

*Exploring the Universe with **Dark Light** Scalars*

Based on 2010.10880

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Outline

Motivation

- light scalars
- form of the axion scalar potential (multi branch structure)
 - gluo-thermodynamics (energy & entropy)

Parts of the results

- multi-component dark matter
(strongly interacting glueball DM & feebly interacting axion DM)

Implications for the supermassive black hole formation

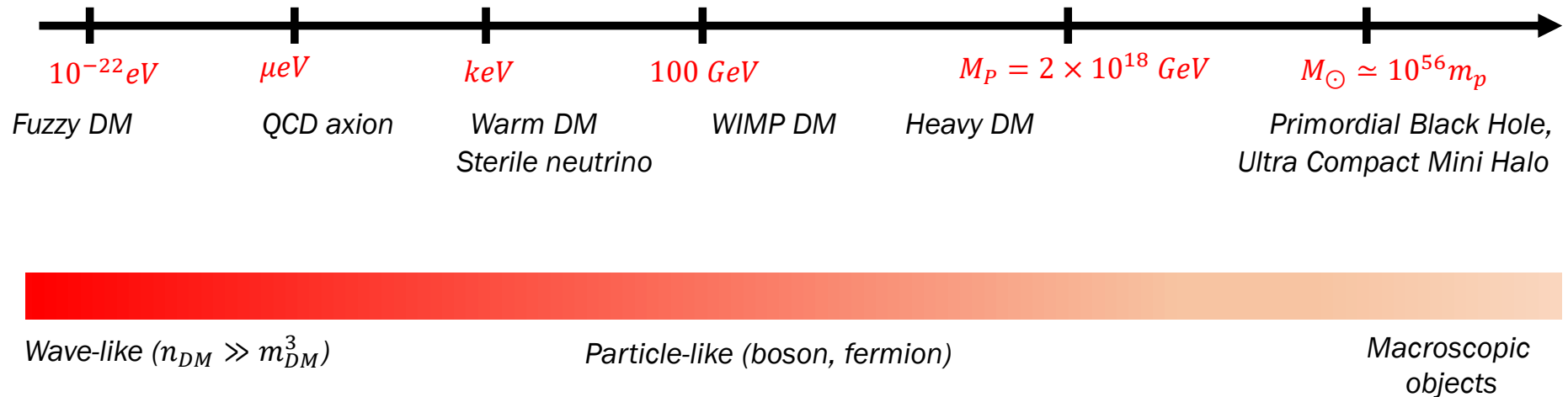
- intro (SMBHs at $z \sim 7$)
- dark matter interpretation (gravo-thermal collapse)
 - our model and more realistic history

Motivation

The nature of dark matter is still shrouded in mystery

$$\bar{\rho}_{DM} \simeq 1.2 \times 10^{-6} \text{ GeV/cm}^3 = m_{DM} \bar{n}_{DM}$$

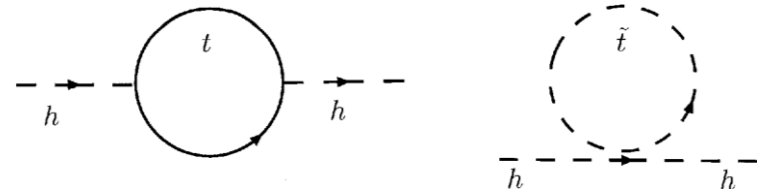
$$\begin{aligned} m_p &= 1 \text{ GeV} \\ m_e &= 0.5 \text{ MeV} \\ m_\nu &= 0.1 \text{ eV} \end{aligned}$$



Unfortunately, there is no good guiding principle for the mass of dark matter these days

Origin of the mass → Hints for new physics

Ex) Higgs boson (the weak scale is radiatively unstable)



$$\delta_{qc} m_H^2 \sim \Lambda_{UV}^2 \gg m_W^2$$

Supersymmetry, Composite Higgs, Relaxion, Scale invariance etc.

Motivation

Considering *scalar* dark matter: Need to explain *the origin of its mass*

→ It also determines its interaction structure, too

Natural scalar dark matter candidates

1) **Axion-like particle** (ϕ): for the compact field ($\phi \rightarrow \phi + 2\pi f_a$),

Approximate global symmetry: $\phi \rightarrow \phi + c$ is broken non-perturbatively

(by instanton, confinement, flux, etc.)

e.g.
$$\frac{\phi}{16\pi^2 f_a} \text{Tr}[G_{\mu\nu}^a \tilde{G}^{a\mu\nu}] \rightarrow V(\phi) = \sum a_n \Lambda^4 \cos\left(\frac{n\phi}{f_a}\right)$$

2) **Glueball-like particle** (φ_g): At high scales, there is no scalar degree of freedom

Confining gauge symmetry: $\text{Tr}[G_{\mu\nu}^a G^{a\mu\nu}] \rightarrow \varphi_g$: $m_g \simeq (5 - 6)\Lambda$ (confinement scale)

Both 1) & 2) can make scalars light. But the interaction strengths among them are quite different.
What if we consider both mechanisms simultaneously as a dark sector?

$$L_{\text{dark}} = -\frac{1}{4g^2} G_{\mu\nu}^a G^{a\mu\nu} + \frac{1}{2} (\partial_\mu \phi)^2 + \frac{\phi}{32\pi^2 f_a} G_{\mu\nu}^a \tilde{G}^{a\mu\nu}$$

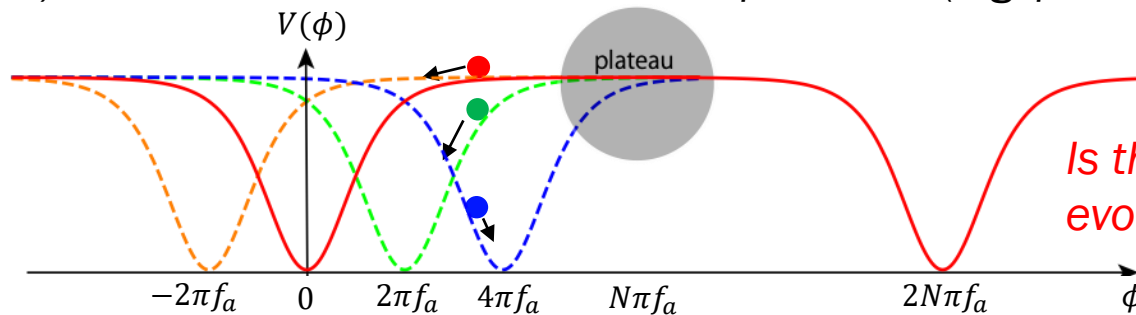
Motivation

Some issues about the set-up (considering $SU(N)$ hidden gauge symmetry)

$$L_{dark} = -\frac{1}{4g^2} GG + \frac{1}{2}(\partial_\mu \phi)^2 + \frac{\phi}{32\pi^2 f_a} G\tilde{G}$$

$$\frac{\theta}{32\pi^2} G\tilde{G}$$

1) Multi branch structure of the axion potential? (e.g. pure natural inflation 1706.08522,1711.10490)

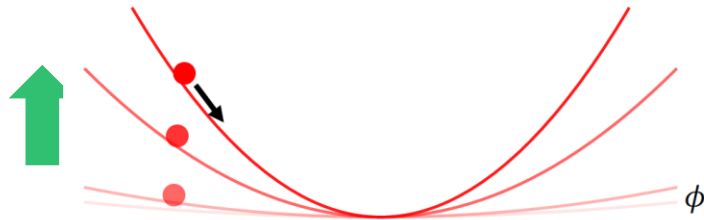


$$e^{iS[\phi+2\pi f_a]} = e^{iS+2in\pi} = e^{iS[\phi]}$$

$$V(\phi + 2\pi f_a) \neq V(\phi)$$

Is there any nontrivial evolution of the axion DM?

2) Energy flow from the dark gluon to axion as the gluon temperature decreases (➡)



$$V(T_g, \phi) \sim \frac{1}{2} m_a^2 \left(\frac{T_{g,c}}{T_g} \right)^{2\eta} \phi^2 + \dots \text{ for } T_g > T_{g,c} \sim \Lambda$$

Is there also the flow of the entropy? What is the correct form of the entropy for coupled fluids?

3) What is the phenomenological consequence of the multi-component dark matter?

(ϕ : very *feely* interacting, ϕ_g : very *strongly* interacting as their mass becomes small)

Results: Dark sector with axion-gluon DM

$$L_{dark} = -\frac{1}{4g^2} GG + \frac{1}{2}(\partial_\mu \phi)^2 + \frac{\phi}{32\pi^2 f_a} G\tilde{G}$$

Confining phase transition
at $T_g \sim T_{g,c} = r T_{SM,c}$

Axion + Glueball
Multi-component dark matter

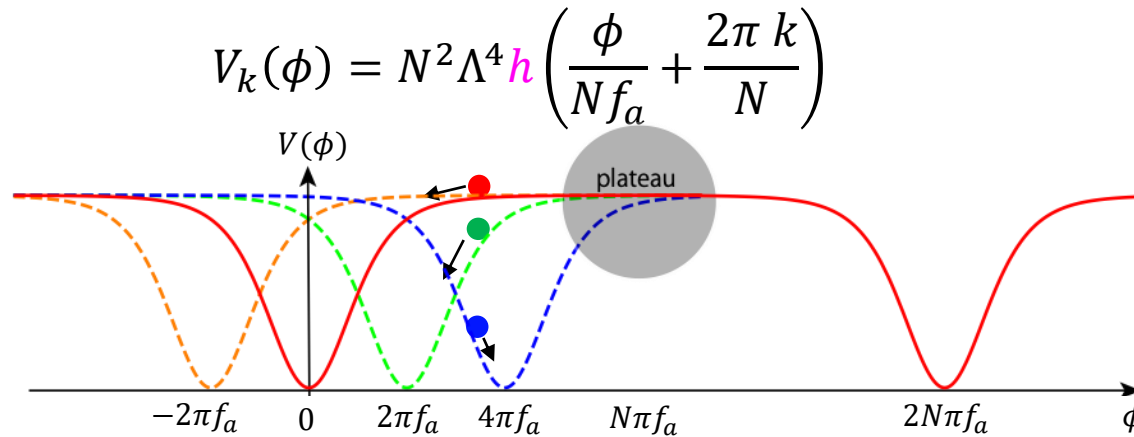
$$\begin{aligned} L_{eff} = & \frac{1}{2}(\partial_\mu \varphi_g)^2 - \frac{1}{2}m_g^2 \varphi_g^2 + c_1 \frac{4\pi}{N} m_g \varphi_g^3 + c_2 \left(\frac{4\pi}{N}\right)^2 \varphi_g^4 + c_3 \frac{1}{m_g} \left(\frac{4\pi}{N}\right)^3 \varphi_g^5 + \dots \\ & + \frac{1}{2}(\partial_\mu \phi)^2 - N^2 \Lambda^4 \left(d_1 \left(\frac{\phi}{N f_a}\right)^2 + d_2 \left(\frac{\phi}{N f_a}\right)^4 + \dots \right) \quad m_g \simeq (5-6)\Lambda \\ & + \left(\frac{\phi}{N f_a}\right)^2 \left(\frac{e_1}{4\pi N} m_g^3 \varphi_g + e_2 m_g^2 \varphi_g^2 + \dots \right) + \sum L_{eff}(\phi, \varphi_g(J^{PC})) \end{aligned}$$

$$m_a \sim 10^{-12} \text{eV} \left(\frac{\Lambda}{\text{MeV}}\right)^2 \left(\frac{10^{15} \text{GeV}}{f_a}\right)$$

$$\tau_{\varphi_g} \sim 10^{17} \text{Gyr} \left(\frac{f_a}{10^{13} \text{GeV}}\right)^4 \left(\frac{\text{GeV}}{m_g}\right)^5$$

Results: Evolution of the axion potential

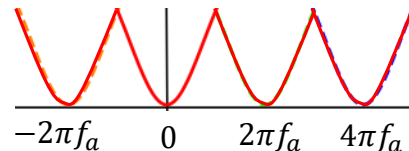
The axion potential for k -th branch ($k=1,\dots,N$) with $h(x) = h(x + 2\pi)$



For the branch structure, comparing lattice results and the analytic result in holographic YM theory, we find $(g^2 N)_{\text{dual}} \sim 10 - 20$. *Tunneling rate from k -th branch to $(k-1)$ -th branch*
 1105.3740, Dubovsky, Lawrence, and Roberts

$$\Gamma_k \propto \exp \left[-O(10^{-11}) N \frac{(N/k)^3}{(1 + O((N/k)^2))} \right]$$

This is very large compared to the Hubble expansion rate unless $N > 10^3$. Therefore, tunneling from branches with higher energies to the branches with lower energies happens instantaneously as the confining phase transition occurs.



