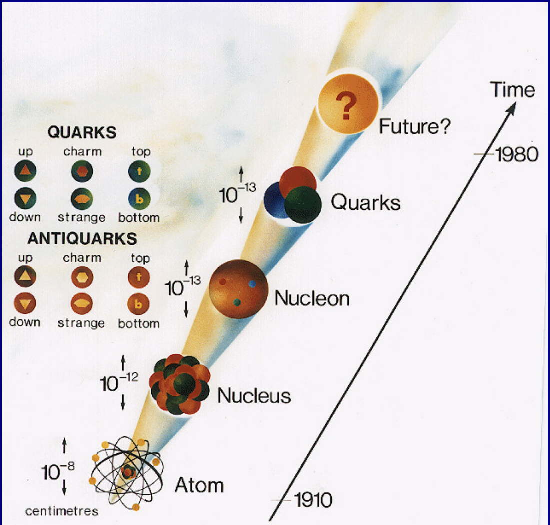
[5] http://www.symmetrymagazine.org/article/the-deconstructed-standard-model-equation

Lagrangian of the Standard Model of Particle Physics



We concentrate on Quantum Chromodynamics (QCD)

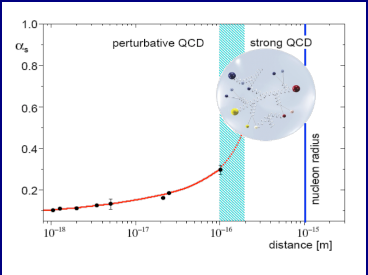
Particle Size

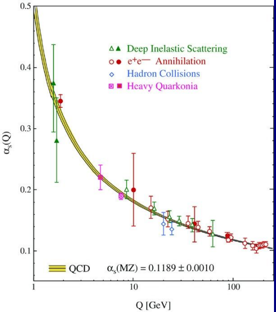


We focus on the interaction quarks inside hadrons with scale around 1 fm

QUANTUM CHROMODYNAMICS (QCD)

QCD start out with almost massless quarks + massless gluons + gauge bosons, their interactions generate the mass of the hadrons more than 95%.

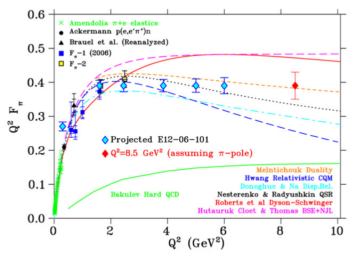


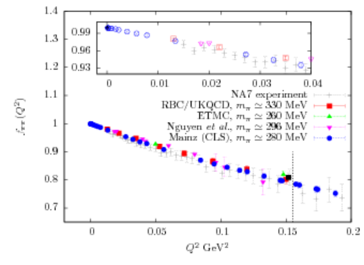


- In short distance scales ($r < 0.1 fm$) $\Rightarrow$ QCD is a theory of weakly coupled quark and gluon $\Rightarrow$ perturbative QCD applicable
- In the low energy limit, at momentum below 1 GeV ($r > 1 fm$) $\Rightarrow$ QCD is governed by quark (color) confinement and dynamical breaking of chiral symmetry $\Rightarrow$ nonperturbative QCD applicable

MOTIVATION: INTERNAL STRUCTURE OF HADRONS

- QCD, as underlying theory of strong interaction, is unable to *directly* predict structure of hadrons. *The solution*:
  - Lattice QCD: Large momentum effective theory (LAMET) [6]
  - QCD inspired models (mimicking features of QCD) such as NJL model, DSE model, QCD Sum rules, Instanton model, Covariant non-local chiral quark model (NLChQM) and other chiral effective models, Credit: Garth Huber Slide, EIC Meeting 2018 & Ref. [7]



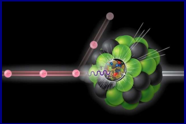


[6] X. Ji, Phys. Rev. Lett. 110

[7] Bastian. B. Brant, Int. J. Mod. Phys. E22 (2013)

Motivation: internal structure of hadrons

- To understand the structure of strongly interacting matter, parton distribution function (PDF), elastic form factor (EFF), fragmentation function (FF), generalized parton distribution function and transverse momentum dependent (TMD) are of fundamental importance and provide complementary information
- From experimental side, next experimental data for the hadron will be coming soon from JLAB (expected to be available soon), J-PARC as well as (CERN-SPS) COMPASS and future experiment Electron-Ion Collider (EIC)



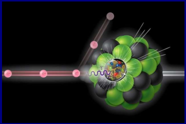


Figure: In an EIC, a beam of electrons (e^-) would scatter off a beam of protons or atomic nuclei, generating virtual photons (λ) — particles of light that penetrate the proton or nucleus to tease out the structure of the quarks and gluons within. Credit: Brookhaven National Laboratory.

Motivation: Meson properties

- The properties of vector meson from quark model

Vector mesons

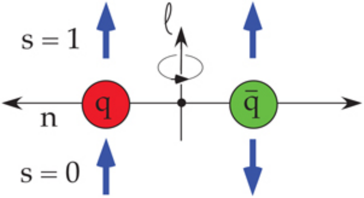
Particle name	Particle symbol	Antiparticle symbol	Quark content	Rest mass (MeV/c^2)	I^G	J^{PC}	S	C	B'	Mean lifetime (s)	Commonly decays to (>5% of decays)
Charged rho meson[27]	$\rho^+(770)$	$\rho^-(770)$	$u\bar{d}$	775.11 ±0.34	1^+	1^-	0	Sort ascending	0	$(4.41 \pm 0.02) \times 10^{-24}$[f][g]	$\pi^\pm + \pi^0$
Neutral rho meson[27]	$\rho^0(770)$	Self	$\frac{u\bar{u}-d\bar{d}}{\sqrt{2}}$	775.26 ±0.25	1^+	1^{--}	0	0	0	$(4.45 \pm 0.03) \times 10^{-24}$[f][g]	$\pi^+ + \pi^-$
Omega meson[28]	$\omega(782)$	Self	$\frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$	782.65 ±0.12	0^-	1^{--}	0	0	0	$(7.75 \pm 0.07) \times 10^{-23}$[f]	$\pi^+ + \pi^0 + \pi^-$ or $\pi^0 + \gamma$
Phi meson[29]	$\phi(1020)$	Self	$s\bar{s}$	1 019.461 ±0.019	0^-	1^{--}	0	0	0	$(1.54 \pm 0.01) \times 10^{-22}$[f]	$K^+ + K^-$ or $K^0_S + K^0_L$ or $(\rho + \pi) / (\pi^+ + \pi^0 + \pi^-)$
J/Psi[30]	J/ψ	Self	$c\bar{c}$	3 096.916 ±0.011	0^-	1^{--}	0	0	0	$(7.09 \pm 0.21) \times 10^{-21}$[f]	See J/ψ(1S) decay modes
Upsilon meson[31]	$\Upsilon(1S)$	Self	$b\bar{b}$	9 460.30 ±0.26	0^-	1^{--}	0	0	0	$(1.22 \pm 0.03) \times 10^{-20}$[f]	See ϒ(1S) decay modes
Kaon[32]	K^{*+}	K^{*-}	$u\bar{s}$	891.66 ±0.26	$\frac{1}{2}$	1^-	1	0	0	$(3.26 \pm 0.06) \times 10^{-23}$[f][g]	See K*(892) decay modes
Kaon[32]	K^{*0}	$\bar{K}^{*0}$	$d\bar{s}$	895.81 ±0.19	$\frac{1}{2}$	1^-	1	0	0	$(1.39 \pm 0.02) \times 10^{-23}$[f]	See K*(892) decay modes
D meson[33]	$D^{*+}(2010)$	$D^{*-}(2010)$	$c\bar{d}$	2 010.26 ±0.07	$\frac{1}{2}$	1^-	0	+1	0	$(7.89 \pm 0.17) \times 10^{-21}$[f]	$D^0 + \pi^+$ or $D^+ + \pi^0$
D meson[34]	$D^{*0}(2007)$	$\bar{D}^{*0}(2007)$	$c\bar{u}$	2 006.96 ±0.10	$\frac{1}{2}$	1^-	0	+1	0	$>3.1 \times 10^{-22}$[f]	$D^0 + \pi^0$ or $D^0 + \gamma$
Strange D meson[35]	D^{*+}_s	D^{*-}_s	$c\bar{s}$	2 112.1 ±0.4	0	1^-	+1	+1	0	$>3.4 \times 10^{-22}$[f]	$D^{*+} + \gamma$ or $D^{*+} + \pi^0$
B meson[36]	B^{*+}	B^{*-}	$u\bar{b}$	5 325.2 ±0.4	$\frac{1}{2}$	1^-	0	0	+1	Unknown	$B^+ + \gamma$
B meson[36]	B^{*0}	$\bar{B}^{*0}$	$d\bar{b}$	5 325.2 ±0.4	$\frac{1}{2}$	1^-	0	0	+1	Unknown	$B^0 + \gamma$
Strange B meson[37]	B^{*0}_s	$\bar{B}^{*0}_s$	$s\bar{b}$	$5\,415.4^{+2.4}_{-2.1}$	0	1^-	−1	0	+1	Unknown	$B^0_s + \gamma$
Charmed B meson†	B^{*+}_c	B^{*-}_c	$c\bar{b}$	Unknown	0	1^-	0	+1	+1	Unknown	Unknown

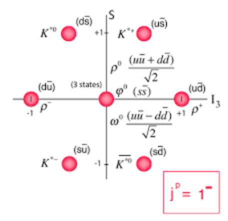
[f] ^ PDG reports the resonance width (Γ). Here the conversion $\tau = \hbar/\Gamma$ is given instead.
[g] ^ The exact value depends on the method used. See the given reference for detail.

