# Axion coupling hierarchies from axion landscape

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# *Outline*

- *Introduction*
- *Well-motivated axion coupling hierarchies*
- *Hierarchies from axion landscape*
- *Conclusion*

# *Introduction*

Axions or axion-like particles (ALP) are one of the most compelling candidates for BSM physics

- Axions may solve the naturalness problems

  * QCD axion for strong CP problem
  * Relaxion for gauge hierarchy problem
  * Inflaton for natural inflation

- Axions may constitute (part of) the dark side of the Universe

  Dark matter, Dark radiation, Dark energy

- Axions may explain certain astrophysical anomalies

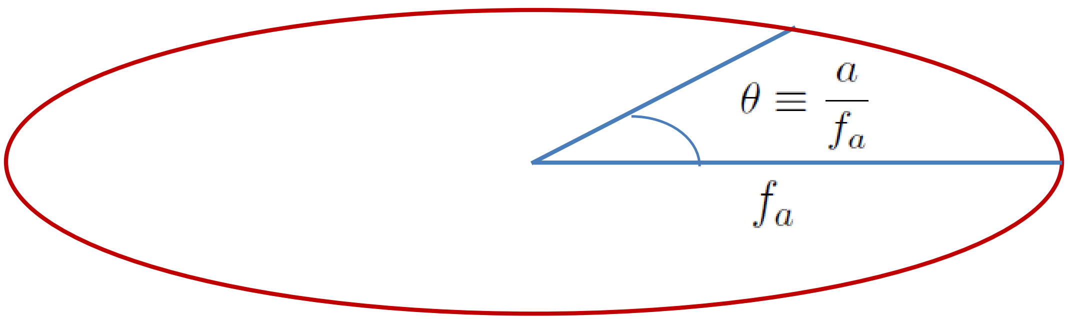
  White dwarf cooling anomaly, $\gamma$-ray transparency, $\gamma$-ray modulations, ..

- Axions arise naturally in compactified string theory

## *Axion couplings or scales*

Axions are periodic scalar field:

$a \cong a + 2\pi f_a$ ( $f_a$ = axion decay constant)




$$\text{Couplings} \propto \frac{1}{f_a}, \qquad \text{Scales} \propto f_a$$

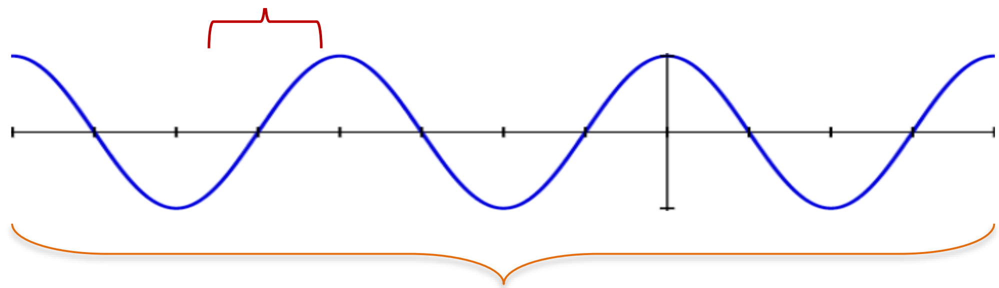
## *Axion couplings*

- Coupling to generate the leading potential: $g_{aV} = \dfrac{N_{\rm DW}}{f_a}$

$$V(a) \simeq \Lambda^4 \cos\left(N_{\rm DW}\frac{a}{f_a}\right)$$

\# of degenerate vacua
= domain wall number

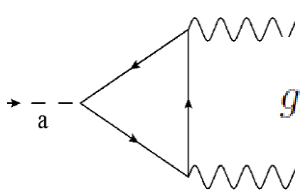
$$\Delta a = \frac{1}{g_{aV}} = \frac{f_a}{N_{\rm DW}}$$ = monotonic field range



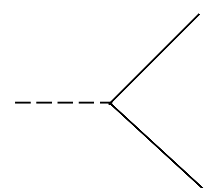
$2\pi f_a$ = full field range

- Couplings to the SM particles

$$\mathcal{L}_{\rm in} = \frac{1}{4}\sum_{A=\gamma,G} g_{aA} F^A \tilde{F}^A + \frac{1}{2}\sum_{\Psi=N,e} g_{a\Psi}\partial_\mu a \bar{\Psi}\gamma^\mu\gamma_5\Psi$$




$$g_{aA} = \frac{g_A^2}{8\pi^2}\frac{c_A}{f_a} \quad (A = \gamma, \text{gluons})$$



$$g_{a\Psi} = \frac{c_\Psi}{f_a} \quad (\Psi = N, e)$$

- Coupling suggested by *the weak gravity conjecture* (WGC): $g_{a\mathrm{WGC}} = \dfrac{N_{\mathrm{WGC}}}{f_a}$

  For axions in theories compatible with quantum gravity, there exist associated instantons whose couplings to axions are stronger than gravity.