Results: Gluo-thermodynamics

Gluons in thermal equilibrium with T_g ; Thermodynamic relations hold during the cosmic expansion.

$$\rho = Ts - p, \quad f = \rho - Ts = -p, \quad s = -\frac{df}{dT}$$

where ρ = energy density, s = entropy density, f = free energy density,
 p = pressure, T = temperature

When $\phi = 0$, the free energy density is given by

$$f_g(T_g) = -\frac{T_g}{V} \ln Z_g = -\frac{T_g}{V} \ln \text{Tr}[e^{-H_g/T_g}] = -\frac{T_g}{V} \ln \int dA_a \exp \left[- \int_0^{1/T_g} dt d^3\vec{x} \left(\frac{1}{4g^2} G^a G^a \right) \right]$$

When $\phi \neq 0$, i.e. nonzero theta-term ($\theta \equiv \phi/f_a$) as

$$f_{g+\theta} = -\frac{T_g}{V} \ln \int dA_a \exp \left[\int_0^{1/T_g} dt d^3\vec{x} \left(\frac{1}{4g^2} G^a G^a + i \frac{\theta}{32\pi^2} G^a \tilde{G}^a \right) \right]$$

$$\simeq f_g(T_g) + V(T_g, \phi) \equiv -p_g(T_g) + V(T_g, \phi)$$

$$V(T_g, \phi) \simeq \frac{1}{2} m_a^2 \left(\frac{T_{g,c}}{T_g} \right)^{2\eta} \phi^2$$

We can show that the “new” entropy $s_{g+\theta}$ is conserved for the adiabatic evolution:

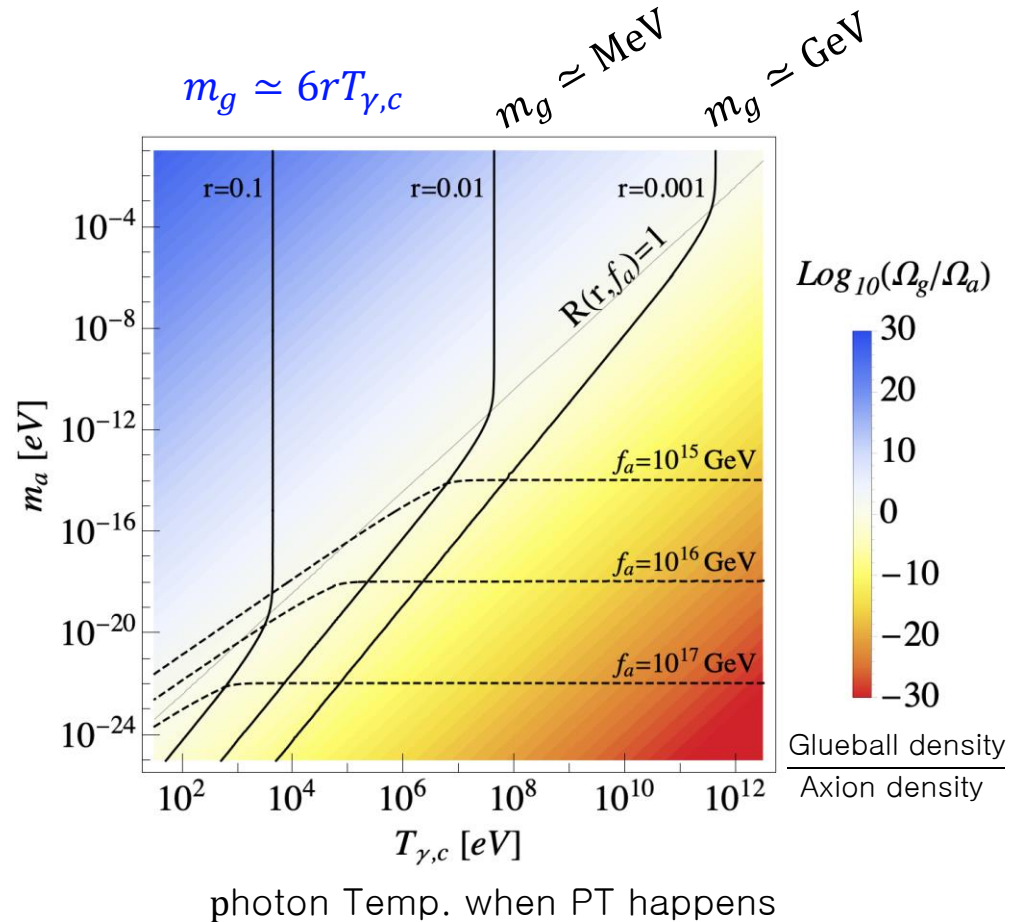
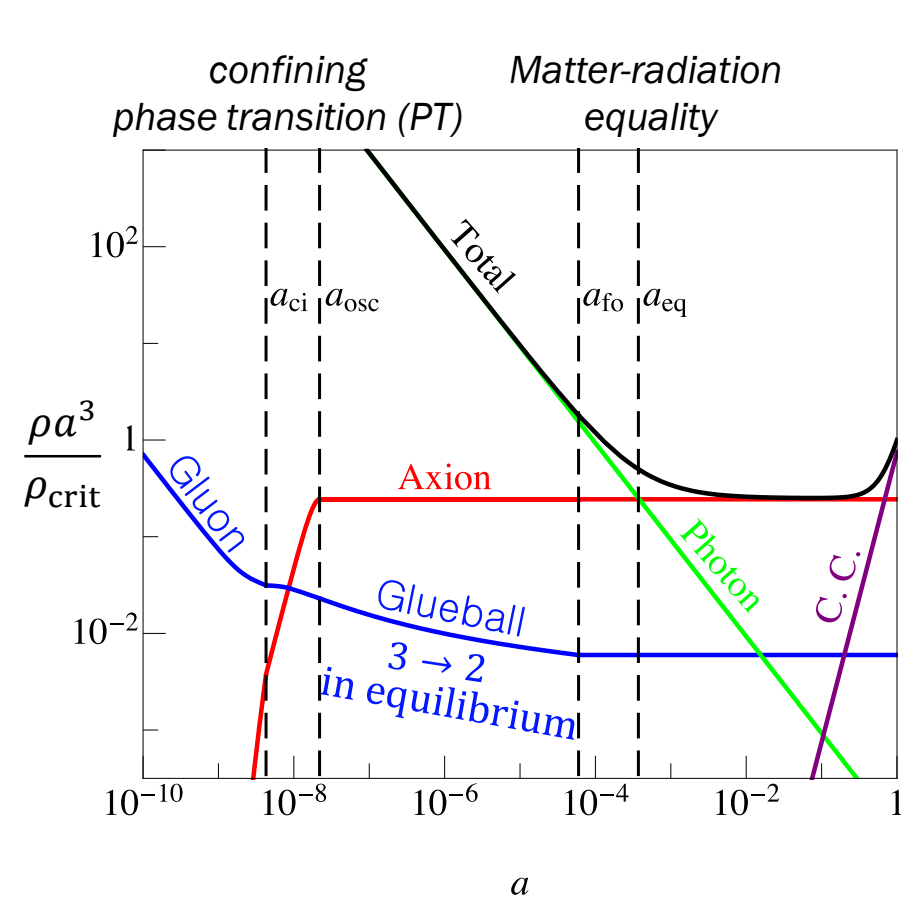
$$s_{g+\theta} = -\frac{df_{g+\theta}}{dT_g} = \frac{dp_g}{dT_g} - \frac{\partial V(T_g, \phi)}{\partial T_g} = \frac{\rho_g + p_g}{T_g} \propto \frac{1}{a^3}!$$

$$\rho_g \equiv \rho_{g+\theta} - V(T_g, \phi)$$

Results: Densities

$$L_{dark} = -\frac{1}{4g^2} GG + \frac{1}{2}(\partial_\mu \phi)^2 + \frac{\phi}{32\pi^2 f_a} G\tilde{G}$$

$$r = \left(\frac{T_g}{T_{SM}} \right)$$



Supermassive black holes at high z

Observations of the quasars lead to the discovery of supermassive black holes in the 2010s.

Around the redshift $z = 7$,

J1342+0928 ($z = 7.54, M_{BH} = 0.8 \times 10^9 M_\odot, 1712.01860$)

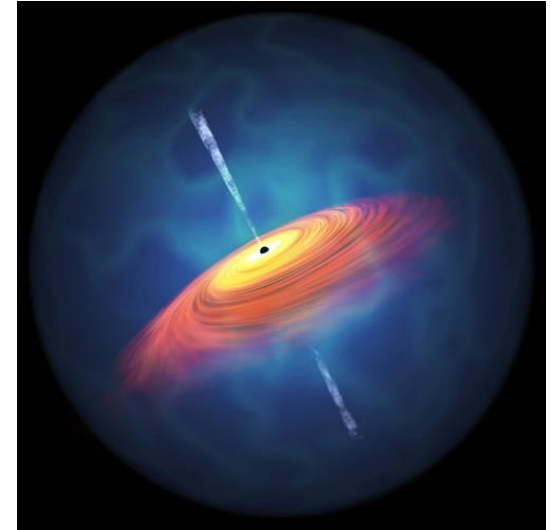
J1120+0641 ($z = 7.09, M_{BH} = 2.0 \times 10^9 M_\odot, 1106.6088$)

J2348-3054 ($z = 6.89, M_{BH} = 2.1 \times 10^9 M_\odot, 1311.3260$)

J0109-3047 ($z = 6.75, M_{BH} = 1.5 \times 10^9 M_\odot, 1311.3260$)

J0305-4150 ($z = 6.61, M_{BH} = 1.0 \times 10^9 M_\odot, 1311.3260$)

J0100+2802 ($z = 6.3, M_{BH} = 1.2 \times 10^{10} M_\odot, 1502.07418$)



The origin of these supermassive black holes is not clear.