Credit: Wikipedia

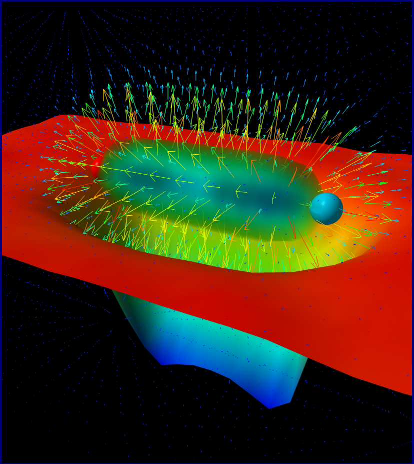
Motivation: Light meson masses

- We believe that meson mass depends on the constituent quark mass, i.e. $m_{K(u\bar{s})}(0.495\text{ GeV}) > m_{\pi(u\bar{d})}(0.140\text{ GeV})$
- However, this does not happen for $m_{\rho(u\bar{d})}(0.776\text{ GeV}) > m_{\pi(u\bar{d})}(0.140\text{ GeV})$
- This is because of the difference quark spin orientation (spin-spin interactions),i.e. $\vec{S} = \vec{S}_1 + \vec{S}_2$ and then $\vec{S}_1.\vec{S}_2 = \frac{1}{2}[\vec{S}^2 - \vec{S}_1^2 - \vec{S}_s^2] = \frac{1}{2}[\vec{S}^2 - \frac{3}{4}]$
- For vector mesons $\vec{S}^2 = S(S+1) = 2 ==> \vec{S}_1.\vec{S}_2 = +1/4$ and for pseudoscalar mesons $\vec{S}^2 = 0 ==> \vec{S}_1.\vec{S}_2 = -3/4$





MESONS PROPERTIES IN THE BSE-NJL MODEL



Credit: Derek Leinweber, CCSM, University of Adelaide

Mesons in the BSE-NJL Model

The three flavor NJL Lagrangian [8]– containing only four fermion interactions

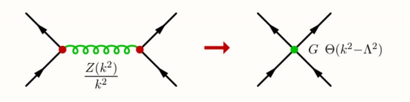
$$
\begin{aligned}
\mathscr{L}_{NJL} &= \bar{\psi}[i\partial\!\!\!/ - \hat{m}_q]\psi + G_\pi \sum_{a=0}^{8} \left[(\bar{\psi}\lambda_a\psi)^2 + (\bar{\psi}\lambda_a\gamma_5\psi)^2\right] \\
&- G_\rho \sum_{a=0}^{8} \left[(\bar{\psi}\lambda_a\gamma^\mu\psi)^2 + (\bar{\psi}\lambda_a\gamma^\mu\gamma_5\psi)^2\right] \\
&- G_\omega(\bar{\psi}\gamma^\mu\psi)^2
\end{aligned}
\tag{1}
$$

- $\psi = (u, d, s)^T$ denotes the quark field with the flavor components
- G_π and G_ρ and G_ω are four-fermion coupling constants
- $\lambda_1, \cdots, \lambda_8$ are Gell-Mann matrices in flavor space and $\lambda_0 \equiv \sqrt{\frac{2}{3}}1$
- $\hat{m}_q = \mathrm{diag}(m_u, m_d, m_s)$ denotes the current quark matrix

[8] **PH**, Ian Cloet, and A. Thomas, PRC**94** (2016), 035201

Mesons in the BSE-NJL Model

- In the NJL model, the gluon exchange is replaced by four-fermion contact interaction by integrating out the gluon field and absorbing into the coupling constant $\Longleftrightarrow$ *quark effective theory*



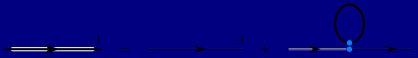
- NJL model has a lack of confinement (it can be simply seen quark propagator has a pole). Therefore we regularize using the proper time regularization to simulate confinement (IC.Cloet, PRC90 (2014), PH, PRC94 (2016))

$$\frac{1}{\mathscr{X}^n} = \frac{1}{(n-1)!}\int_0^\infty d\tau\tau^{(n-1)}e^{-\tau\mathscr{X}} \rightarrow \frac{1}{(n-1)!}\int_{1/\Lambda_{UV}^2}^{1/\Lambda_{IR}^2} d\tau\tau^{(n-1)}e^{-\tau\mathscr{X}} \quad (2)$$

where $\Lambda_{IR} \sim \Lambda_{QCD} \sim 0.24$ GeV and Λ_{UV} is determined.

Mesons in the BSE-NJL Model

- NJL Gap Equation is determined using quark propagator in momentum space $S_q^{-1}(p) = \not{p} - M_q + i\epsilon$



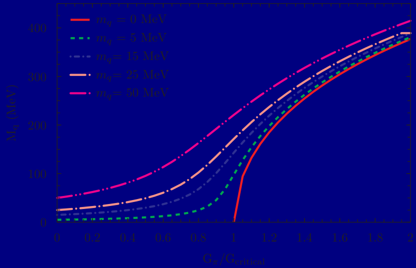
$$
\begin{aligned}
M_q &= m_q + M_q \frac{3G_\pi}{\pi^2} \int d\tau \frac{e^{-\tau M_q^2}}{\tau^2} \\
&= m_q - 2G_\pi \langle \bar{\psi}\psi \rangle
\end{aligned} \tag{3}
$$

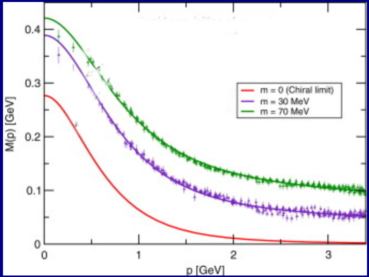
- Chiral quark condensates is defined by $\langle \bar{\psi}\psi \rangle = -\frac{3M_q}{2\pi^2} \int d\tau \frac{e^{-\tau M_q^2}}{\tau^2}$
- Mass is generated through interaction vacuum $\rightarrow \langle \bar{\psi}\psi \rangle \neq 0$

NJL Gap Equation

NJL [9] and DSE gap equations [10]

- The NJL constituent quark mass is a constant up to certain $p \sim 0.6$ GeV and it drops in higher p region



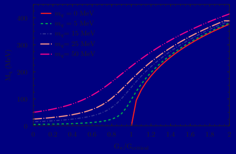


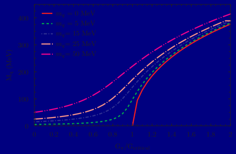
- The NJL model can be used for low momentum p and low energy E

[9] PH *et al.*, PRC**94** (2016)

[10] C.D.Roberts, PPNP **61** (2008)

NJL Gap Equation

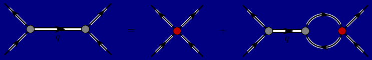




- In the chiral limit ($m_q \sim 0$), the constituent quark mass has non-trivial solution where $M \neq 0$, provided $G_\pi > G_{critical}$. This corresponds to DCSB and the Nambu-Goldstone phase. The Nambu-Goldstone phase occurs when the chiral has been dynamically broken
- when $m_q = 0$, the critical coupling has a form $G_{critical} = \frac{\pi^2}{3(\Lambda_{UV}^2 - \Lambda_{IR}^2}$
- From gap equation figure, we also clearly that when $M \neq m_q$ then $\langle \bar{\psi}\psi \rangle \neq 0$. This indicates that the dynamical mass generation is also associated with the generation of a non-zero chiral condensate.
- The chiral condensate are zero when $G_\pi < G_{critical}$. This is well know as the Wigner-Weyl phase

Bethe Salpeter Equation for the mesons

Mesons in the NJL model are quark-antiquark bound sates whose properties are determined by solving the BSE



- In the NJL model, $\mathscr{T}$-matrix is given by

$$\mathscr{T}(q) = \mathscr{K} + \int \frac{d^4k}{(2\pi)^4} \mathscr{K} S(q+k) \mathscr{T}(q) S(k)$$

- The solution to the BSE in the mesons

$$\mathscr{T}_\alpha(q)_{ab,cd} = [\Gamma\lambda_\alpha]_{ab}\, t_\alpha(q) \left[\bar{\Gamma}\lambda_\alpha^\dagger\right]_{cd} \cdots \Gamma = \{\gamma_5, \gamma^\mu, \gamma_5\gamma^\mu\} \tag{4}$$

- The reduced t-matrix in this channel take a form

$$\begin{aligned} t_\alpha(q) &= \frac{-2iG_\pi}{1+2G_\pi\Pi_\pi(q^2)} \\ t_\beta^{\mu\nu}(q) &= \frac{-2iG_\rho}{1+2G_\rho\Pi_\beta(q^2)} \left(g^{\mu\nu} + 2G_\rho\Pi_\beta(q^2)\frac{q^\mu q^\nu}{q^2}\right) \end{aligned} \tag{5}$$

Bethe Salpeter Equation of the mesons

- The bubble diagrams appearing read

$$
\begin{aligned}
\Pi_\pi(q^2) &= 6i\int \frac{d^4k}{(2\pi)^4} Tr_D\left[\gamma_5 S_l(k)\gamma_5 S_l(k+q)\right], \\
\Pi_K(q^2) &= 6i\int \frac{d^4k}{(2\pi)^4} Tr_D\left[\gamma_5 S_l(k)\gamma_5 S_s(k+q)\right], \\
\Pi_\rho(q^2)\left(g^{\mu\nu}-\frac{q^\mu q^\nu}{q^2}\right) &= 6i\int \frac{d^4k}{(2\pi)^4} Tr_D\left[\gamma^\mu S_a(k)\gamma^\nu S_a(k+q)\right]
\end{aligned} \tag{6}
$$