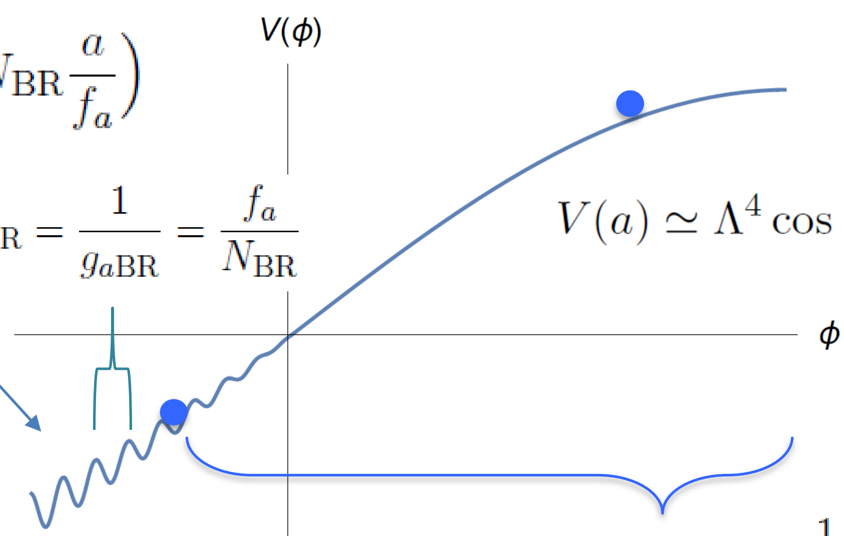
$$\mathcal{A}_{\mathrm{ins}} \propto \exp\left(-S_{\mathrm{ins}} + iN_{\mathrm{WGC}}\frac{a}{f_a}\right) \quad \left(g_{a\mathrm{WGC}} = \frac{N_{\mathrm{WGC}}}{f_a} \gtrsim \frac{S_{\mathrm{ins}}}{M_{\mathrm{Pl}}}\right)$$

- *Relaxion* coupling to generate the barrier potential: $g_{a\mathrm{BR}} = \dfrac{N_{\mathrm{BR}}}{f_a}$

  (= axion-like particle proposed to solve the weak scale hierarchy problem)

$$V_{\mathrm{BR}} = \Lambda_{\mathrm{BR}}^4(|H|)\cos\left(N_{\mathrm{BR}}\frac{a}{f_a}\right)$$

$$(\Delta a)_{\mathrm{BR}} = \frac{1}{g_{a\mathrm{BR}}} = \frac{f_a}{N_{\mathrm{BR}}}$$




$$V(a) \simeq \Lambda^4\cos\left(N_{\mathrm{DW}}\frac{a}{f_a}\right)$$

$$\Delta a = \frac{1}{g_{aV}} = \frac{f_a}{N_{\mathrm{DW}}}$$

There can be *technically natural* hierarchies among the axion couplings.

**Axion couplings:**

* Derivative couplings invariant under $U(1)_{PQ}$ : $a \rightarrow a + constant$

* PQ-breaking non-derivative couplings:

*Quantized* (integer-valued in the unit of $\frac{1}{f_a}$ ) to be invariant under $a \rightarrow a + 2\pi f_a$

➢ No quantum correction to the quantized axion couplings, so any hierarchy among the quantized PQ-breaking couplings are stable against quantum corrections:

$$\frac{2\pi}{\alpha_{\rm em}} g_{a\gamma} \gg g_{aV},\ \frac{2\pi}{\alpha_{\rm QCD}} g_{aG} \quad (c_\gamma \gg N_{\rm DW}, c_G) \quad \text{(Photophilic)}$$

$$g_{a{\rm WGC}} \gg g_{aV} \quad (N_{\rm WGC} \gg N_{\rm DW}) \quad \text{(Large axion field excursion)}$$

$$g_{a{\rm BR}} \gg g_{aV} \quad (N_{\rm BR} \gg N_{\rm DW}) \quad \text{(Relaxation of the Higgs boson mass)}$$