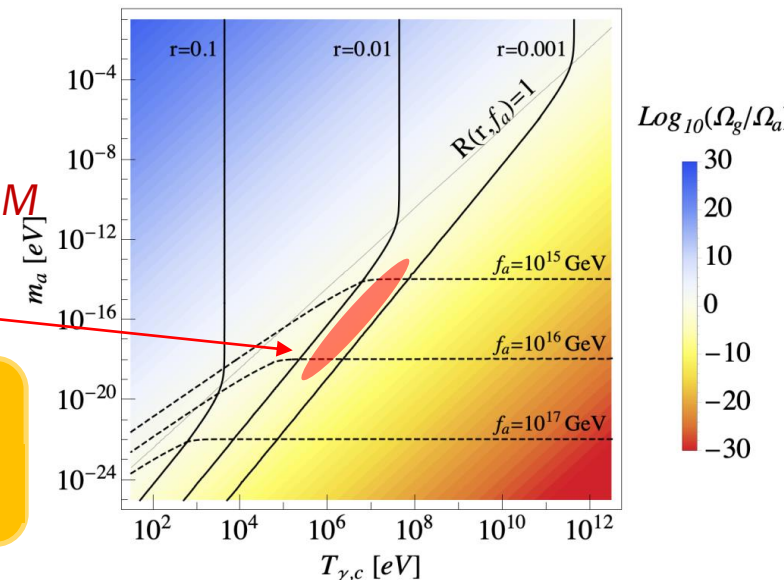
It may originate from strongly interacting subcomponent DM

(dark glueball subcomponent DM)

$$\sigma_{g2 \rightarrow 2} \sim \frac{1}{16\pi} \left(\frac{4\pi}{N} \right)^4 \frac{1}{m_g^2}$$

$$m_g \sim 0.01 - 1 \text{ MeV}$$

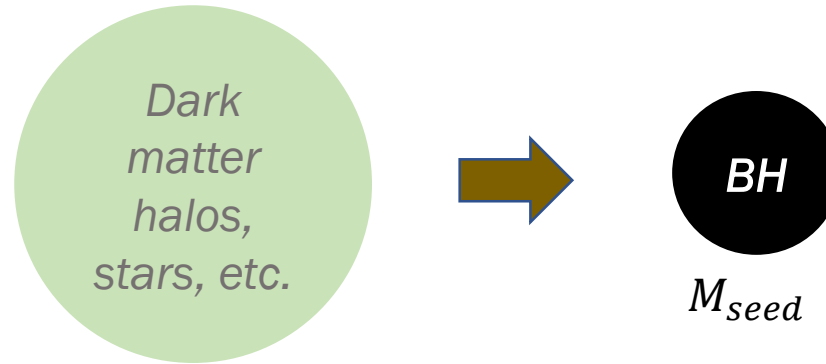
$$m_a \sim 10^{-18} - 10^{-14} \text{ eV}$$



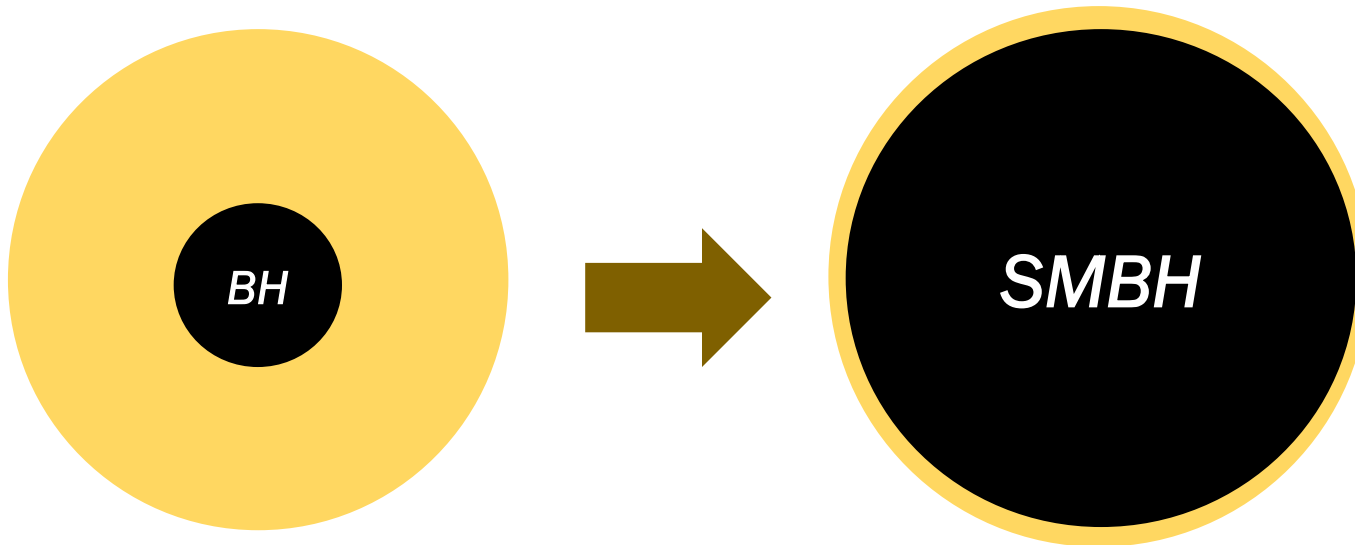
Formation and growth of the BHs

Basic process

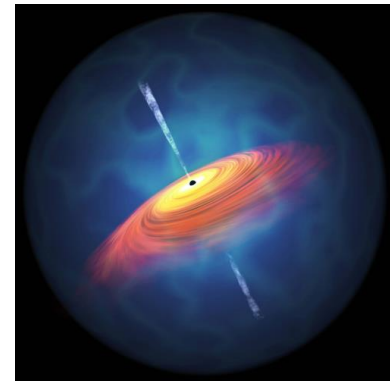
- 1) Formation of the seed black hole (e.g. stellar evolution, direct formation from halo collapse)



- 2) Growth by *accretion* of baryons and dark matters or *mergers* with other black holes



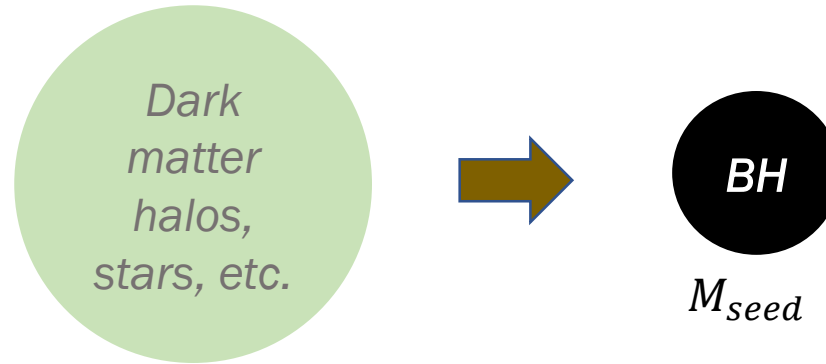
$$M_{BH}(t_c) = M_{seed} \rightarrow M_{BH}(t_{obs}) = M_{SMBH} \gg M_{seed}$$



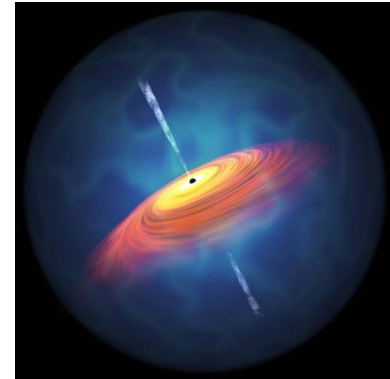
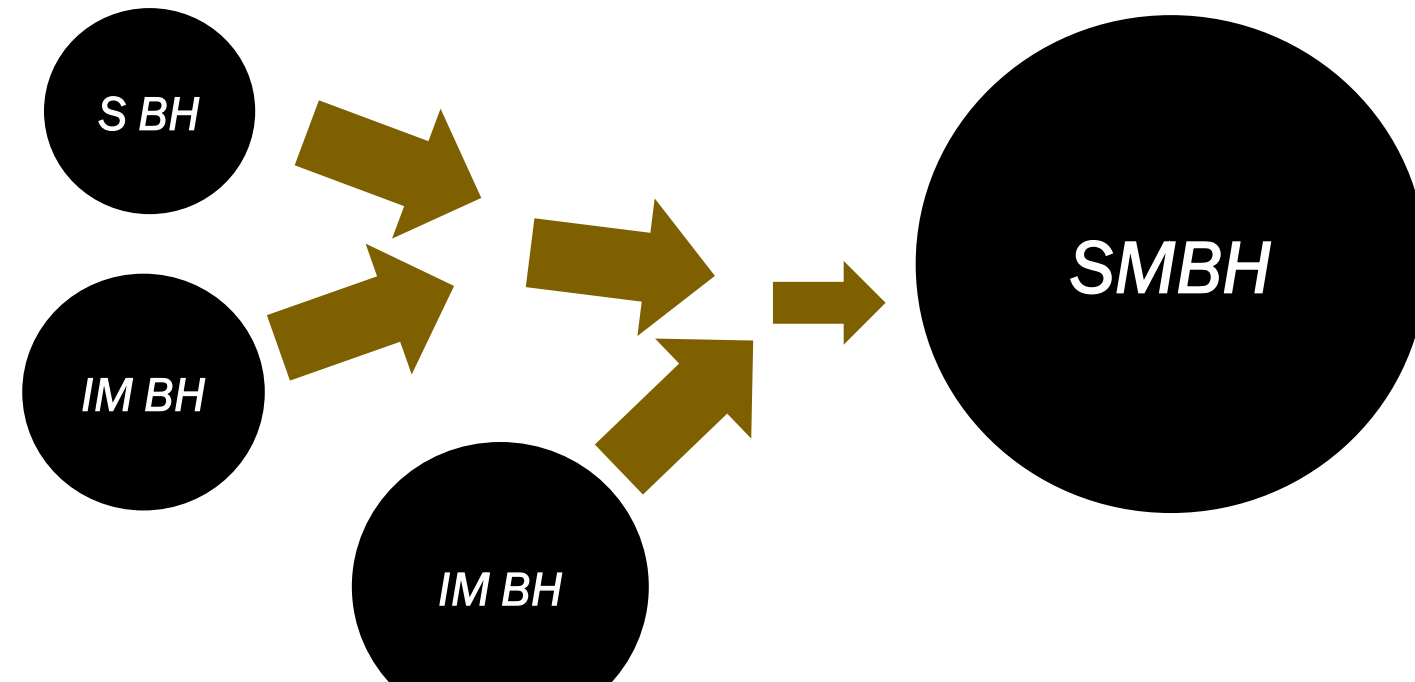
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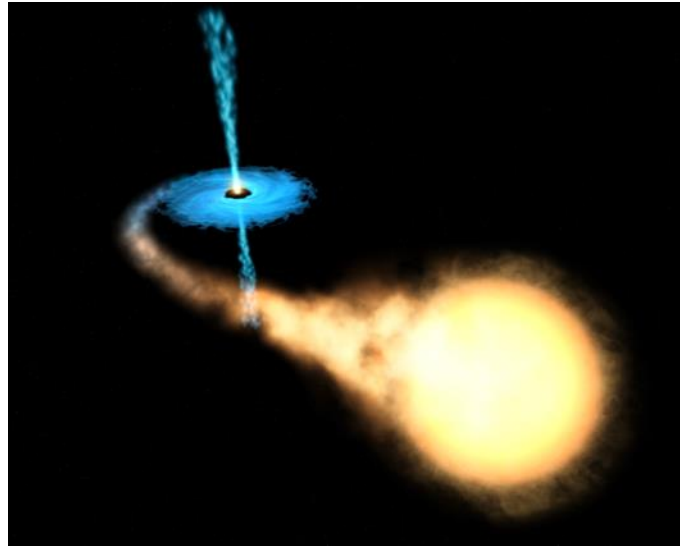