- The meson masses is given by the pole of the t-matrix

$$
\begin{aligned}
1+2G_\pi\Pi_\pi(k^2=m_\pi^2) &= 0 \\
1+2G_\pi\Pi_K(k^2=m_K^2) &= 0 \\
1+2G_\rho\Pi_\rho(k^2=m_\rho^2) &= 0
\end{aligned} \tag{7}
$$

Meson masses

The meson masses are defined by the pole in the corresponding t-matrix and therefore the meson masses are given by

$$\begin{aligned} m_\pi^2 &= \left[\frac{m}{M_l}\right] \frac{2}{G_\pi \mathscr{I}_{ll}(m_\pi^2)} \\ m_K^2 &= \left[\frac{m_s}{M_s} + \frac{m}{M_l}\right] \frac{1}{G_\pi \mathscr{I}_{ls}(m_K^2)} + (M_s - M_l)^2 \,, \\ m_\rho^2 &= \left[\frac{m}{M_l}\right] \frac{2}{G_\rho \mathscr{I}_{ll}(m_\rho^2)}, \end{aligned} \tag{8}$$

where $\mathcal{I}_{ll}$ and $\mathcal{I}_{ls}$ in the proper time regularization scheme are defined by

$$\mathcal{I}_{ab}(k^2) = \frac{3}{\pi^2} \int_0^1 dx \int \frac{d\tau}{\tau} e^{-\tau(x(x-1)k^2 + xM_b^2 + (1-x)M_a^2)} \tag{9}$$

The meson-quark-quark coupling constants and meson decay constants

The residue at a pole in the $\bar{q}q$ $t-$matrix defines the effective meson-quark-antiquark coupling constants:

$$
\begin{aligned}
Z_\pi = g^2_{\pi qq} &= -\frac{\partial \Pi_\pi(q^2)}{\partial q^2}\Big|_{q^2=m^2_\pi} \\
Z_K = g^2_{Kqq} &= -\frac{\partial \Pi_K(q^2)}{\partial q^2}\Big|_{q^2=m^2_K} \\
Z_\rho = g^2_{\rho qq} &= -\frac{\partial \Pi_\rho(q^2)}{\partial q^2}\Big|_{q^2=m^2_\rho}
\end{aligned}
\tag{10}
$$

Meson decay constant in the proper time regularization is given by

$$
\begin{aligned}
f_\pi &= \frac{N_C\sqrt{Z_\pi}M}{4\pi^2}\int_0^1 dx \int \frac{d\tau}{\tau} e^{-\tau(k^2(x^2-x)+M^2)} \\
f_K &= \frac{N_C\sqrt{Z_K}}{4\pi^2}[(1-x)M_2+xM_1]\int_0^1 dx \int \frac{d\tau}{\tau} e^{-\tau(k^2(x^2-x)+xM_2^2-(x-1)M_1^2)}
\end{aligned}
\tag{11}
$$

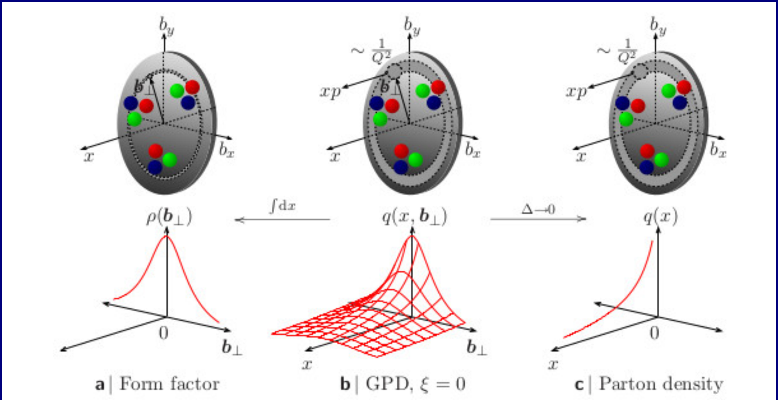
The NJL Parameters Result

The parameters of our NJL model are:

1. The Coupling constants in the NJL Lagrangian G_π and G_ρ
2. The regularization parameters, Λ_{IR} and Λ_{UV}. In QCD the confinement scale is set by Λ_{QCD} and therefore we fix $\Lambda_{IR} = 240$ MeV and choose the dressed light quark mass as $M_l = 400$ MeV
3. The u/d and s dressed quark masses (current quark masses)
4. The remaining parameters are then fit to the physical pion ($m_\pi = 140$ MeV), kaon ($m_K = 495$ MeV and ρ ($m_\rho = 776$ MeV) masses, together with the pion decay constant ($f_\pi = 93$ MeV)
5. This gives $G_\pi = 19.04$ GeV$^-2$, $G_\rho = 11.04$ GeV^{-2}, $\Lambda_{UV} = 645$ MeV, and $M_s = 611$ MeV. Note that for the ϕ mass we obtain $m_\phi = 1020$ MeV. Results for Z_α, f_α and the quark condensates:

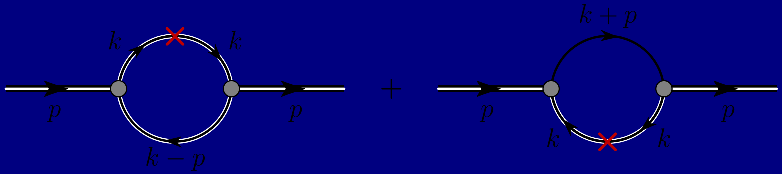
Z_π	Z_K	Z_ρ	Z_ω	Z_ϕ	f_K	$\langle \bar{u}u \rangle^{1/3}$	$\langle \bar{s}s \rangle^{1/3}$
17.85	20.89	6.80	6.63	12.08	0.097	- 0.171	- 0.150

PARTON DISTRIBUTION FUNCTIONS IN THE BSE-NJL MODEL



Parton distribution functions

The valence quark distribution functions of the mesons are given by the two Feynman diagrams



The operator insertion $\gamma^+\delta\left(k^+ - xp^+\right)\hat{P}_q$, where $\hat{P}_q$ is the projection operator for quarks of flavor q:

$$\begin{aligned} \hat{P}_{u/d} &= \frac{1}{2}\left(\frac{2}{3}1 \pm \lambda_3 + \frac{1}{\sqrt{3}}\lambda_8\right) \\ \hat{P}_s &= \frac{1}{3}1 - \frac{1}{\sqrt{3}}\lambda_8 \end{aligned} \qquad (12)$$

Parton distribution functions

The valence quark and anti-quark distributions in the mesons are given by

$$\begin{aligned} q_\alpha(x) &= iZ_\alpha \int \frac{d^4k}{(2\pi)^4}\delta\left(k^+ - xp^+\right) \\ &\times \quad Tr\left[\gamma_5\lambda_\alpha^\dagger S(k)\gamma^+\hat{P}_q S(k)\gamma_5\lambda_\alpha S(k-p)\right] \\ \bar{q}_\alpha(x) &= -iZ_\alpha \int \frac{d^4k}{(2\pi)^4}\delta\left(k^+ + xp^+\right) \\ &\times \quad Tr\left[\gamma_5\lambda_\alpha S(k)\gamma^+\hat{P}_q S(k)\gamma_5\lambda_\alpha^\dagger S(k+p)\right] \end{aligned} \tag{13}$$

To evaluate these expression we first take the moments

$$\mathcal{A}_n = \int_0^1 dx x^{n-1} q(x) \tag{14}$$

where $n = 1, 2, 3, \cdots$ is an integer.

Parton distribution functions

Using the Ward-like identity $S(k)\gamma^+S(k) = \frac{-\partial S(k)}{\partial k_+}$ and introducing the Feynman parameterization, the quark and anti-quark distributions can then be straightforwardly determined. For the valence quark and anti-quark distributions of the K^+ we find:

$$\begin{aligned} q_K(x) &= \frac{3Z_K}{4\pi^2}\int d\tau e^{-\tau\left[x(x-1)m_K^2+xM_s^2+(1-x)M_l^2\right]} \\ &\times \left[\frac{1}{\tau}x(1-x)\left[m_K^2-(m_l-M_s)^2\right]\right] \\ \bar{q}_K(x) &= \frac{3Z_K}{4\pi^2}\int d\tau e^{-\tau\left[x(x-1)m_K^2+xM_l^2+(1-x)M_s^2\right]} \\ &\times \left[\frac{1}{\tau}x(1-x)\left[m_K^2-(m_l-M_s)^2\right]\right] \end{aligned} \tag{15}$$

$\Rightarrow$ Results for the π^+ are obtained by $M_s \to M_l$ and $Z_K \to Z_\pi$, giving the result $u_\pi(x) = \bar{d}_\pi(x)$

Parton distribution functions

The quark distributions satisfy the baryon number and momentum sum rules, which for the K^+ read:

$$\int_0^1 dx\,[u_{K^+}(x) - \bar{u}_{K^+}(x)] \;=\; \int_0^1 [\bar{s}_{K^+}(x) - s_{k^+}(x)] = 1 \qquad (16)$$

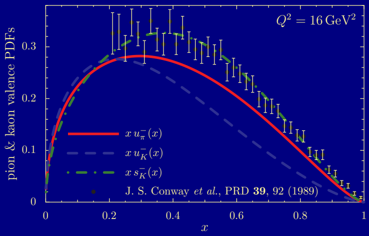
for the number sum rules and at the model scale the momentum sum rules is given by

$$\int_0^1 dxx\,[u_{K^+}(x) + \bar{u}_{K^+}(x) + \bar{s}_{K^+}(x) + s_{k^+}(x)] \;=\; 1 \qquad (17)$$

Analogous results holds for the remaining kaons and the pions.

Meson PDFs results

Results for the valence quark distributions of the meson evolved from model scale to $Q^2 = 16\ GeV^2$ using NLO DGLAP equations[11] and compared empirical data for the pion meson valence PDF.