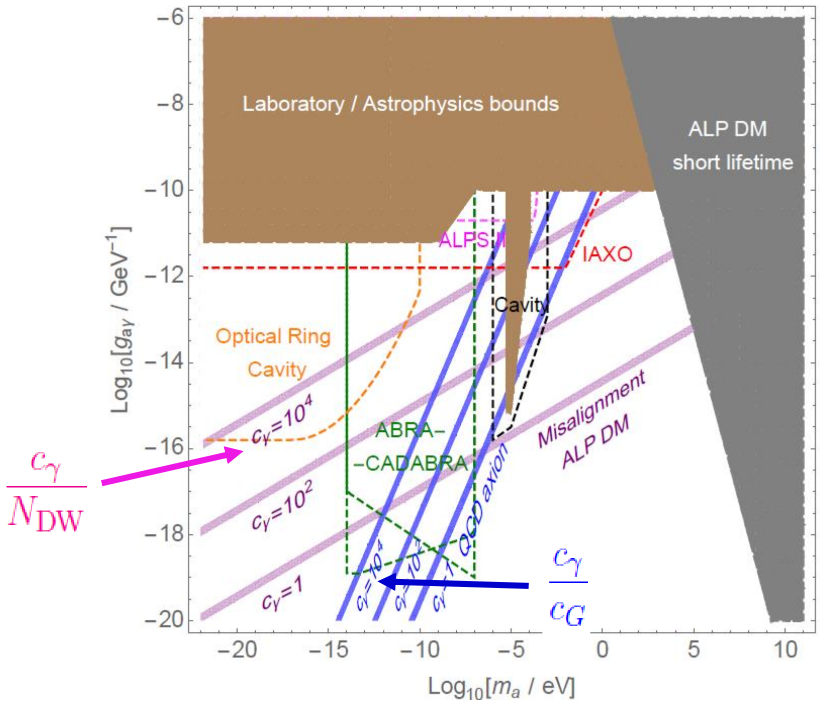
➢ PQ-invariant couplings >> PQ-breaking couplings are stable also.

$$g_{a\Psi} \gg \frac{2\pi}{\alpha_{\rm em}} g_{a\gamma} \quad (c_\Psi \gg c_\gamma) \quad \text{(Fermiophilic)}$$

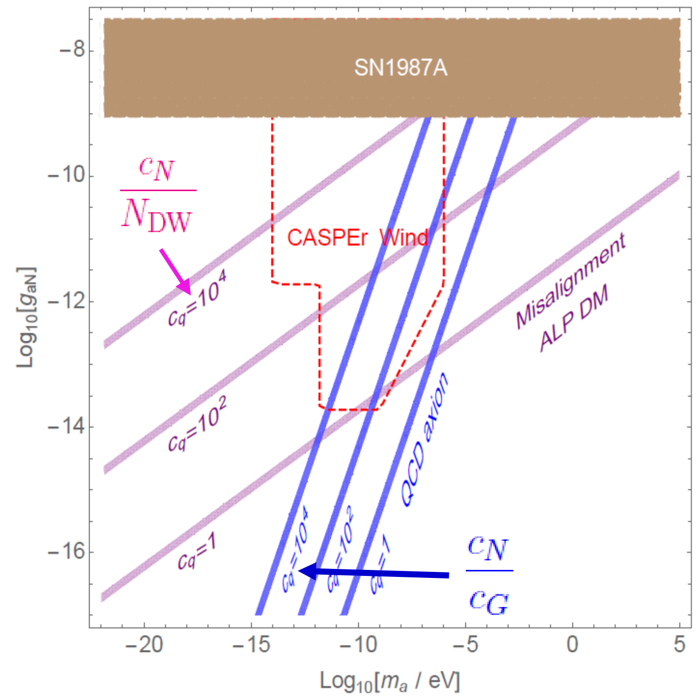
# *Well-motivated axion coupling hierarchies*

- Hierarchy for axion search experiments

Photophilic QCD axion
or ALP dark matter




Nucleophilic QCD axion
or ALP dark matter




Most easily accessible region is the one with hierarchical couplings!