- 2) Growth by *accretion* of baryons and dark matters or *mergers* with other black holes



Growth mechanism of the BH from accretion material

If the black hole is surrounded by the accretion material, that material will be gradually absorbed by the black hole: the black hole mass and spin will grow



accretion = release of gravitational energy from infalling matter (by dissipation)

→ accreting material gets closer to the BH as it loses its kinetic energy

$$\Delta M_{BH} = M_{acc} (= \text{rest mass}) - \Delta E_{\text{release}} (= \epsilon M_{acc}) = (1 - \epsilon) M_{acc}$$

$$\Delta E_{\text{release}} = L_{acc} \Delta t = \epsilon M_{acc} = \frac{\epsilon}{1 - \epsilon} \Delta M_{BH}$$

$$\Delta E_{\text{release}} \sim \frac{GM_{BH} M_{acc}}{r_{acc}}$$

$$\rightarrow L_{acc} \sim \frac{\epsilon}{1 - \epsilon} \dot{M}_{BH}$$

Growth mechanism of the BH from accretion material

During the absorption, radiations are emitted from the accretion material: slow down the absorption of the material by gravity

The energy released as electromagnetic (or other) radiation with a luminosity, L

$$\rightarrow F_{rad} = \frac{L\sigma_T}{4\pi r^2}$$

for on an electron (and a coupled proton)

$$F_{grav} = \frac{GM_{BH}m_p}{r^2}$$

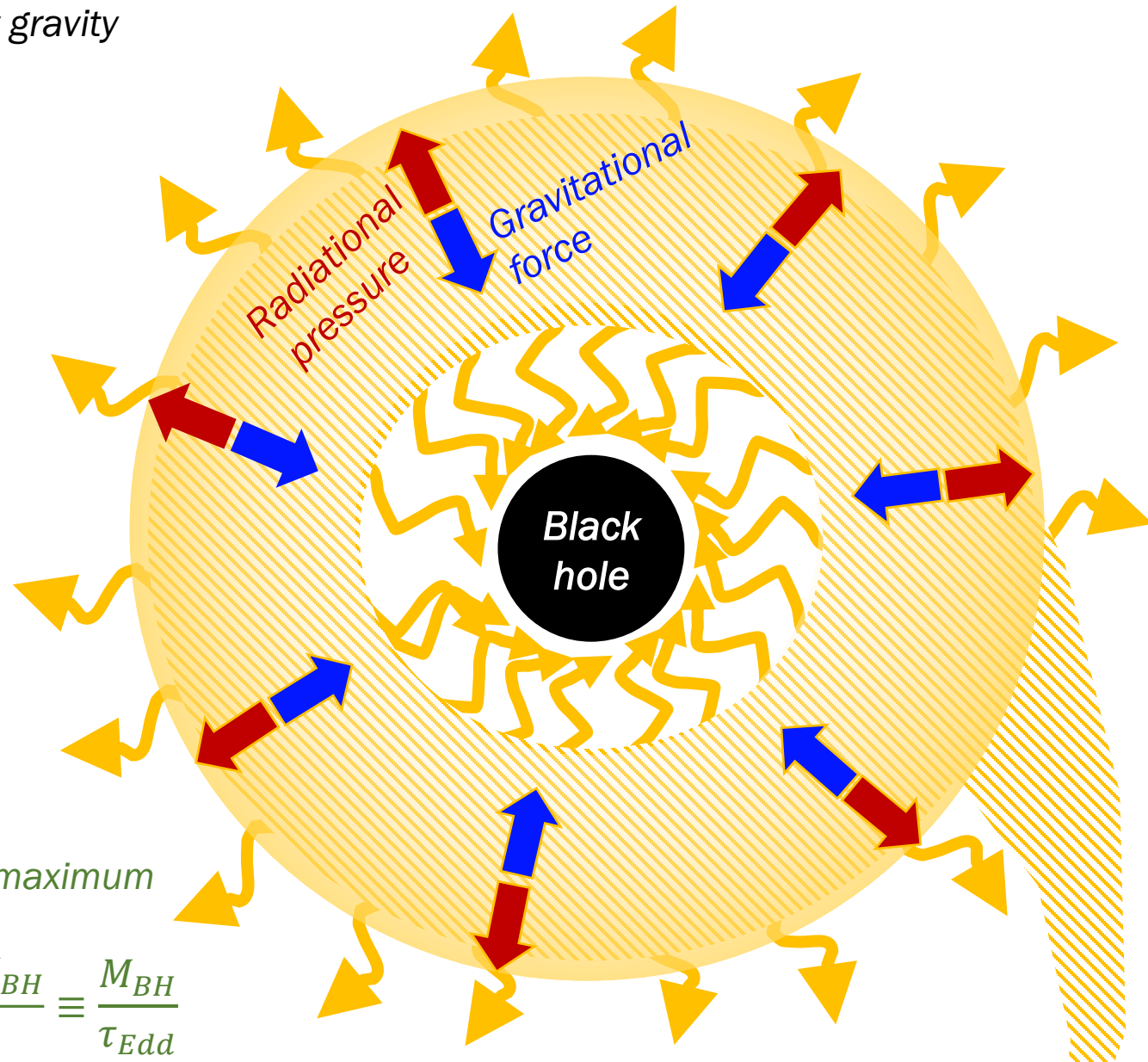
gravitational force on electron-proton pair

→ Accretion is forbidden if

$$F_{rad} \geq F_{grav}$$

Therefore accretion has the maximum luminosity :

$$L_{max} = L_{Edd} = \frac{4\pi Gm_p M_{BH}}{\sigma_T} \equiv \frac{M_{BH}}{\tau_{Edd}}$$



Growth mechanism of the BH from accretion material

During the absorption, radiations are emitted from the accretion material: slow down the absorption of the material by gravity

$$L_{acc} \leq L_{max} = L_{Edd} = \frac{4\pi G m_p M_{BH}}{\sigma_T} \equiv \frac{M_{BH}}{\tau_{Edd}}$$

The Eddington time can be defined as

$$\tau_{Edd} = \frac{\sigma_T}{4\pi G m_p} \simeq 4.5 \times 10^8 \text{ years}$$

On one hand, the accretion luminosity also provides the increasing rate of the black hole mass as

$$L_{acc} = \frac{\epsilon}{1 - \epsilon} \dot{M}_{BH} \leq \frac{M_{BH}}{\tau_{Edd}}$$

$\epsilon \sim GM_{BH}/r_{acc}$ is the radiative efficiency for the accretion disk around the BH and $r_{acc} \gtrsim 3R_{BH}$.

$$L_{acc} = \frac{\epsilon}{1 - \epsilon} \dot{M}_{BH} \leq \frac{M_{BH}}{\tau_{Edd}} \rightarrow \frac{dM_{BH}}{dt} \simeq (\epsilon \tau_{Edd})^{-1} M_{BH} \equiv \tau_{Sal}^{-1} M_{BH}$$

$$\tau_{Sal} (\text{Salpeter time}) = \epsilon \tau_{Edd} = \left(\frac{\epsilon}{0.1} \right) 45 \text{ Myr}$$

$$M_{BH}(t) \leq M_{seed} e^{t/\tau_{Sal}}$$

Accretion rate is enough for SMBHs at high z ?

If the seed black hole is formed inside the virialized massive halo, it is reasonable that it happens $z < 20 - 30$.

$$M_{BH}(t) = M_{seed} \exp \left[\frac{t(z_{obs}) - t(z_c)}{\left(\frac{\epsilon}{0.1}\right) 45 \text{ Myr}} \right] \equiv M_{seed} \mathcal{A}_{acc}$$

During matter domination, the age of the Universe, $t(z) = 550 \text{ Myr} \left(\frac{10}{1+z}\right)^{3/2}$

For $z_{obs} = 7$, $z_c = 15$, $\mathcal{A}_{acc} \sim (2 - 6)10^4$, so that $M_{seed} \sim 10^5 M_\odot$ at $z_c = 15$

For $z_c = 30$, $\mathcal{A}_{acc} \sim (6 - 10)10^5$, so that $M_{seed} \sim 10^4 M_\odot$ at $z_c = 30$

Within the CDM framework

As the remnants of the Pop III stars ($z \sim 20$), $M_{seed} = O(100) M_\odot$ (Madau & Rees 2001; Heger et al. 2003; Wise & Abel 2005)

The larger growth rate within a certain period? (super-Eddington accretion)

$$M_{BH}(t) = M_{seed} e^{f_{Edd} t/\tau_{sal}}, \quad (f_{Edd} > 1)$$

Direct collapse of gas into the BH ? $M_{seed} = O(10^{4-6}) M_\odot$ (Loeb & Rasio 1994; Eisenstein & Loeb 1995, and so on.) but should prevent fragmentation, star formation before the collapse

Collapsing star cluster? From mergers of Pop II stars, $M_{seed} = O(10^3) M_\odot$ (Devecchi & Volonteri 2009) but maybe useful only for explaining observed quasars at $z < 5$.