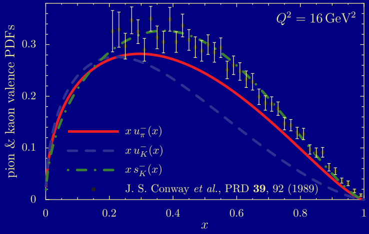
⇒ At model scale, the momentum fraction by the u and s quarks in the K^+, $< xu >= 0.42$ and $< xs >= 0.58$

⇒ The flavor breaking effects of $[< xs > - < xu >]/[< xs > + < xu >] \sim$ 16% which is similar to that seen in masses $[M_s - M_u]/[M_s + M_u] \sim 21\%$.

[11] M. Miyama and S. Kumano, Comput.Pys.Commun. **94**, 185

Meson PDFs results

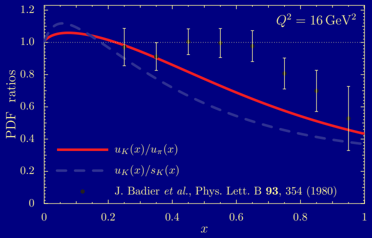
Results for the valence quark distributions of the π^+ and K^+, evolved from the model scale using NLO DGLAP equations.



$\rightarrow$ $SU(3)$ flavor breaking at the model scale $u_K(x)$ peaks at $x_u = 0.237$ and $\bar{s}_K$ peaks at the $x_s = 1 - x_u = 0.763$
$\rightarrow$ This implies flavor breaking effects of around $[x_s - x_u]/[x_s + x_u] \sim 53\%$. For the pion the peak at $x = 0.5$ when $m_u = m_d$.

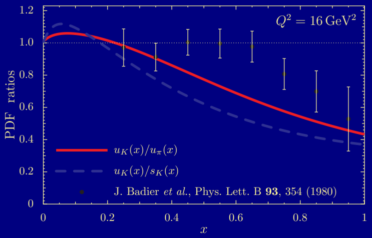
Meson PDFs results

The ratio of the u quark distribution in the K^+ to the u quark distribution in the π^+, after NLO evolution to $Q^2 = 16$ GeV^2



$\rightarrow$ The ratio of $u_K/u_\pi \rightarrow 0.434 \sim M_u/M_s$ as $x \rightarrow 1$, which is in a good agreement with existing data.
$\rightarrow$ The x-dependence differs from much of data in the valence region. This may lie with the absence of the momentum dependence in the NJL Bethe Salpeter vertices, or with data itself.

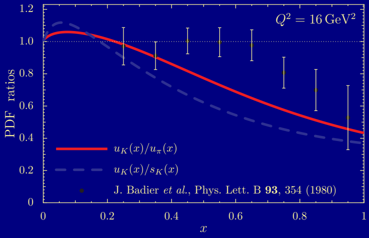
Meson PDFs results



$\rightarrow$ The ratio $u_K(x)/s_K(x)$ approaches 0.37 as $x \rightarrow 1$. It is evident that the flavor breaking effects have a sizable x dependence, being maximal at large x and becoming negligible at small x where the perturbative effects from DGLAP evolution dominate.

$\rightarrow$ The Drell-Yan-West (DYW) relation, $F(Q^2) \sim \frac{1}{Q^{2n}} \leftrightarrow q(x) \sim (1-x)^{2n-1}$. For the pion, $F_\pi \sim 1/Q^2$ and the DYW relation implies $q_\pi(x) \sim (1-x)$. Kaon PDF do behave as the pion.

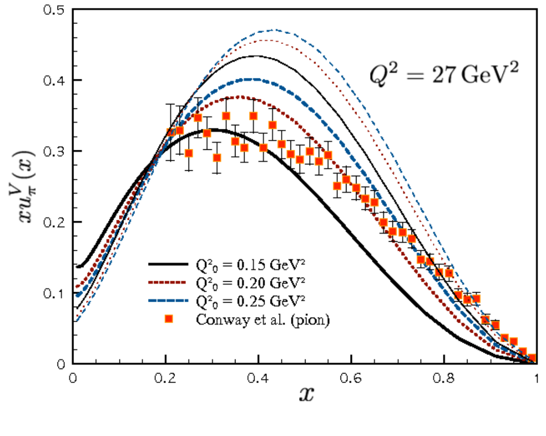
Meson PDFs results



→ As reflection of the expectations of may be expected by DYW like relations, $u_K/s_K < 1$ as $x \to 1$ and $\mid F_K^u/F_K^s \mid < 1$ for $Q^2 >> \Lambda_{QCD}$.

Meson PDFs results from NLChQM

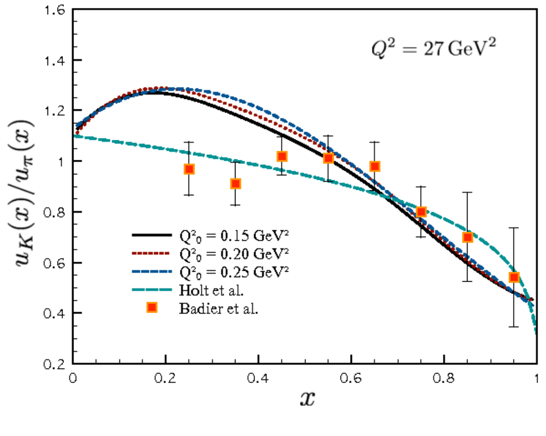
PDF of the pion and kaon in the nonlocal chiral quark model [12]



[12] SiNam, Phys. Rev. D **86**, 074005 (2012)

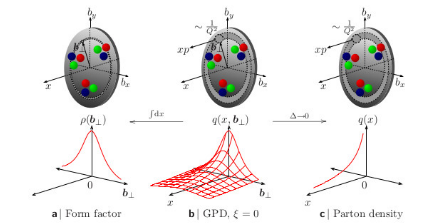
Meson PDFs Results from NLChQM

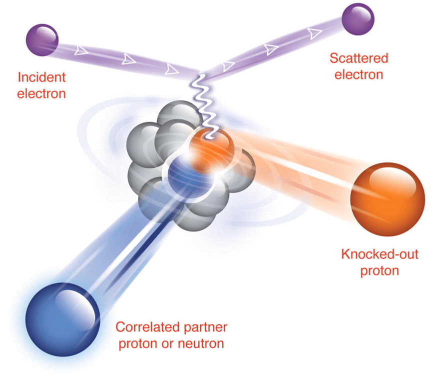
A ratio of the PDF of the pion and kaon in the nonlocal chiral quark model [13]



[13] SiNam, Phys. Rev. D **86**, 074005 (2012)

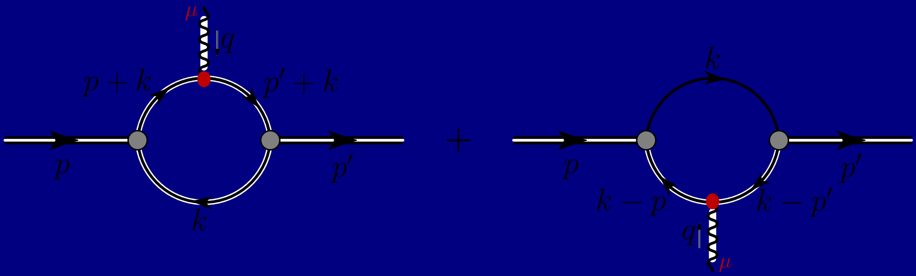
FORM FACTOR IN THE BSE-NJL MODEL





Form Factor in NJL model

Diagrammatic representation of the electromagnetic current for the pion and kaon [14]



$\Rightarrow$ Feynman diagram for quark [left] and for the anti quark [right]

[14] PH, Ian Cloet, and A. Thomas, PRC94 (2016), 035201

Form Factor in NJL model

The matrix element of the electromagnetic current for a pseudoscalar mesons reads

$$J^{\mu}_{\alpha}(p', p) = \left(p'^{\mu} + p^{\mu}\right) F_{\alpha}(Q^2), \quad \alpha = \pi, K \tag{18}$$

where p and $p^{'}$ denote the initial and final four momentum of the state, $q^2 = (p^{'} - P)^2 = -Q^2$ and $F_{\alpha}(Q^2)$ is the pion or kaon form factor. The pseudoscalar meson form factor in the NJL model are given by the sum of the two Feynman diagrams, which are respectively given by

$$\begin{aligned} j^{\mu}_{1,\alpha}(p^{'}, p) &= iZ_{\alpha} \int \frac{d^4k}{(2\pi)^4} Tr\left[\gamma_5 \lambda^{\dagger}_{\alpha} S(p^{'} + k) \hat{Q}\gamma^{\mu} S(p + k) \gamma_5 \lambda_{\alpha} S(k)\right] \\ j^{\mu}_{2,\alpha}(p^{'}, p) &= iZ_{\alpha} \int \frac{d^4k}{(2\pi)^4} Tr\left[\gamma_5 \lambda_{\alpha} S(k - p) \hat{Q}\gamma^{\mu} S(k - p^{'}) \gamma_5 \lambda^{\dagger}_{\alpha} S(k)\right] \end{aligned} \tag{19}$$

where the Tr is over Dirac, color and flavor indices. The index α labels the state and the λ_{α} are the corresponding flavor matrices

Form Factor in NJL model

We will focus on the quark sector and total form factors for π^+, K^+ and K^0, we find

$$\begin{aligned} F^{(bare)}_{\pi^+}(Q^2) &= (e_u - e_d) f^{ll}_{\pi}(Q^2) \\ F^{(bare)}_{K^+}(Q^2) &= e_u f^{ls}_{K}(Q^2) - e_s f^{sl}_{K}(Q^2) \\ F^{(bare)}_{K^0}(Q^2) &= e_d f^{ls}_{K}(Q^2) - e_s f^{sl}_{K}(Q^2) \end{aligned} \tag{20}$$