- Hierarchy for relaxion couplings

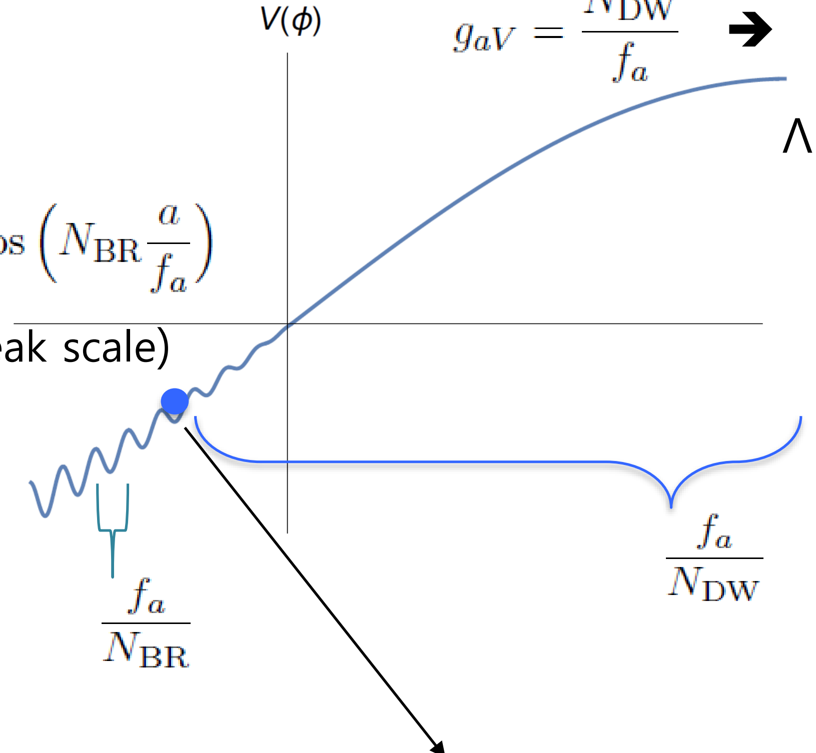
$$g_{aV} = \frac{N_{\rm DW}}{f_a} \quad \Rightarrow \quad V(a) \simeq \Lambda^4 \cos\left(N_{\rm DW}\frac{a}{f_a}\right)$$

$\Lambda$ = Higgs mass cutoff scale
>> Weak scale

$$g_{a\rm BR} = \frac{N_{\rm BR}}{f_a}$$

$$\Rightarrow \quad V_{\rm BR} = \Lambda_{\rm BR}^4(|H|)\cos\left(N_{\rm BR}\frac{a}{f_a}\right)$$

( $\Lambda_{\rm BR}$ < Weak scale)




$$\text{Slope of } V \sim \frac{N_{\rm DW}}{f_a}\Lambda^4 \sim \text{Slope of } V_{\rm BR} \sim \frac{N_{\rm BR}}{f_a}\Lambda_{\rm BR}^4$$

$$\Rightarrow \quad \frac{g_{a\rm BR}}{g_{aV}} = \frac{N_{\rm BR}}{N_{\rm DW}} \sim \frac{\Lambda^4}{\Lambda_{\rm BR}^4} \gtrsim \left(\frac{\text{Higgs mass cutoff scale}}{\text{Weak scale}}\right)^4 \gg 1$$

- Hierarchy for large field excursion

$$\Delta a = \frac{1}{g_{aV}} \sim \begin{cases} \sqrt{N_e} M_P & \text{(large field axion inflation)} \\ M_P & \text{(quintessence (DE) axion)} \\ \frac{1}{20} M_P \left(\frac{\Omega_a h^2}{0.1}\right)^{1/2} \left(\frac{10^{-22}\,\mathrm{eV}}{m_a}\right)^{1/4} & \text{(ultralight ALP dark matter)} \end{cases}$$

Axion weak gravity conjecture (WGC): $g_{a\mathrm{WGC}} = \dfrac{N_{\mathrm{WGC}}}{f_a} \gtrsim \dfrac{S_{\mathrm{ins}}}{M_P}$