Ideas in 1501.00017 (Pollack, Spergel, Steinhardt)

Beyond the CDM framework : multi-component with strongly interacting sub-comp. DM

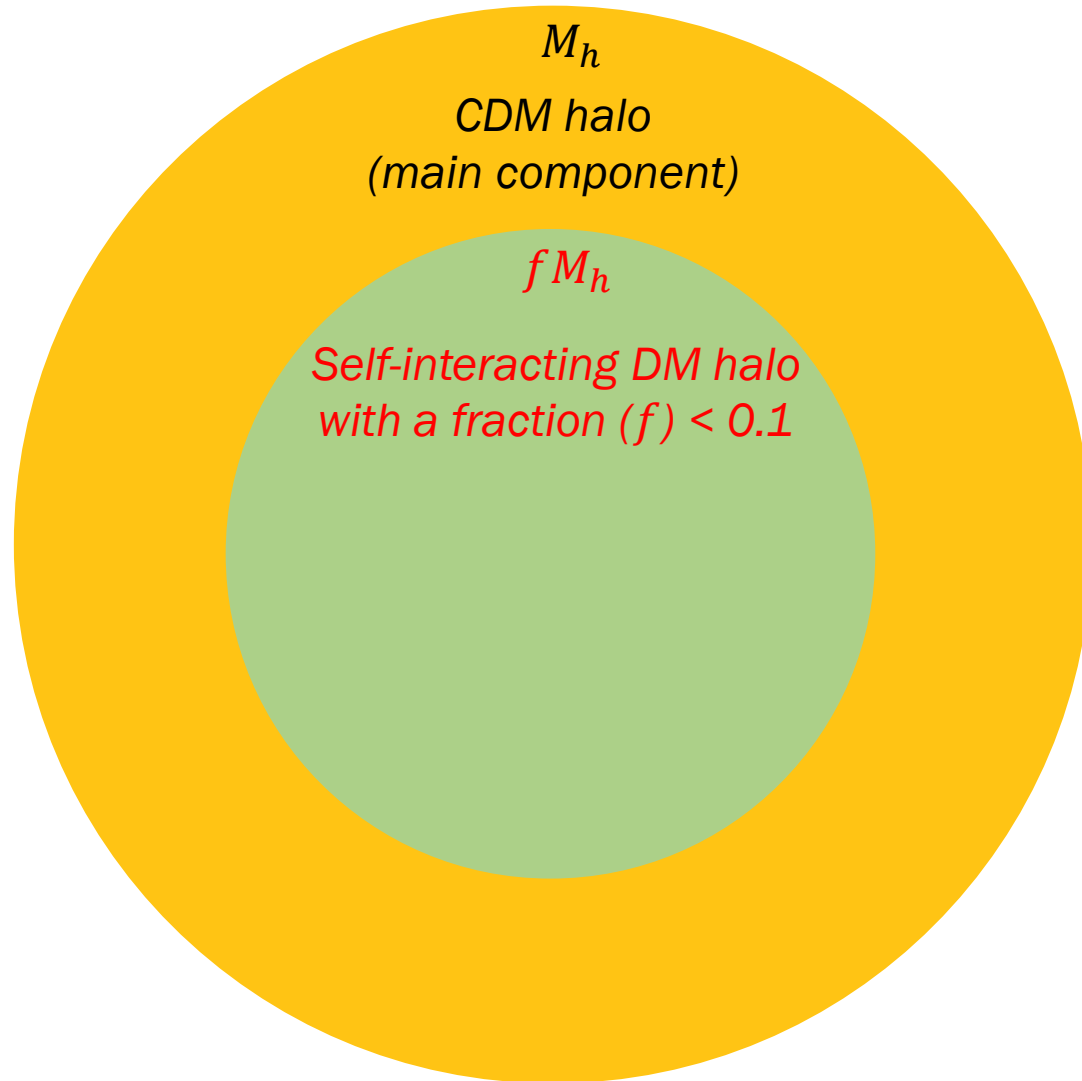
Supermassive Black Holes from Ultra-Strongly Self-Interacting Dark Matter

Jason Pollack,^{1,*} David N. Spergel,² and Paul J. Steinhardt³

We consider the cosmological consequences if a small fraction ($f \lesssim 0.1$) of the dark matter is ultra-strongly self-interacting, with an elastic self-interaction cross-section per unit mass $\sigma \gg 1 \text{ cm}^2/\text{g}$. This possibility evades all current constraints that assume that the self-interacting component makes up the majority of the dark matter. Nevertheless, even a small fraction of ultra-strongly self-interacting dark matter (uSIDM) can have observable consequences on astrophysical scales. In particular, the uSIDM subcomponent can undergo gravothermal collapse and form seed black holes in the center of a halo. These seed black holes, which form within several hundred halo interaction times, contain a few percent of the total uSIDM mass in the halo. For reasonable values of σf , these black holes can form at high enough redshifts to grow to $\sim 10^9 M_\odot$ quasars by $z \gtrsim 6$, alleviating tension within the standard Λ CDM cosmology. The ubiquitous formation of central black holes in halos could also create cores in dwarf galaxies by ejecting matter during binary black hole mergers, potentially resolving the “too big to fail” problem.

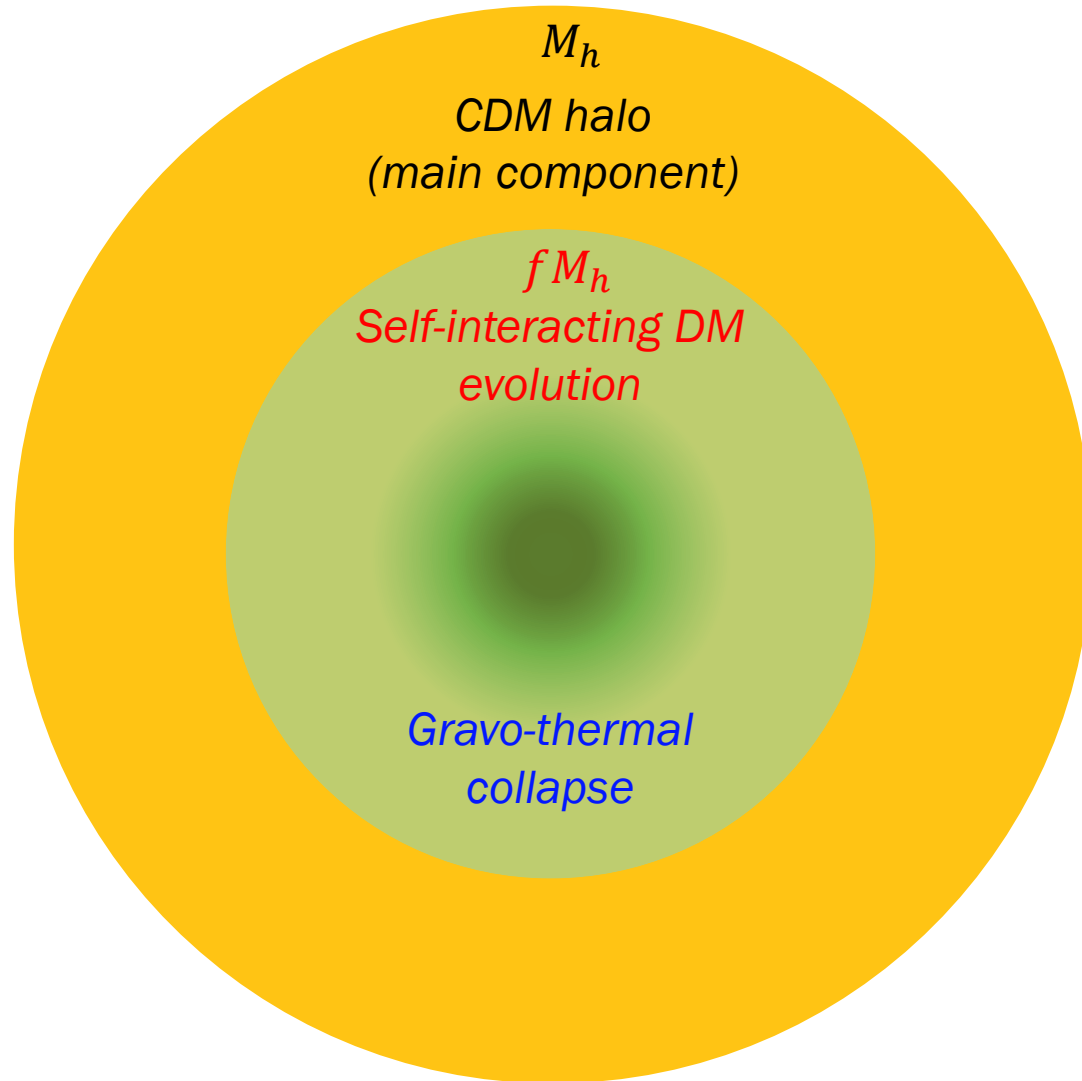
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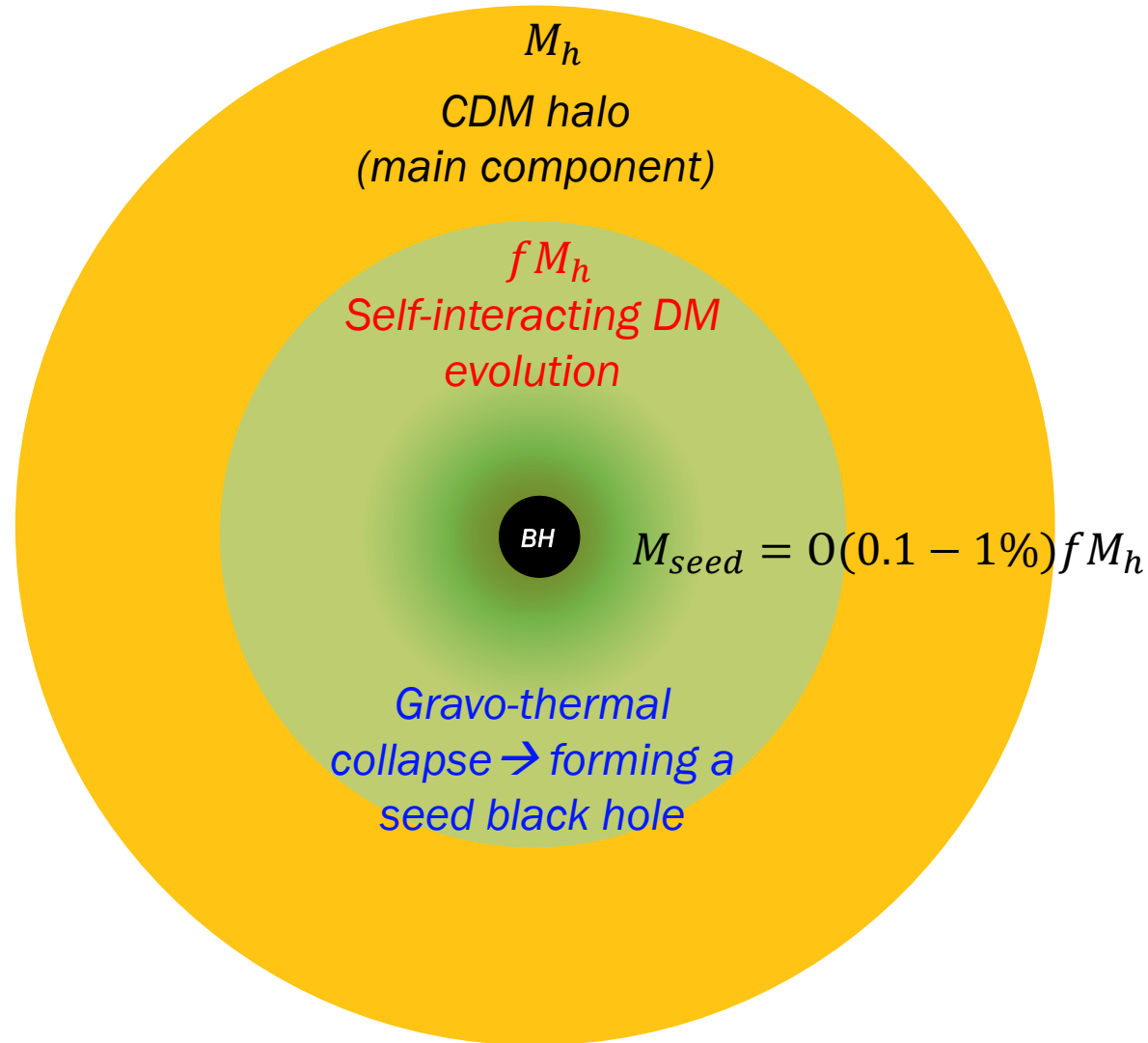
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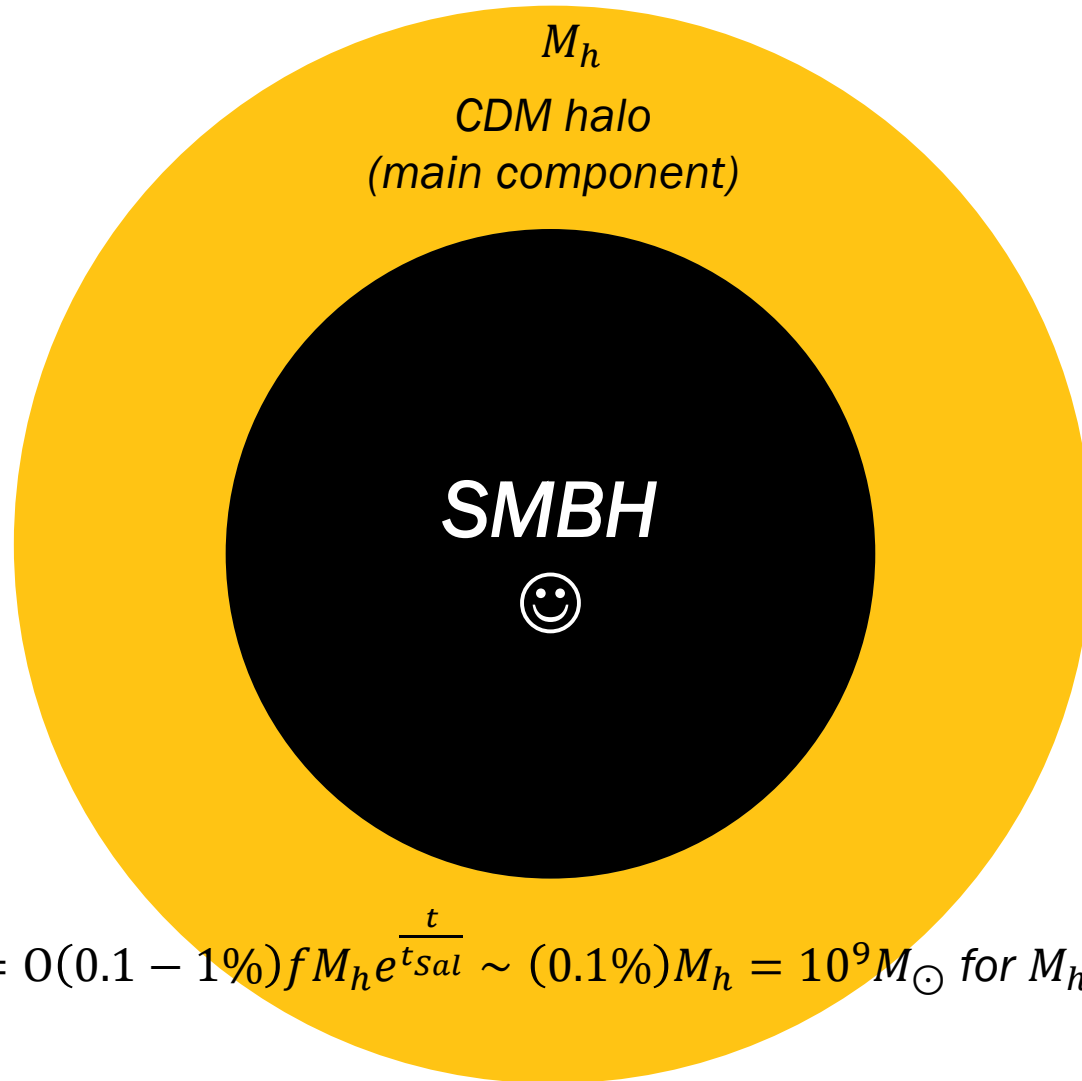
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Beyond the CDM framework : multi-component with strongly interacting sub-comp. DM