The results are denoted as "*bare*" because the quark-photon vertex is elementary result, that is, $\Lambda^{\mu(bare)}_{\gamma q} = \hat{Q}\gamma^{\mu}$. The quark-sector form factors for a hadron α are defined by

$$F_{\alpha}(Q^2) = e_u F^u_{\alpha}(Q^2) + edF^d_{\alpha}(Q^2) + e_s F^s_{\alpha}(Q^2) + \cdots \tag{21}$$

therefore the "*bare*" pseudoscalar meson quark-sector form factors are easily read from the total form factor equation above

Form Factor in NJL model

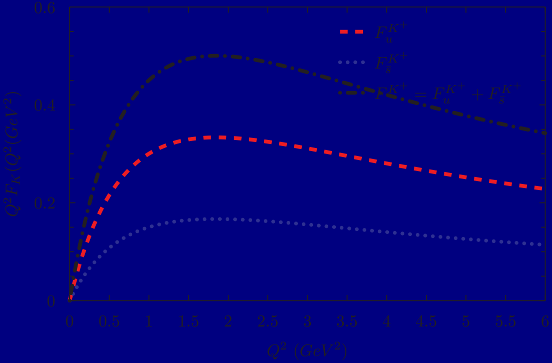
The first superscript on the body form factors, $f_\alpha^{ab}(Q^2)$, indicates the struck quark and the second the spectator, where

$$
\begin{aligned}
f_\alpha^{ab}(Q^2) &= \frac{3Z_\alpha}{4\pi^2}\int_0^1 dx \int \frac{d\tau}{\tau} e^{-\tau(M_a^2+x(1-x)Q^2)} \\
&+ \frac{3Z_\alpha}{4\pi^2}\int_0^1 dx \int_0^{1-x} dz \int d\tau \\
&\times \; e^{-\tau((x+z)(x+z-1)m_\alpha^2+(x+z)M_a^2+(1-x-z)M_b^2+xzQ^2)} \\
&\times \left[(x+z)m_\alpha^2 + (M_a - M_b)^2(X+Z) + 2M_b(M_a - M_b)\right]
\end{aligned}
\tag{22}
$$

$\Rightarrow$ Importantly, these expression satisfy charge conservation.

Elastic Form Factor Results

Results for the kaon form factor – from the bare quark-photon vertex



$\Rightarrow$ We find that our results are not excellent agreement with the perturbative QCD that predicts $F_K(Q^2) \sim \frac{\alpha_S(Q^2)}{Q^2}$.

Form Factor in NJL model : Modify the Quark-Photon vertex

⇒ The limit $Q^2 >> m_\alpha^2$ of the body form factors can be obtained by noting the Feynman parameter domains which dominate the integrals giving

$$\begin{aligned} Q^2 f_\alpha^{ab}(Q^2) &= \frac{3Z_\alpha}{2\pi^2}\int\frac{d\tau}{\tau^2}e^{-\tau M_a^2} + \frac{3Z_\alpha}{2\pi^2}M_b(M_a - M_b)\int\frac{d\tau}{\tau}e^{-\tau M_b^2} \\ &\times \left[\gamma_E + \log(M_b^2\tau) + \log(Q^2/M_b^2)\right] \end{aligned} \tag{23}$$

Therefore the form factors receive log correction at large Q^2 only if $M_a \neq M_b$
⇒ In general the quark-photon vertex is not elementary ($\Lambda_{\gamma q}^{\mu(bare)} = \hat{Q}\gamma^\mu$) but instead **dressed**, with this dressing given by the inhomogeneous BSE. The general solution for the dressed quark-photon vertex for a quark of flavor q has the form

$$\Lambda_{\gamma Q}^{\mu}(p', p) = e_q\gamma^\mu + \left(\gamma^\mu - \frac{q^\mu \not{q}}{q^2}\right)F_Q(Q^2) \to \gamma^\mu F_{1Q}(Q^2) \tag{24}$$

Form Factor in NJL model

where the final result is used because the $\frac{q^\mu \not{q}}{q^2}$ term cannot contribute to a hadron electromagnetic current because of current conservation
The dressed u, d and s quarks are expressed by

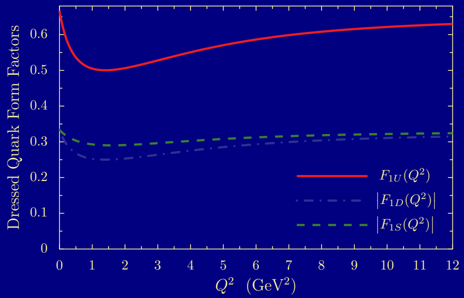
$$\begin{aligned} F_{1U/D}(Q^2) &= e_{u/d}\frac{1}{1+2G_\rho \Pi_v^{ll}(Q^2)} \\ F_{1S}(Q^2) &= e_s\frac{1}{1+2G_\rho \Pi_v^{ss}(Q^2)} \end{aligned} \tag{25}$$

where the explicit form of the bubble diagram is

$$\Pi_v^{qq}(Q^2) = \frac{3Q^2}{\pi^2}\int_0^1 dx \int \frac{d\tau}{\tau} x(1-x)e^{-\tau\left[M_q^2+x(1-x)Q^2\right]} \tag{26}$$

Form Factor in NJL model

The dressed quark form factors obtained as solutions to the inhomogeneous BSE:



$\Rightarrow$ In the limit $Q^2 \to \infty$ these form factors reduce to the elementary quark charges, as expected because of asymptotic freedom in QCD. For small Q^2 these results are similar to expectations from vector meson dominance, where the dressed u and d quarks are dressed by ρ and ω mesons and the dressed s quark by ϕ meson.

Form Factor in NJL model

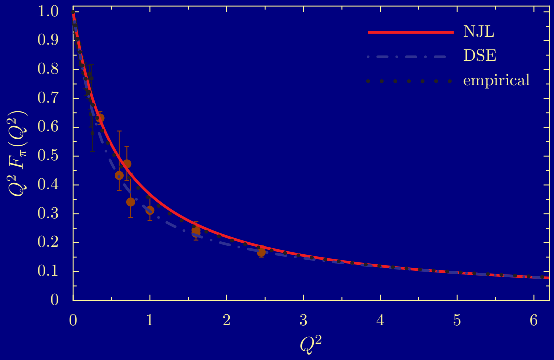
The complete results for the pseudoscalar meson form factor – with a dressed quark-photon vertex – read

$$\begin{aligned} F_{\pi^+}(Q^2) &= \Big[F_{1U}(Q^2) - F_{1D}(Q^2)\Big] f_\pi^{ll}(Q^2) \\ F_{K^+}(Q^2) &= F_{1U}(Q^2) f_K^{ls}(Q^2) - F_{1S}(Q^2) f_K^{sl}(Q^2) \\ F_{K^0}(Q^2) &= F_{1D}(Q^2) f_K^{ls}(Q^2) - F_{1S}(Q^2) f_K^{sl}(Q^2) \end{aligned} \tag{27}$$

$\Rightarrow$ $f_\pi^{ll}(Q^2)$, $f_K^{sl}(Q^2)$ and $f_K^{sl}(Q^2)$ are the same as the *BARE* form factor expressions

Elastic Form Factor Results

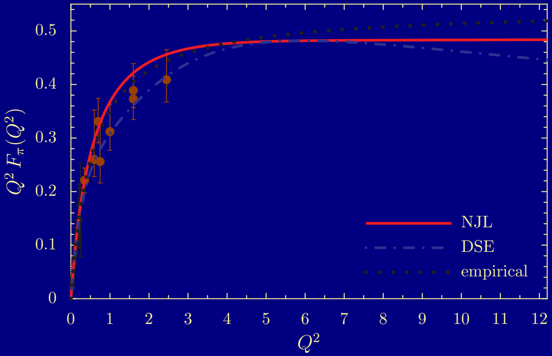
Results for the pion form factor – the dressed quark-photon vertex



$\Rightarrow$ We find that excellent agreement with existing data and the modest difference with the DSE result for $Q^2 \leq 6$ GeV^2

Elastic Form Factor Results

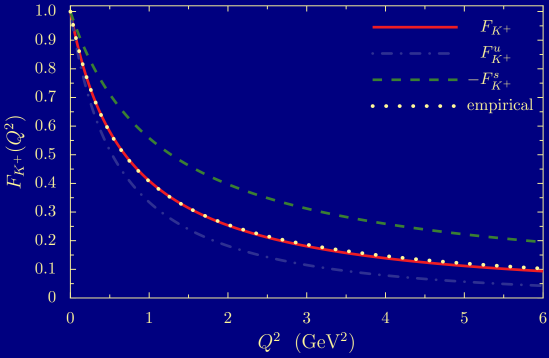
Results for $Q^2 F_\pi(Q^2)$



$\Rightarrow$ Our result for $Q^2 F_\pi(Q^2)$ is very similar to the empirical monopole result but begins to plateau for $Q^2 \geq 6$ GeV^2, where $Q^2 F_\pi(Q^2) \sim 0.49$. This maximum is almost identical to that obtaining using the DSEs, which is not surprising because in both approaches it is driven by DCSB.