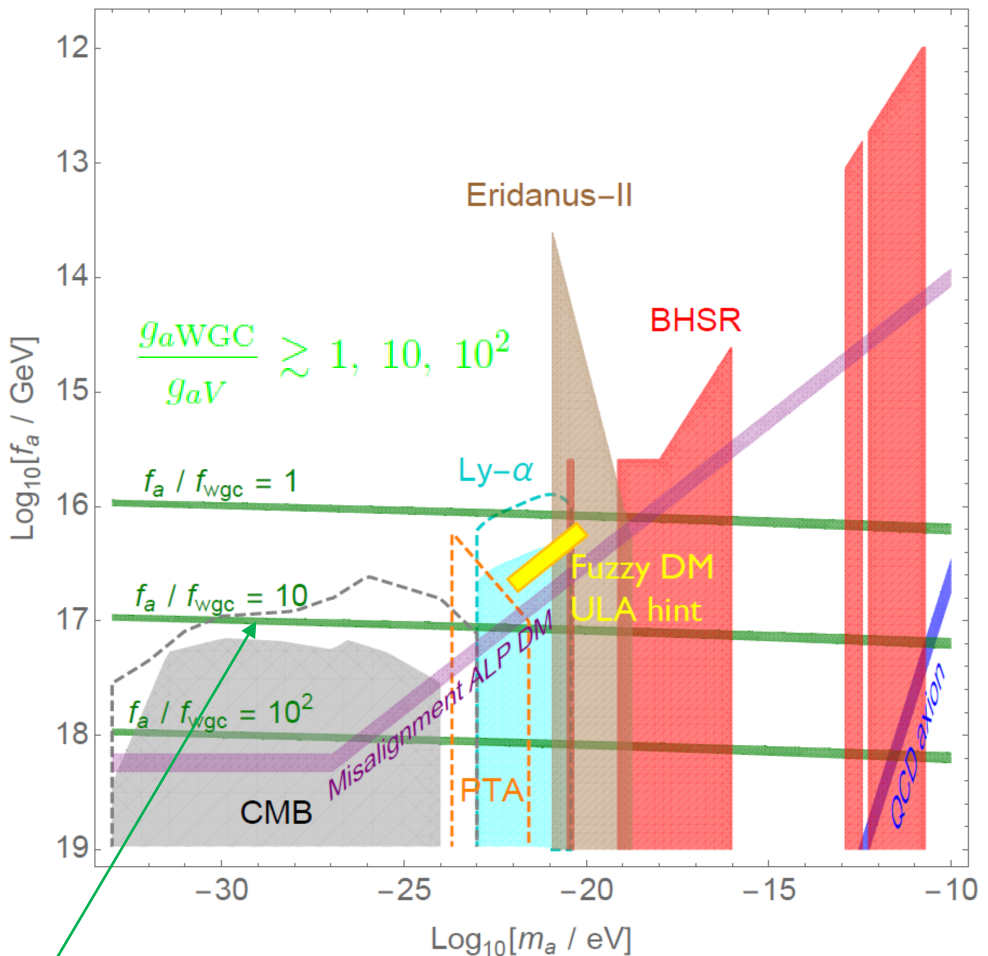
$$\rightarrow \frac{g_{a\mathrm{WGC}}}{g_{aV}} \gtrsim \frac{S_{\mathrm{ins}} \Delta a}{M_P} \sim \begin{cases} \sqrt{N_e} S_{\mathrm{ins}} & \text{(large field axion inflation)} \\ S_{\mathrm{ins}} & \text{(quintessence (DE) axion)} \\ \frac{S_{\mathrm{ins}}}{20} \left(\frac{\Omega_a h^2}{0.1}\right)^{1/2} \left(\frac{10^{-22}\mathrm{eV}}{m_a}\right)^{1/4} & \text{(ultralight ALP dark matter)} \end{cases}$$

Axion mass induced by the WGC instanton:

(assuming that it generate nonperturbative correction to the superpotental)

$$m_a^2 \gtrsim (\delta m_a^2)_{\mathrm{WGC}} \sim \frac{e^{-S_{\mathrm{ins}}} m_{3/2} M_P^3}{f_a^2} \quad \rightarrow \quad S_{\mathrm{ins}} \gtrsim \ln\left(\frac{m_{3/2} M_P}{m_a^2}\right)$$

# Gravitational probe of ultralight DE-like or DM axions




Future CMB and PTA observations will probe the parameter region with $\frac{g_{a\mathrm{WGC}}}{g_{aV}} \gtrsim 10$ .

## *Hierarchies from axion landscape*

In the landscape, usually river does not flow in straight line, but has multiple windings, making the excursion distance longer than the geodesic distance.