$$M_{BH}(t) = 0(0.1 - 1\%)fM_h e^{\frac{t}{\tau_{sal}}} \sim (0.1\%)M_h = 10^9 M_\odot \text{ for } M_h = 10^{12} M_\odot$$

Gravo-thermal collapse

Thermally equilibrated system which is bound by *gravity*

If the system is in equilibrium with gravity, the system has a negative specific heat capacity

$$c_T = \frac{dE}{dT} < 0$$

Why? Thermal equilibrium \rightarrow thermal energy (kinetic energy) is virialized by potential energy

$$\langle V \rangle = -2\langle K \rangle \rightarrow E = Nm + \langle V \rangle + \langle K \rangle = Nm - \langle K \rangle = Nm - NT \rightarrow \frac{dE}{dT} = -N \simeq -\frac{E}{m}$$

(c.f. black hole: $E = M_{BH}$, $T = \frac{M_P^2}{M_{BH}} = \frac{M_P^2}{E} \rightarrow \frac{dE}{dT} = -\frac{E}{T} < 0$)

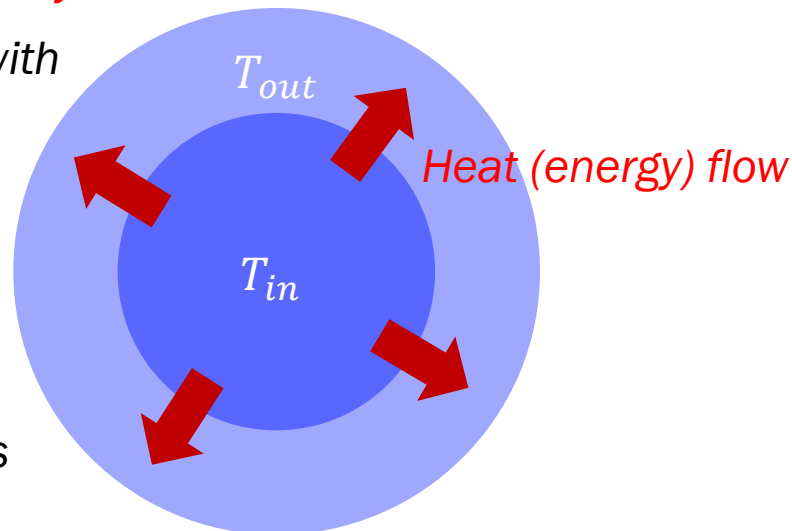
Negative heat capacity \rightarrow *instability*

Considering the bound system with
initial temperature gradient

$$T_{in} > T_{out}$$

For the positive c_T case,

T_{in} decreases, T_{out} increases
and meet at T_{eq} . Heat flow stops



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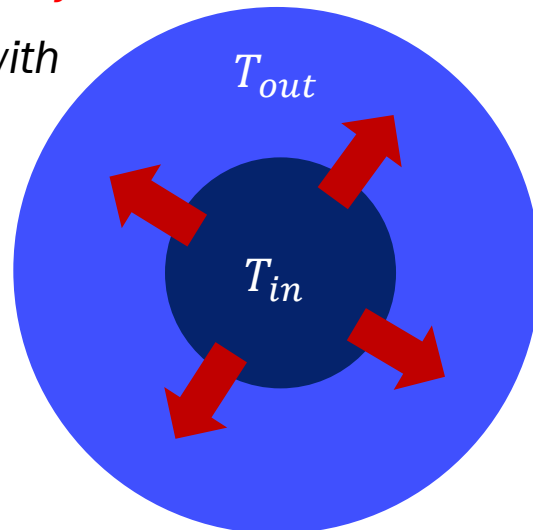
Negative heat capacity \rightarrow *instability*

Considering the bound system with
initial temperature gradient

For the negative c_T case

Heat (energy) flow continues!

$T_{in} > T_{out}$ maintains forever!



Strongly bound system
with high virial velocity
 \rightarrow leads to gravitational collapse
 \rightarrow forming a black hole

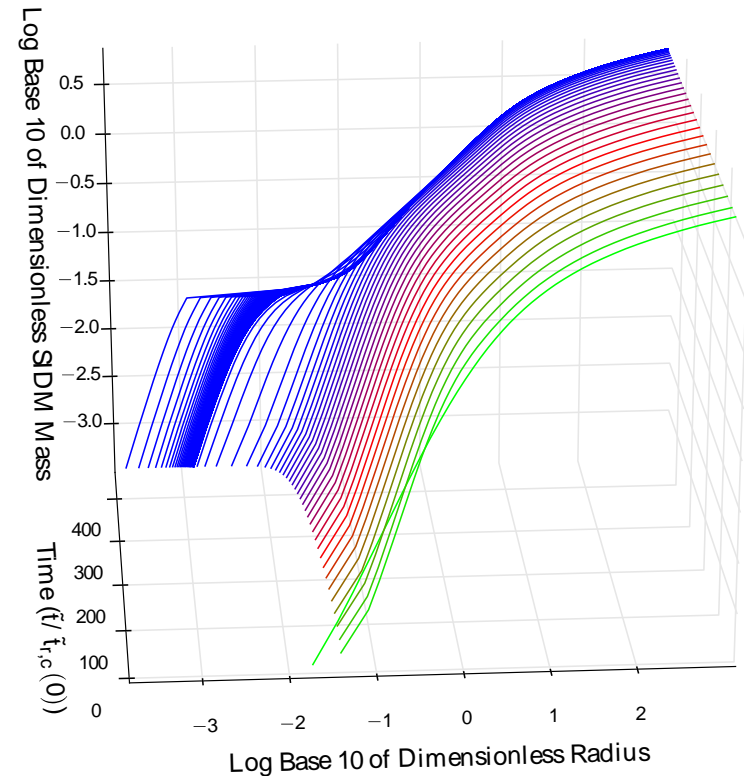
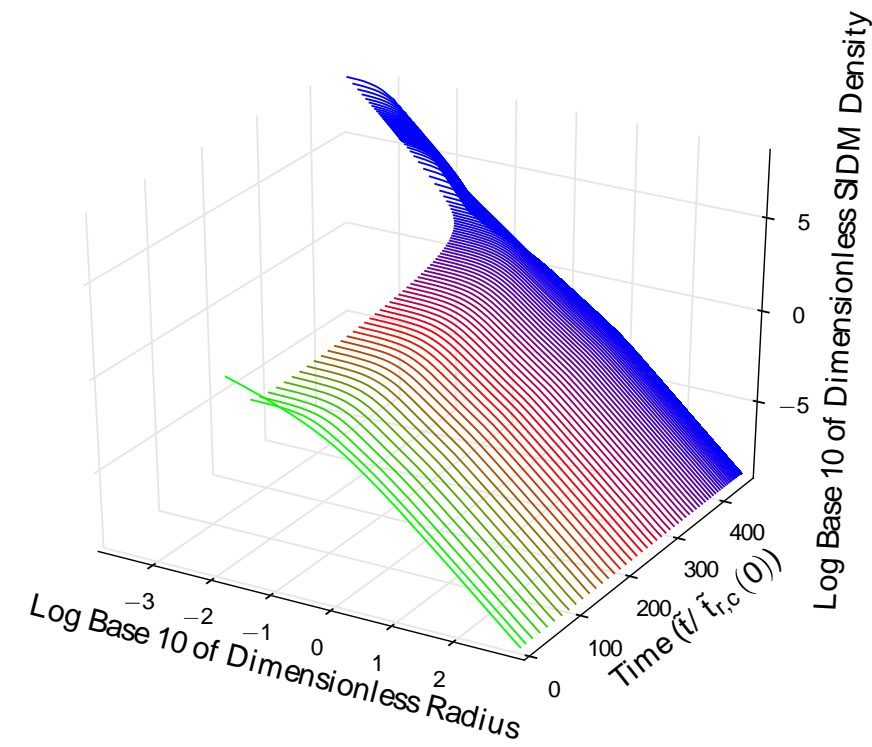
Gravo-thermal collapse

The thermally equilibrated system bound by gravity

For the gravo-thermal collapse, maintaining thermal equilibrium is an important condition.