Elastic Form Factor Results

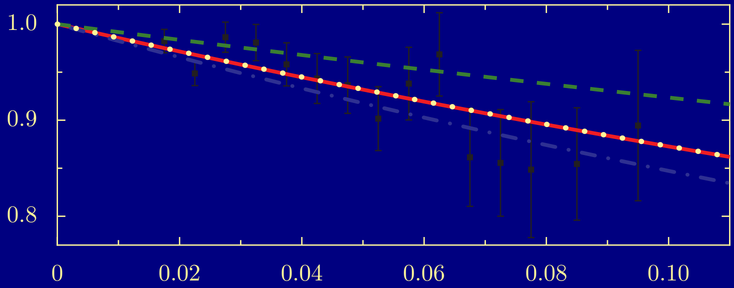
Results for the K^+ form factor and the quark sector components – each including effects from the dressed quark-photon vertex



$\Rightarrow$ we find an excellent agreement with the data and the empirical monopole $F_K(Q^2) = \left[1 + \frac{Q^2}{\Lambda_K^2}\right]^{-1}$ determined by the charge radius.

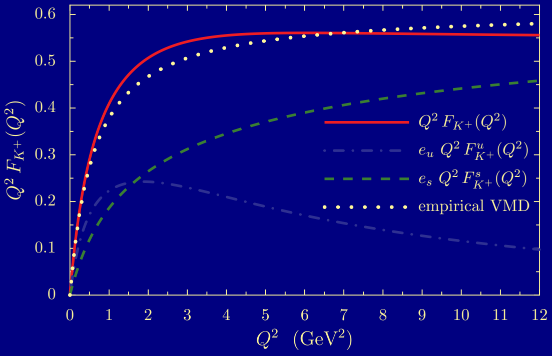
Elastic Kaon Form Factor Results

Results for the K^+ form factor – each including effects from the dressed quark-photon vertex – compare with the available experimental data



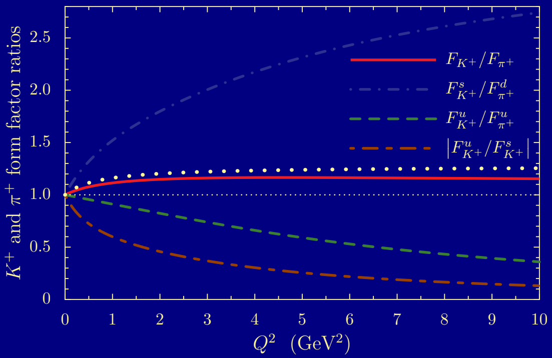
Elastic Form Factor Results

Results for the $Q^2 F_{K^+}(Q^2)$ form factor and the quark sector components – each including effects from the dressed quark-photon vertex



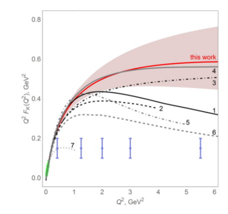
Elastic Form Factor Results

Results for the form factor and the quark sector components ratio



Elastic Form Factor Results

Our model prediction compares with other model predictions/calculations [15]

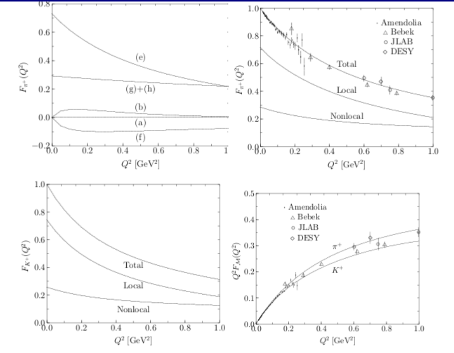


$\Rightarrow$ Model predictions for $F_K(Q^2)$. The red line is the best fit and pink shaded area is the 68% CL range and 1 (full black):The 3 forms of relativistic kinematics model; 2(dashed black):DSE 2000 model; 3 (dot-dashed black): NJL 2015 model, 4(full gray):BSE-NJL model, 5 (dot-dashed gray): DSE 1996 model; 6 (dashed gray):CGNPSS model, 7(dotted gray):LF model; 8(dotted black):RQM model and Points with error bars are projected uncertainties of the E12-09-011 JLAB experiment[27]

[15] A.F.Krutov, EPJC **77** (2017),464

ELASTIC FORM FACTOR RESULTS FROM NLCHQM

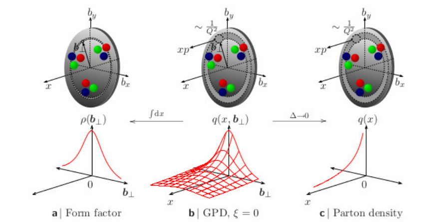
A form factor result in the nonlocal chiral quark model (NLChQM) [16]

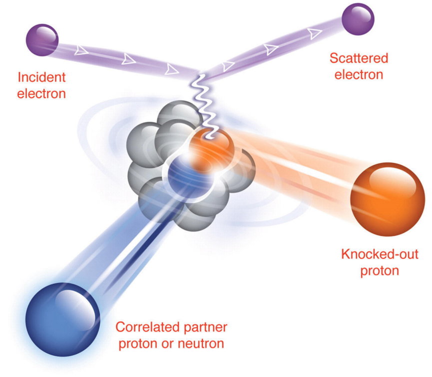


$\Rightarrow$ A nonlocal contributions turn out to be crucial to reproduce the experimental data and $\langle r^2\rangle_{\pi^+} = 0.455$ fm^2 and $\langle r^2\rangle_{K^+} = 0.537$ fm^2.

[16] Seung-il Nam and Hyun-Chul Kim, PRD77 (2008)

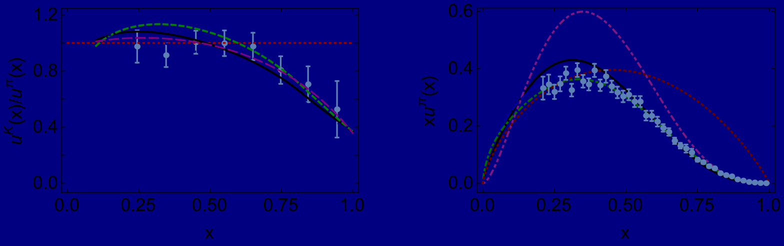
LINKING TO ELECTRON-ION COLLIDER (EIC)





Parton distribution function of the mesons linking to EIC

- Using DSE model, they include the gluon and sea quark distribution in the pion and kaon [17], but there is no experimental data for the gluon and sea quark distributions. Therefore, we need a data from EIC or CERN-SPS COMPASS to fully understand the pion and kaon structures



- How large the valence, gluon and sea quark carry the pion momentum? or in the other word, how large the non-local contribution affect the structure of the pion and kaon?

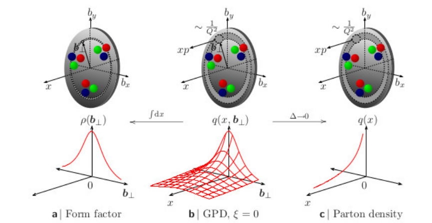
[17] Chen Chen, PRD **93** (2016) 7, 074021

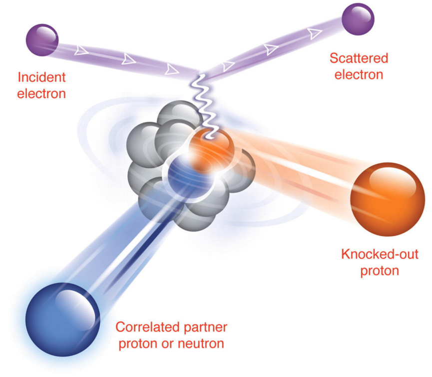
Parton distribution function of the mesons linking to EIC

- For the valence quark distribution in the pion and kaon, we still need more data to explain the discrepancy (tension) among theoretical models, where DSE model predict $(1-x)^2$ at x $\rightarrow$ 1, whereas the newest result from global data analysis for the quark distribution of the pion at x $\rightarrow$ 1 behaves $(1-x)^1$ which is similar as our BSE-NJL model. They also include the gluon which contributes by 30% and sea quark contributes by 15% [18]
- From Drell-Yan-West (DRW) relation, the relation between form factor at $Q^2 \rightarrow \infty$, $F_\pi = \frac{1}{Q^{2n}}$ and pdf at $x \rightarrow q$ has $(1-x)^{(2n-1)}$, where n is the number of quark inside the pion. We also need the EIC data to answer this discrepancy.

[18] P.C.Barry, PRL **121** (2018) 15, 152001

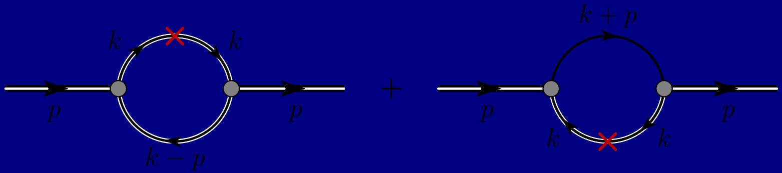
VECTOR MESON IN THE BSE-NJL MODEL





Parton Distribution of the vector meson in the BSE-NJL model

The valence quark distribution functions of the ρ-meson in the NJL model are given by the two Feynman diagrams



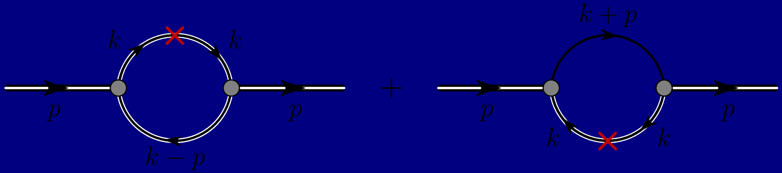
- ρ-meson Bethe-Salpeter vertex function is given by

$$\bar{\Gamma}^{(\lambda),i}_{\gamma\delta}\Gamma^{(\lambda),i}_{\alpha\beta} = \left[iZ_\rho\tau_i\gamma^\mu\epsilon^*_{\lambda\mu}\right]_{\gamma\delta}\left[iZ_\rho\tau_i\gamma^\nu\epsilon_{\lambda\nu}\right]_{\alpha\beta} \tag{28}$$