Models with multiple axions at UV scale,

$$\mathcal{L} = \frac{1}{2} f_{ij}^2 \partial_\mu \theta^i \partial^\mu \theta^j + \Lambda_X^4 \cos(q_i^X \theta^i + \delta^X) + \frac{k_{Ai}\theta^i}{32\pi^2} F^A \tilde{F}^A + c_{\psi i} \partial_\mu \theta^i \bar{\psi} \bar{\sigma}^\mu \psi + \cdots$$

(Summation over the axion flavors $i$, independent potentials $X$, gauge fields $A$, and fermions $\psi$)

$$\theta^i \cong \theta^i + 2\pi \ \ (i = 1, \ldots, N) \ \Rightarrow \ q_i^X, k_{Ai} \in \mathbb{Z}$$

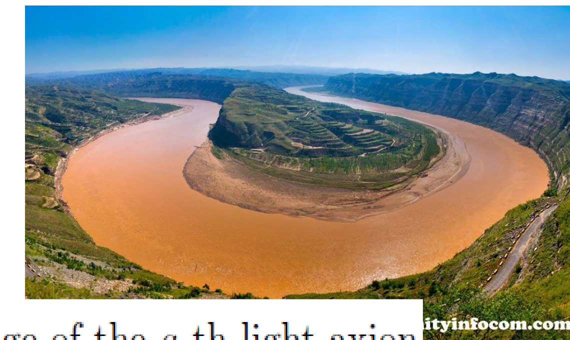
Generically different axion potentials have hierarchically different magnitudes as some (or all) of them are generated by nonperturbative effects, so exponentially suppressed.

Then, at a given energy scale of our interest, some axions are heavy enough to be integrated out, while others are light enough to be regarded as nearly massless.

$$\sum_X \Lambda_X^4 \cos(q_i^X \theta^i + \delta^X) = \underbrace{\sum_{\alpha=1}^{N_H} \Lambda_\alpha^4 \cos(q_i^\alpha \theta^i)}_{\text{Potentials for heavy axions}} + \underbrace{\sum_{a=1}^{N_L} \Lambda_a^4 \cos(q_i^a \theta^i)}_{\text{Leading potential of the light axions which correspond to the flat directions of the heavy axion potentials}} + \ldots \quad \Big(\Lambda_\alpha \gg \Lambda_a, \ \ N_H + N_L = N\Big)$$

Field space of light axions corresponds to the degenerate vacuum manifold of the heavy axion potential $V_H = \sum_\alpha \Lambda_\alpha^4 \cos(q_i^\alpha \theta^i)$ which defines the shape of the axion landscape.

Vacuum solution parametrized by the $N_L$ angles $\theta_L^a$ and the $N_L \times N$ integer-valued coefficients $\hat{n}_a^i$:



$$\theta^i = \sum_a \hat{n}_a^i \theta_L^a \quad \left(\theta_L^a \cong \theta_L^a + 2\pi,\ \ \hat{n}_a^i q_i^\alpha = 0,\ \ \text{g.c.d}(\hat{n}_a^i) = 1\right)$$

$\hat{n}_a^i = \langle \theta^i | \theta_L^a \rangle$ = Number of windings of $\theta^i$ over the full field range of the $a$-th light axion

= Wavefunction of the $a$-th light axion in the discrete axion flavor space labelled by $i$

Smith normal decomposition of the integer-valued $N \times N_H$ matrix:

$$q_i^\alpha = \hat{U}_\beta^\alpha q_\beta \hat{Q}_i^\beta \quad \Big(q_\alpha \in \mathbb{Z},\ \hat{U} = [\hat{U}_\beta^\alpha] \in GL(N_H, \mathbb{Z}),\ \hat{Q} = [\hat{Q}_i^\alpha, \hat{Q}_i^a] \in GL(N, \mathbb{Z})\Big)$$

$$\Rightarrow \quad \hat{Q}^{-1} = \left[\hat{n}_\alpha^i,\ \hat{n}_a^i\right]^T$$

Field range and effective couplings of the light axionsare crucially depend on the integer-valued vacuum solution coefficients $\hat{n}_a^i$ :

$$\mathcal{L}_{\rm eff} = \frac{1}{2}(f_L^2)_{ab}\partial_\mu\theta_L^a\partial^\mu\theta_L^b + \Lambda_a^4\cos(q_i^a\hat{n}_b^i\theta_L^b) + \frac{1}{32\pi^2}k_{Ai}\hat{n}_a^i\theta_L^a F^A\tilde{F}^A + c_{\psi i}\hat{n}_a^i\partial_\mu\theta_L^a\bar{\psi}\bar{\sigma}^\mu\psi + \cdots$$

$(f_L^2)_{ab} = \hat{n}_a^i f_{ij}^2 \hat{n}_b^j$ = Effective decay constant of light axions

Even when the underlying model parameters ($\ni q_i^\alpha, q_i^a, k_{Ai}, c_{\psi i}$) do not involve any hierarchy, there might be hierarchies among the effective scales and couplings induced by the hierarchical structure of $\hat{n}_a^i$.