Therefore the “relaxation time” should be shorter than the age of the Universe for a given z .

How short? Numerical calculation is necessary



1501.00017 for the isolated halo with $f=1$

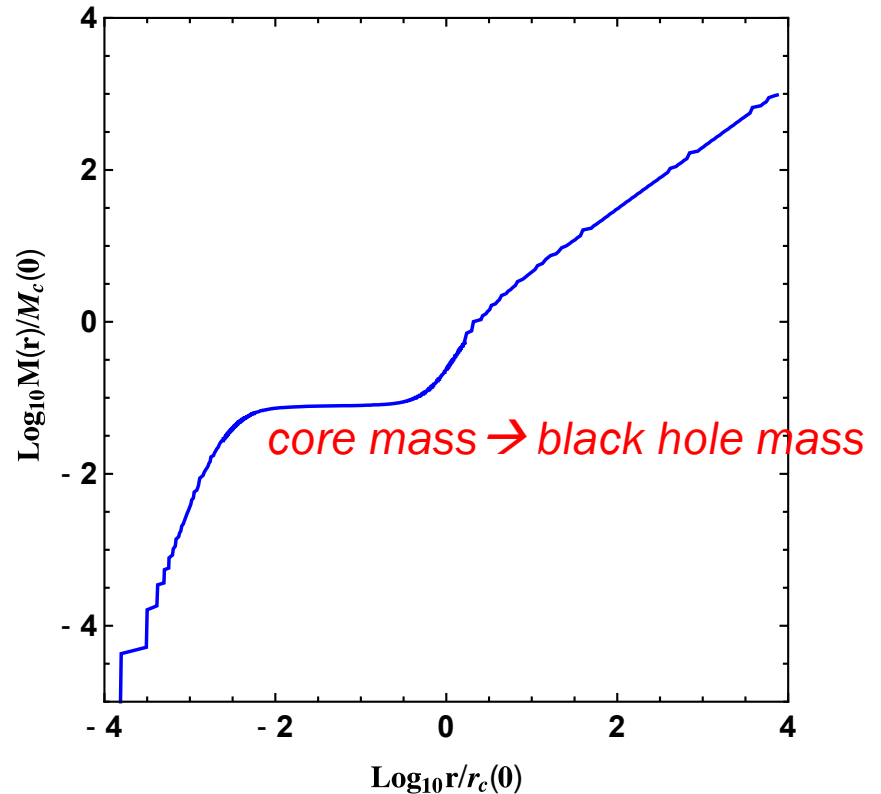
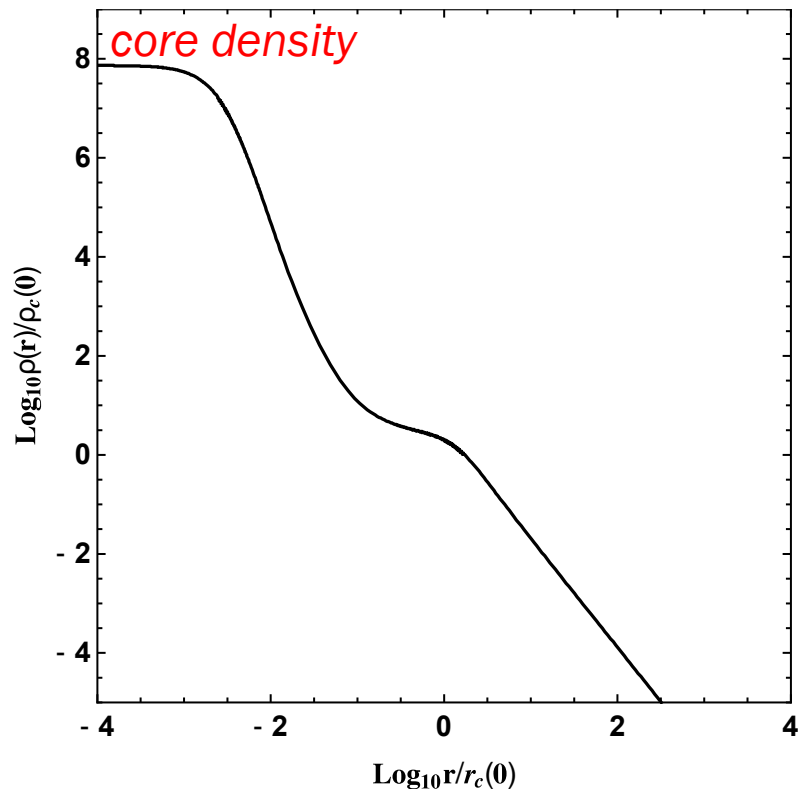
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Balberg et.al. 0110561 for the isolated halo with $f=1$

Gravo-thermal collapse

Numerical calculations estimate the collision time as

$$\Delta t_{col} \simeq 480 t_{relax}(t_i)$$

$$t_{relax}(t_i) = \frac{m_{DM}}{\sigma \rho_s(t_i) v_s(t_i)}$$

*It is non-trivial to estimate the collision time for the collapse of **sub**-component dark matter.*

*There are two papers to estimate the collision time and the seed black hole mass for the isolated halo with a small fraction (**f**) of self-interacting DM*

$$\Delta t_{col} \simeq 480 t_{relax}(t_i), \quad M_{seed} \simeq \frac{0.02}{\ln c} \mathbf{f} M_h$$

1501.00017, fluid approximation

$$t_{relax}(t_i) = \frac{m_{SDM}}{\sigma \mathbf{f} \rho_s(t_i) v_s(t_i)}$$

$$\Delta t_{col} \simeq \frac{480}{\mathbf{f}^2} t_{relax}(t_i), \quad M_{seed} \simeq 0.006 \mathbf{f} M_h$$

1812.05088 Choquette, Cline, Cornell, N-body simulation up to $f=0.1$

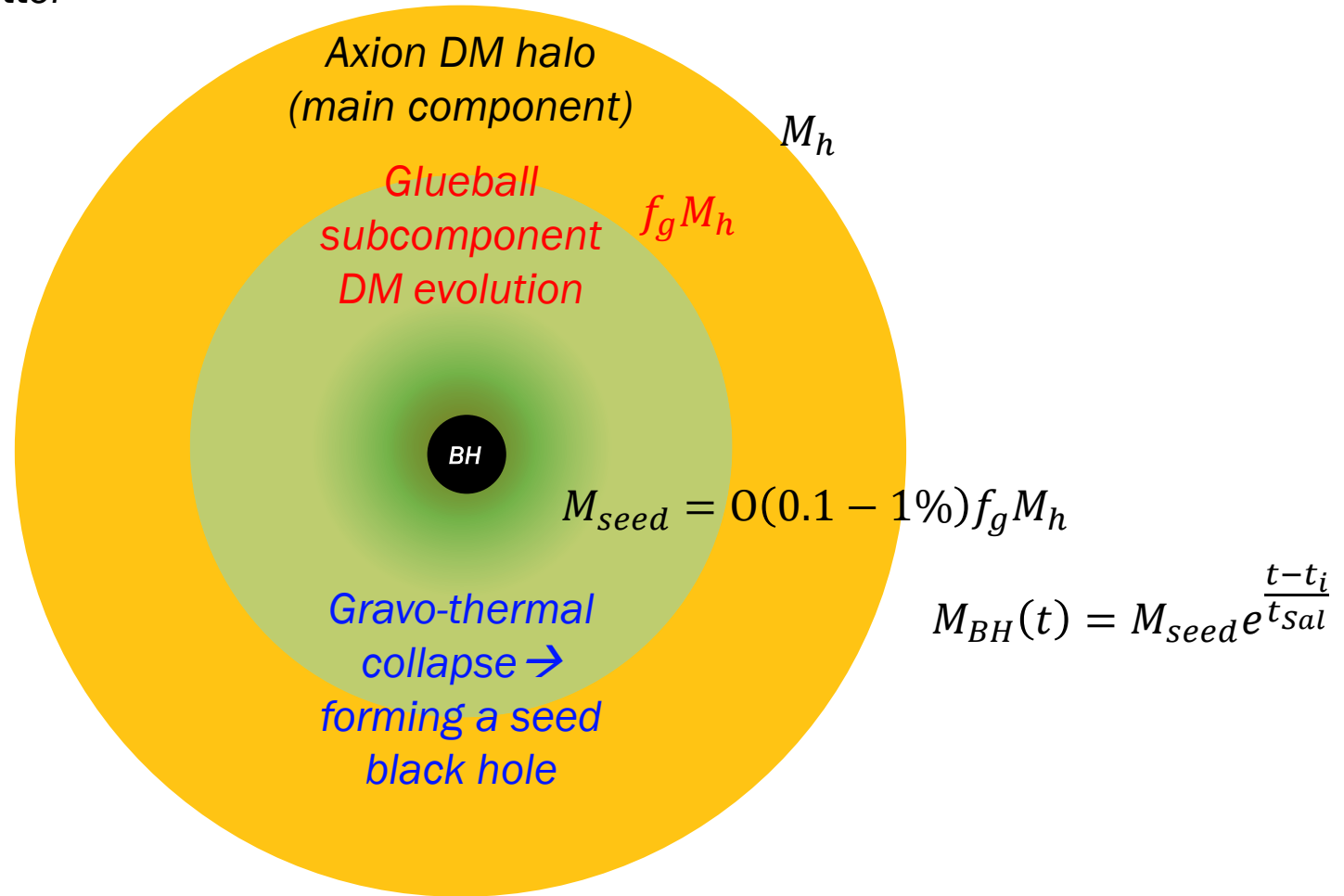
In order to explain the SMBH at $z=7$, ($\Delta t_{col} < t(z_c) < t(z = 7)$)

$$\mathbf{f} \sigma / m_{SDM} \sim 1 - 10 \text{ cm}^2/\text{g}$$

$$\mathbf{f}^3 \sigma / m_{SDM} \sim 1 - 10 \text{ cm}^2/\text{g}$$

Based on the idea of 1501.00017 & 1812.05088

The large seed black hole can be made by the gravo-thermal collapse of the subcomponent glueball dark matter