- Operator insertion has a form $\Gamma\delta\left(x - \frac{k^+}{p^+}\right)$, where $\Gamma \equiv \{\gamma^+, \gamma^+\gamma^1\gamma_5, \gamma^+\gamma^2\gamma_5, \gamma^+\gamma_5\}$

Parton Distribution of the vector meson in the BSE-NJL model

The valence quark distribution functions of the ρ-meson in the NJL model are given by the two Feynman diagrams, where in the NJL model, the valence antiquark distribution are the same as the valence quark distribution so we just calculate for the valence quark one



$$\langle\Gamma\rangle^{\mu\nu}(x,\mathbf{k}_T) = -\frac{3iZ\rho^2}{p^+}\int\frac{dk^+dk^-}{(2\pi)^4}\delta\left(x-\frac{k^+}{p^+}\right) \times Tr_D\left[\gamma^\mu S(k)\Gamma\gamma^\nu S(k-p)\right] \quad (29)$$

Parton Distribution of the vector meson in the BSE-NJL model

The methods of covariant integration for expectation values of local operators can be used within the proper-time regularization scheme by first applying a Mellin transformation, which gives the n-th moment of $\langle\Gamma\rangle^{\mu\nu}$ as

$$\langle\Gamma\rangle_n^{\mu\nu}(\mathbf{k}_T) \equiv \int_0^1 dx\, x^{n-1}\, \langle\Gamma\rangle^{\mu\nu}(x, \mathbf{k}_T), \tag{30}$$

- The coefficient functions that contribute at leading-twist are

$$\begin{aligned}
\langle\gamma^+\rangle_S^{(\lambda)}(x, \mathbf{k}_T) &= \frac{1}{2} Tr_D\left[\gamma^+\Phi^{(\lambda)S}(x, \mathbf{k}_T)\right], \\
\langle\gamma^+\gamma_5\rangle_S^{(\lambda)}(x, \mathbf{k}_T) &= \frac{1}{2} Tr_D\left[\gamma^+\gamma_5\Phi^{(\lambda)S}(x, \mathbf{k}_T)\right], \\
\langle\gamma^+\gamma^i\gamma_5\rangle_S^{(\lambda)}(x, \mathbf{k}_T) &= \frac{1}{2} Tr_D\left[-i\sigma^{+i}\Phi^{(\lambda)S}(x, \mathbf{k}_T)\right]
\end{aligned} \tag{31}$$

Parton Distribution of the vector meson in the BSE-NJL model

The coefficient functions that contribute at leading-twist are

$$\begin{aligned}
\langle\gamma^+\rangle_S^{(\lambda)}(x,\mathbf{k}_T) &= \epsilon^*_{(\lambda)\mu}(p)\,\langle\gamma^+\rangle^{\mu\nu}(x,\mathbf{k}_T)\epsilon_{(\lambda)\nu}(p),\\
\langle\gamma^+\gamma_5\rangle_S^{(\lambda)}(x,\mathbf{k}_T) &= \epsilon^*_{(\lambda)\mu}(p)\,\langle\gamma^+\gamma_5\rangle^{\mu\nu}(x,\mathbf{k}_T)\epsilon_{(\lambda)\nu}(p),\\
\langle\gamma^+\gamma^i\gamma_5\rangle_S^{(\lambda)}(x,\mathbf{k}_T) &= \epsilon^*_{(\lambda)\mu}(p)\,\langle\gamma^+\gamma^i\gamma_5\rangle^{\mu\nu}(x,\mathbf{k}_T)\epsilon_{(\lambda)\nu}(p) \qquad (32)
\end{aligned}$$

where,

$$\begin{aligned}
\epsilon^{*\mu}_{(\lambda)}(p)\epsilon^{\nu}_{(\lambda)}(p) &= \frac{1}{3}\left(-g^{\mu\nu}+\frac{p^\mu p^\nu}{m_h^2}\right)-\frac{i\lambda}{2m_h}\epsilon^{\mu\nu\alpha\beta}p_\alpha S_\beta(p)\\
&- \frac{3\lambda^2-2}{2}\left[S^\mu(p)S^\nu(p)-\frac{1}{3}\left(-g^{\mu\nu}+\frac{p^\mu p^\nu}{m_h^2}\right)\right] \qquad (33)
\end{aligned}$$

with $S^\mu(p)=\left(\frac{p^3}{m_h}S_L, S_T, \frac{E_p}{m_h}S_L\right)$ and spin projection $\lambda=\pm1,0$ for $S_T=0$ and $|S_L|=1$ as well as for $S_L=0$ and $|S_T|=1$, there are $\lambda=\pm1,0$

Parton Distribution of the vector meson in the BSE-NJL model

The coefficient functions that contribute at leading-twist are

$$\begin{aligned}
\langle\gamma^+\rangle_S^{(\lambda)}(x,\mathbf{k}_T) &\equiv f(x,\mathbf{k}_T^2) + S_{LL} f_{LL}(x,\mathbf{k}_T^2) \\
&+ \frac{S_{LT}.\mathbf{k}_T}{m_h} f_{LT}(x,\mathbf{k}_T^2) + \frac{\mathbf{k}_T.S_{TT}.\mathbf{k}_T}{m_h^2} f_{TT}(x,\mathbf{k}_T^2), \\
\langle\gamma^+\gamma_5\rangle_S^{(\lambda)}(x,\mathbf{k}_T) &= \lambda\left[S_L g_L(x,\mathbf{k}_T^2) + \frac{\mathbf{k}_T.S_T}{m_h} g_T(x,\mathbf{k}_T^2)\right], \\
\langle\gamma^+\gamma^i\gamma_5\rangle_S^{(\lambda)}(x,\mathbf{k}_T) &= \lambda\left[S_T^i h(x,\mathbf{k}_T^2) + S_L \frac{\mathbf{k}_T^i}{m_h} h_L^\perp(x,\mathbf{k}_T^2)\right. \\
&\left. + \frac{1}{2m_h^2}\left(2k_T^i \mathbf{k}_T.S_T - S_T^i \mathbf{k}_T^2\right) h_T^\perp(x,\mathbf{k}_T^2)\right] \qquad (34)
\end{aligned}$$

where $S_{LL} = (3\lambda^2 - 2)(\frac{1}{6} - \frac{1}{2}S_L^2)$, $S_{LT}^i = (3\lambda^2 - 2)S_L S_T^i$ and $S_{TT}^{ij} = (3\lambda^2 - 2)(S_T^i S_T^j - \frac{1}{2}S_T^2\delta^{ij})$

Parton Distribution of the vector meson in the BSE-NJL model

By integrating $\langle\gamma^+\rangle_S^{(\lambda)}(x, \mathbf{k}_T)$, $\langle\gamma^+\gamma_5\rangle_S^{(\lambda)}(x, \mathbf{k}_T)$, and $\langle\gamma^+\gamma^i\gamma_5\rangle_S^{(\lambda)}(x, \mathbf{k}_T)$ over $\mathbf{k}_T$. it gives the four PDFs for spin-1 target

$$\begin{aligned}
\langle\gamma^+\rangle_S^{(\lambda)}(x) &\equiv f(x) + S_{LL} f_{LL}(x), \\
\langle\gamma^+\gamma_5\rangle_S^{(\lambda)}(x) &\equiv \lambda S_L g(x), \\
\langle\gamma^+\gamma^i\gamma_5\rangle_S^{(\lambda)}(x) &\equiv \lambda S_T^i h(x)
\end{aligned} \tag{35}$$

The relations between the TMDs and PDFs for spin-1 target is given by

$$\begin{aligned}
f(x) &= \int d^2\mathbf{k}_T f(x, \mathbf{k}_T^2), \\
g(x) &= \int d^2\mathbf{k}_T g(x, \mathbf{k}_T^2), \\
h(x) &= \int d^2\mathbf{k}_T h(x, \mathbf{k}_T^2), \\
f_{LL}(x) &= \int d^2\mathbf{k}_T f_{LL}(x, \mathbf{k}_T^2)
\end{aligned} \tag{36}$$

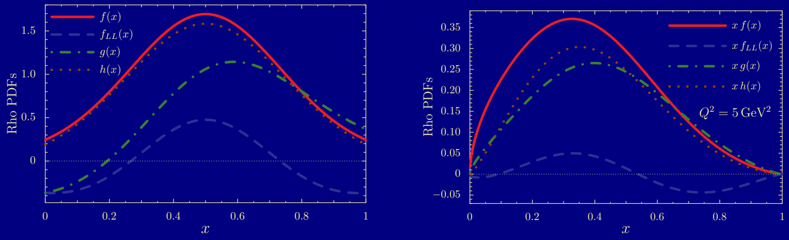
Parton Distribution of the vector meson in the BSE-NJL model

The $f(x)$, $g(x)$, $h(x)$, and f_{LL} in the proper-time regularization scheme are written as

$$
\begin{aligned}
f(x) &= \frac{Z_\rho^2}{2\pi^2}\int\frac{d\tau}{\tau}\left[1+x(1-x)[1+2x(1-x)]m_\rho^2\tau\right] \\
&\times\; e^{-\tau(M_u^2-x(1-x)m_\rho^2)}, \\
g(x) &= \frac{3Z_\rho^2}{4\pi^2}\int d\tau\left[\frac{2x-1}{\tau}+x(1-x)m_\rho^2\right] \\
&\times\; e^{-\tau(M_u^2-x(1-x)m_\rho^2)}, \\
h(x) &= \frac{3Z_\rho^2 M_u}{4\pi^2 m_\rho}\int d\tau\left[\frac{1}{\tau}+2x(1-x)m_\rho^2\right] \\
&\times\; e^{-\tau(M_u^2-x(1-x)m_\rho^2)}, \\
f_{LL}(x) &= -\frac{3Z_\rho^2}{4\pi^2}\int\frac{d\tau}{\tau}\left[1-6x(1-x)+x(1-x)(1-2x)^2m_\rho^2\tau\right] \\
&\times\; e^{-\tau(M_u^2-x(1-x)m_\rho^2)}
\end{aligned}
\tag{37}
$$