This is generically true in the limit $N_H >> N_L = O(1)$:
(many-dimensional landscape)

$$\mathrm{Det}(\hat{n}_a\cdot\hat{n}_b) = \mathrm{Det}(\hat{Q}^\alpha\cdot\hat{Q}^\beta) \sim \mathcal{Q}^{2N_H} \quad \left(\mathcal{Q}^2 = \langle\sum_{\alpha=1}^{N_H}\frac{||\hat{Q}^\alpha||^2}{N_H}\rangle > 1,\ N_H \gg 1\right)$$

$\Rightarrow \quad ||n_a|| \sim \mathcal{Q}^{N_H/N_L} \quad \left(\mathcal{Q} > 1,\ N_H \gg N_L\right)$ (exponentially long solution vector)

➔ Generically exponentially enlarged field range:

$$f_L \sim f||\hat{n}_a|| \sim \mathcal{Q}^{N_H/N_L} f$$

Exponentially enlarged monotonic field range and exponential coupling hierarchies for certain discrete choice of model parameters:

$$\Delta a_L = \frac{f_L}{N_{\rm DW}(a_L)} = \frac{f_L}{\vec{q}^a \cdot \hat{n}_b}, \quad k_A(a_L) = \vec{k}_A \cdot \hat{n}_a, \quad c_\psi(a_L) = \vec{c}_\psi \cdot \hat{n}_a$$

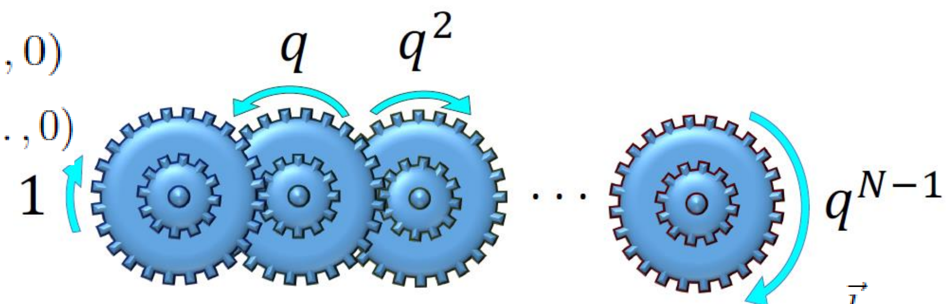
A particularly well-organized example to generate exponential scale and coupling hierarchies: **Clockwork axion model**

$$N_H \gg N_L = 1, \quad f_{ij}^2 = f_0^2 \delta_{ij}$$

$$\vec{q}^{\,\alpha=1} = (q,-1,0,\ldots,0), \quad \vec{q}^{\,\alpha=2} = (0,q,-1,0,\ldots,0), \ldots, \quad \vec{q}^{\,\alpha=N-1} = (0,\ldots,0,q,-1)$$

$$\vec{q}^{\,a} = (1,0,\ldots,0)$$

$$\vec{k}_{A=G} = (1,0,\ldots,0)$$




$$\vec{k}_{A=\gamma} = (0,\ldots,0,1)$$

$$\Rightarrow \quad \hat{n}_a = (1,q,q^2,\ldots,q^{N-1}) \quad (q \geq 2)$$

$$\Rightarrow \quad f_L = \Delta a_L = \sqrt{\frac{q^{2N}-1}{q^2-1}} f_0 \ \Big(N_{\rm DW}(a_L) = 1\Big), \qquad \frac{g_{a\gamma}}{g_{aG}} = \frac{e^2}{g_c^2} q^{N-1}$$

# Conclusion

- There are a variety of well-motivated axion coupling hierarchies.

- Axion landscape can result in such hierarchical axion couplings in low energy limit without introducing any hierarchy of parameters in the UV theory.