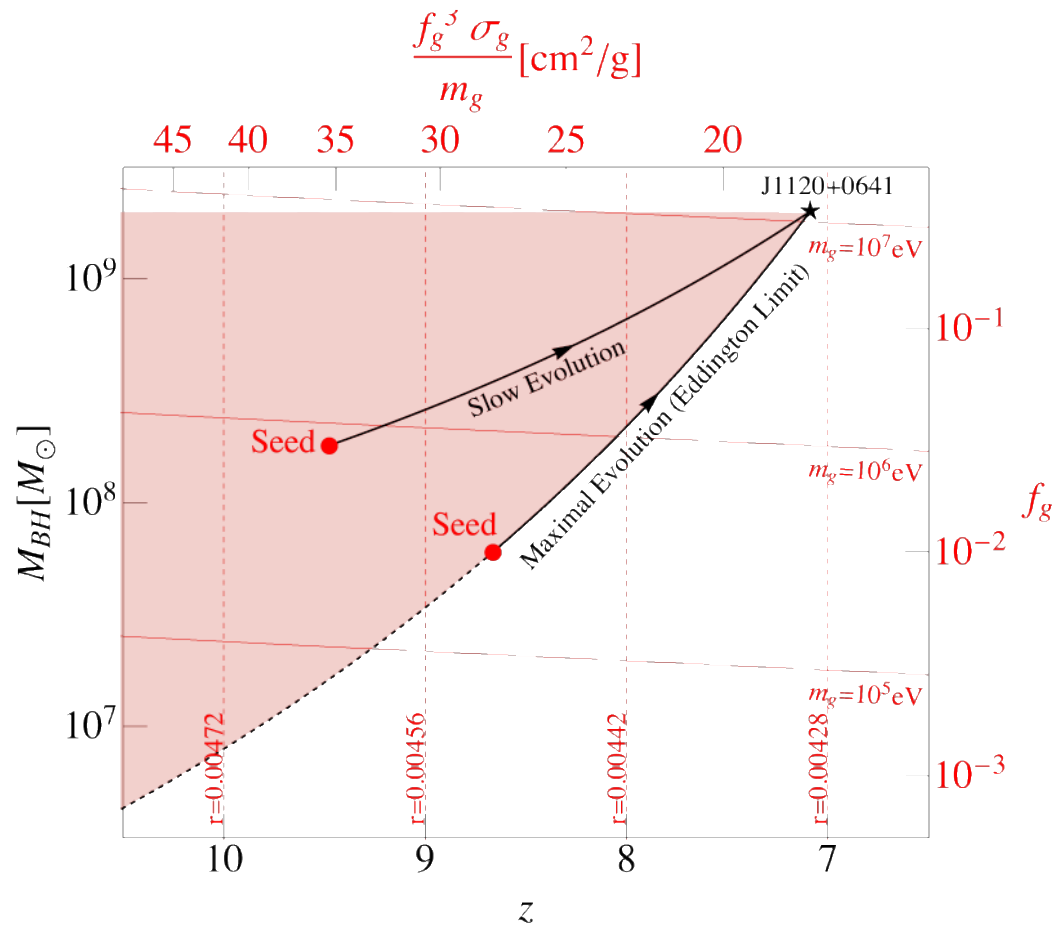


$$\Delta t_{col} \simeq \frac{480}{f_g^2} t_{relax}(t_i), \quad M_{seed} \simeq 0.006 f_g M_h$$

$$t_{relax}(t_i) = \frac{m_g}{\sigma_g f_g \rho_s(t_i) v_s(t_i)}$$

SMBH at high z for a isolated host halo

The large seed black hole can be made by the gravo-thermal collapse of the subcomponent glueball dark matter



for $M_h = 10^{12} M_\odot$

Caveats: History of the host halo mass

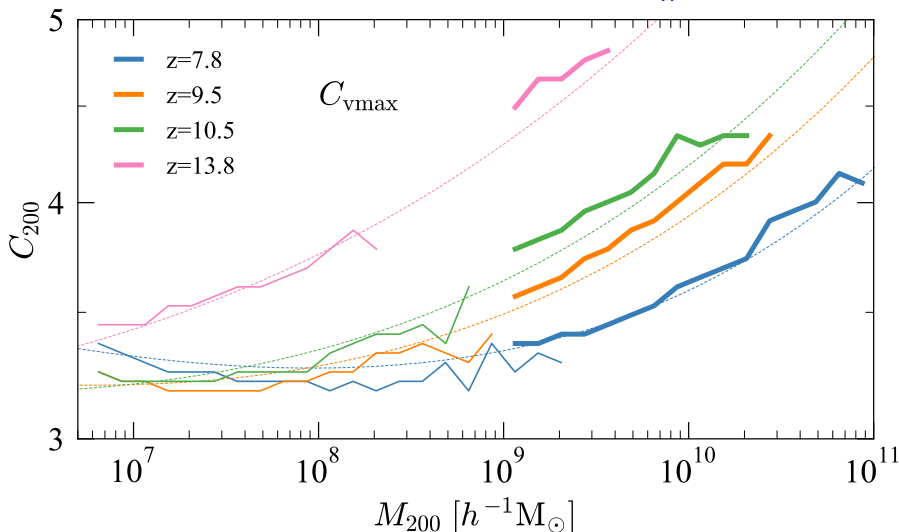
The assumption of the isolated host halo with a mass $M_h = 10^{12} M_\odot$ for $z = 15 - 10$ is difficult to be accepted in realistic cases. The merger history of the host halo (e.g. heaviest halos for a given z) and its time-dependent profile should be considered: $M_h(z), \rho_h(z), \dots$

Our approach: at $z = z_i$, seed black hole forms with $M_{seed} = 0.006 f_g M_h(z_i)$.

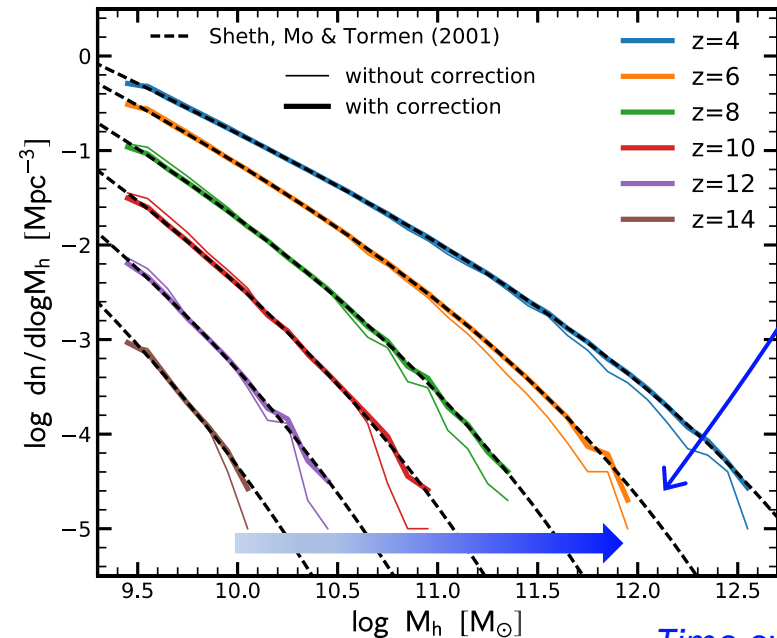
Both the black hole and the host halo grow such that

$$M_{BH}(z = 7) = M_{seed} e^{\frac{t(7) - t(z_i)}{t_{sal}}} \sim 10^9 M_\odot, \quad M_h(z = 7) \gtrsim 10^{12} M_\odot$$

concentration parameter (M_h, z)



halo mass function (z)



Time evolution
of the host halo mass

Caveats: Number changing interactions

As the core becomes extremely dense after the gravo-thermal collapse accelerates, the temperature of the core also increases as $\rho_c \propto T_c^\eta \propto (v_c)^{2\eta}$. This could provide nontrivial effects for the interaction rates of the dark matter inside the core.

For the glueball DM case, the number-changing interaction is important for $m_g \lesssim 0(\text{keV})$ before the collapse ($t < \Delta t_{\text{col}}$) \rightarrow generates additional heat to prevent the collapse

$$\Delta t_{\text{col}} \left(\frac{1}{T_g} \frac{dT_g}{dt} \right)_{3 \rightarrow 2} \simeq 0.06 \left(\frac{10^{-3}}{f_g} \right) \left(\frac{10^{-3}}{v_s(t_i)} \right)^2 \left(\frac{\text{keV}}{m_g} \right)^4 \left(\frac{\rho_s(t_i)}{10^{12} M_\odot / \text{kpc}^3} \right)$$

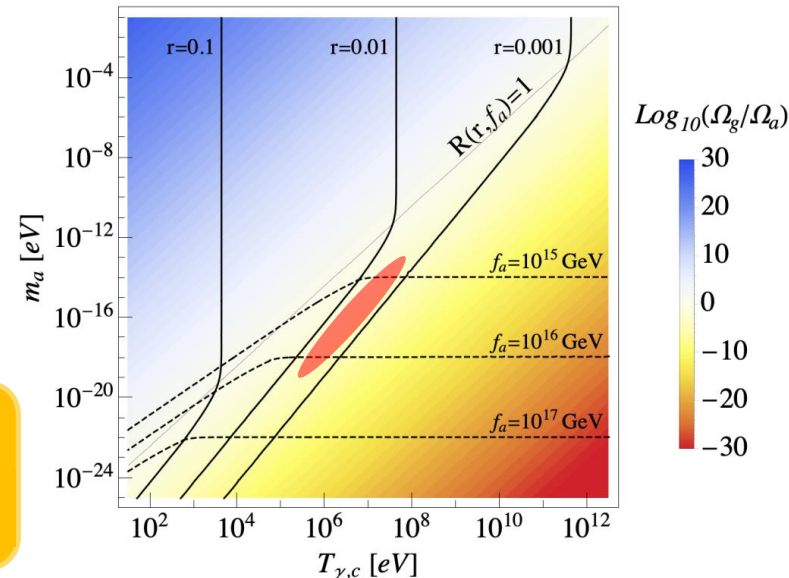
Even for $m_g \gg 0(\text{keV})$, it becomes gradually important ($\Gamma_{3 \rightarrow 2} \propto \rho_c^2$) during the gravo-thermal collapse inside the core. *What's the effect on the final formation of the BH?*

From the cosmological history, the mass of the glueball is also bounded by its free-streaming length.

For $m_g \lesssim 100 \text{ eV}$, the glueball DM will not form a halo at $z = 15 - 10$ because it is too warm

$$m_g \sim 0.01 - 1 \text{ MeV}$$

$$m_a \sim 10^{-18} - 10^{-14} \text{ eV}$$



Summary

The origin of the lightness of the scalar is directly related to its cosmological evolution.

Two well-known mechanisms provide opposite behavior for the relation between the interaction strength and the scalar mass.

We studied the (non-trivial) minimal model of the dark sector that comprises the coupled scalar dark matters: dark axion and dark glueball

Some nontrivial features are clarified.

Strongly interacting subcomponent glueball dark matter can provide a hint on the origin of supermassive black holes at high redshifts.