Results for the vector meson PDFs in the BSE-NJL model

Results for the unpolarized ($f(x)$), helicity ($h(x)$), tranversity ($g(x)$) and $f_{LL}(x)$ tensor polarized PDFs of the ρ-meson. The left panel is result for the model scale $Q_0^2 = 0.16\ GeV^2$. The right panel is respectively the PDFs ($xf(x)$, $xh(x)$, $xg(x)$ and $xf_{LL}(x)$) of the ρ-meson after evolving at the scale $Q^2 = 5\ GeV^2$ using the non-singlet DGLAP evolution [19] and Ref. [20]

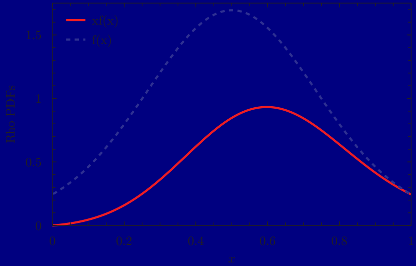


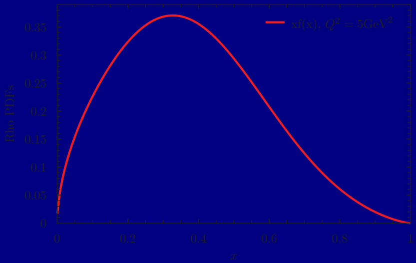
[19] W. Bentz, PRC **96** (2017), 045206

[20] **PH**. Nam and W. Bentz et. al, in preparation (2020)

Results for the vector meson PDFs in the BSE-NJL model

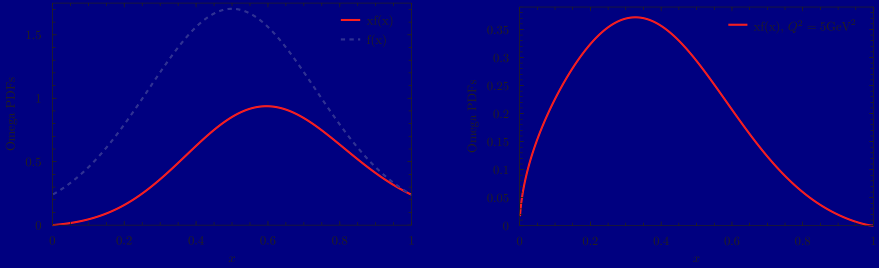
Results for the unpolarized ($f(x)$) of the vector mesons. The left panel is result for the model scale $Q_0^2 = 0.16\ GeV^2$. The right panel is respectively the PDFs of the vector mesons after evolving at the scale $Q^2 = 5\ GeV^2$ using the non-singlet DGLAP evolution.





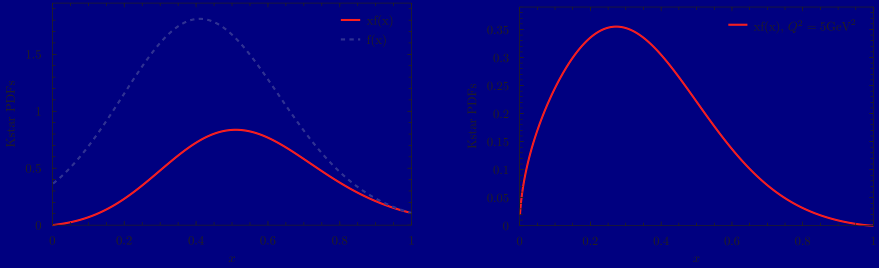
Results for the vector meson PDFs in the BSE-NJL model

Results for the unpolarized ($f(x)$) of the vector mesons. The left panel is result for the model scale $Q_0^2 = 0.16\ GeV^2$. The right panel is respectively the PDFs of the vector mesons after evolving at the scale $Q^2 = 5\ GeV^2$ using the non-singlet DGLAP evolution.



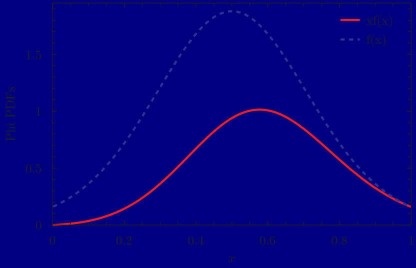
Results for the vector meson PDFs in the BSE-NJL model

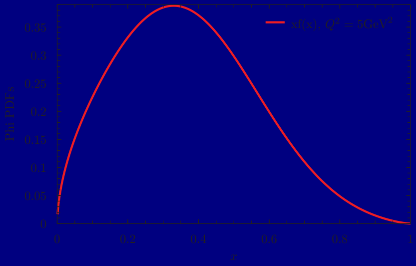
Results for the unpolarized ($f(x)$) of the vector mesons. The left panel is result for the model scale $Q_0^2 = 0.16\ GeV^2$. The right panel is respectively the PDFs of the vector mesons after evolving at the scale $Q^2 = 5\ GeV^2$ using the non-singlet DGLAP evolution.



Results for the vector meson PDFs in the BSE-NJL model

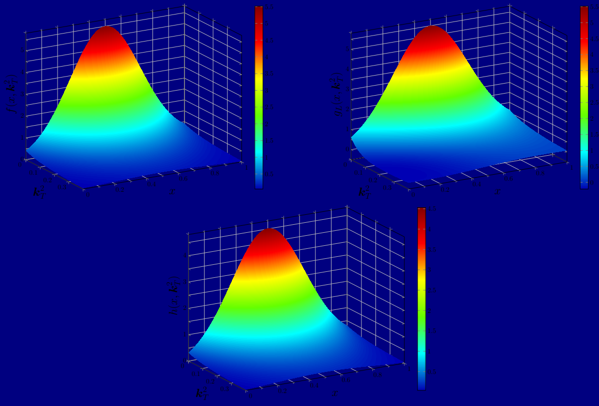
Results for the unpolarized ($f(x)$) of the vector mesons. The left panel is result for the model scale $Q_0^2 = 0.16\ GeV^2$. The right panel is respectively the PDFs of the vector mesons after evolving at the scale $Q^2 = 5\ GeV^2$ using the non-singlet DGLAP evolution.





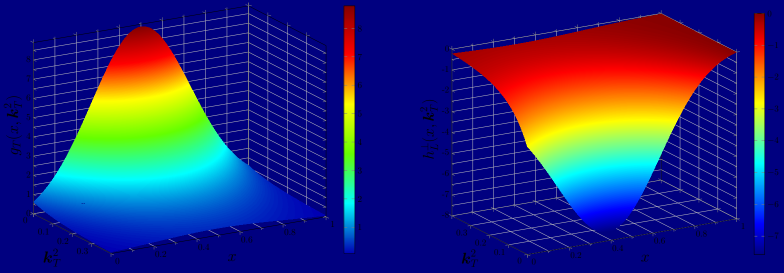
Results for TMDs for the ρ-meson in the BSE-NJL model

Results for the TMDs ($f(x, \mathbf{k}_T^2)$), helicity ($h(x, \mathbf{k}_T^2)$), and tranversity ($g(x, \mathbf{k}_T^2)$) of the ρ-meson.



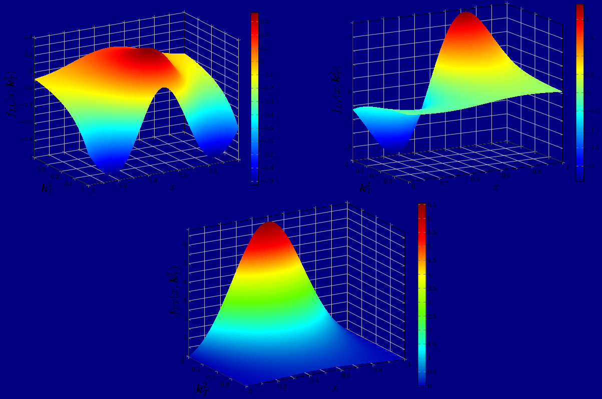
Results for TMDs for the ρ-meson in the BSE-NJL model

Results for the TMDs ($h_L^\perp(x, \mathbf{k}_T^2)$), and ($g_T(x, \mathbf{k}_T^2)$) of the ρ-meson.



Results for TMDs for the ρ-meson in the BSE-NJL model

Results for the TMDs ($f_{LL}(x, \mathbf{k}_T^2)$),($f_{LT}(x, \mathbf{k}_T^2)$), and ($f_{TT}(x, \mathbf{k}_T^2)$) of the ρ-meson.



Summary and Outlook

- We have presented the form factor and parton distribution of the kaon and pion in the confining NJL model. Our results on FF and PDF of the kaon and pion are in excellent agreement with the available experimental data (need the data from EIC to see our prediction to valence quark pdfs)
- We showed our preliminary results on unpolarized valence quark distribution for the light vector mesons. Unfortunately no data to compare at the moment. Hoping that the EIC also may measure the light vector meson unpolarized pdf in the future
- We have to consider the nonlocal terms (the momentum dependence of the effective quark mass) in our PDF and FF calculations, namely the NLChQM model or other satisfactory QCD model
- On-going work, we are now calculating the GPD for the pion and kaon in the NLChQM model, where we are eager to consider the momentum dependent or nonlocal contribution. The result for the pion and kaon GPDs are STILL IN PROGRESS...
- In the future we are also going to calculate the quasi-PDF for the vector mesons and fragmentation functions of vector mesons.

THANK YOU VERY MUCH FOR ATTENTION

Any questions or comments ??