Phenomenological constraints on the mass of family-dependent extra U(1) gauge boson

Bumseok KYAE

Collaboration with K.S. Choi (이화여대) and M.-S. Seo (교원대)

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Anomaly-free U(1) extensions

$$U(1)_{L_i - L_j} \ , \qquad U(1)_{B_i - L_i} \ ,$$

- and their linear combinations.
- Here L and B indicate the lepton and baryon numbers,
- i, j (=1,2,3, i≠j) are family (or generation) indices.
- For example, the $U(1)_{L1\text{-}L2}$ gauge charges are

Fields	$(\nu_e, e)_L,\ \nu_{eR},\ e_R;\ L,\ E^c$	$(\nu_\mu, \mu)_L,\ \nu_{\mu R},\ \mu_R;\ L^c,\ E$	$(\nu_\tau, \tau)_L,\ \nu_{\tau R},\ \tau_R,\ H$, quarks	ϕ	$\tilde{\phi}$
$U(1)_{L_1 - L_2}$	$+q$	$-q$	0	$+2q$	$+q$

Anomaly-free U(1) extensions

$$U(1)_{L_i - L_j}\ , \qquad U(1)_{B_i - L_i}\ ,$$

- **and their linear combinations.**

Since the new interactions through the new gauge boson should be suppressed than the well-known interaction by the SM gauge boson, the new gauge couplings (g_X) and gauge boson mass (M_g) must be constrained as

$$\frac{Q_\psi^2 g_X^2}{M_g^2} \ll \frac{g_2^2}{M_Z^2} \approx (140\,\mathrm{GeV})^{-2}, \tag{2}$$

where Q_ψ denotes the charges of the SM fermions under an extra $\mathrm{U}(1)_X$ gauge symmetry.

$U(1)_{L1\text{-}L2}$ and FCNC

In particular, $U(1)_{L_i-L_j}$s [$U(1)_{B_i-L_i}$s] admit only the [block-] diagonal forms of the Yukawa mass matrices for the SM chiral fermions. Actually non-zero off-diagonal components in the Yukawa textures are indispensable for the flavor mixing effects observed in weak interaction mediated by the W bosons. Moreover, the mixing angles in the lepton sector are known to be almost maximal [1].

- For example, the $U(1)_{L1\text{-}L2}$ gauge charges are

Fields	$(\nu_e, e)_L$, ν_{eR}, e_R; L, E^c	$(\nu_\mu, \mu)_L$, $\nu_{\mu R}$, μ_R; L^c, E	$(\nu_\tau, \tau)_L$, $\nu_{\tau R}$, τ_R, H, quarks	ϕ	$\tilde{\phi}$
$U(1)_{L_1-L_2}$	$+q$	$-q$	0	$+2q$	$+q$

$U(1)_{L1-L2}$ and FCNC

For the non-zero off-diagonal components in the Yukawa textures, the surplus $\mathrm{U}(1)_X$ charges in the required operators need to be compensated properly by a scalar field ϕ, and a non-zero VEV of ϕ should be developed, breaking the $\mathrm{U}(1)_X$. Let us discuss the following form of the non-renormalizable terms in the Lagrangian with $\mathrm{U}(1)_{L_1-L_2}$:

- **Here L and B indicate the lepton and baryon numbers,**
- **i, j (i≠j) are family (or generation) indices.**
- **For example, the $U(1)_{L1-L2}$ gauge charges are**

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$\mathrm{U}(1)_{L_1-L_2}$	$+q$	$-q$	0	$+2q$	$+q$

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$$-\mathcal{L} \supset \kappa_{12}\frac{\phi}{\Lambda_{\rm cut}}\overline{e_R}\mu_L H \;+\; \kappa_{21}\frac{\phi^*}{\Lambda_{\rm cut}}\overline{\mu_R}e_L H \;+\; \text{h. c.}\;,$$

$$-\mathcal{L} \supset \kappa_{13}\frac{\tilde\phi}{\Lambda_{\rm cut}}\overline{e_R}\tau_L H + \kappa_{31}\frac{\tilde\phi^*}{\Lambda_{\rm cut}}\overline{\tau_R}e_L H + \kappa_{23}\frac{\tilde\phi^*}{\Lambda_{\rm cut}}\overline{\mu_R}\tau_L H + \kappa_{32}\frac{\tilde\phi}{\Lambda_{\rm cut}}\overline{\tau_R}\mu_L H \;+\; \text{h. c.}\;,$$

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$U(1)_{L_1-L_2}$	$+q$	$-q$	0	$+2q$	$+q$

$U(1)_{L1\text{-}L2}$ and FCNC

$$V_\phi = -m_\phi^2|\phi|^2 - m_{\tilde{\phi}}^2|\tilde{\phi}|^2 + \frac{\lambda}{2}|\phi|^4 + \frac{\tilde{\lambda}}{2}|\tilde{\phi}|^4 + \cdots$$

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Fields	$(\nu_e, e)_L$, ν_{eR}, e_R; L, E^c	$(\nu_\mu, \mu)_L$, $\nu_{\mu R}$, μ_R; L^c, E	$(\nu_\tau, \tau)_L$, $\nu_{\tau R}$, τ_R, H, quarks	ϕ	$\tilde{\phi}$
$U(1)_{L_1-L_2}$	$+q$	$-q$	0	$+2q$	$+q$

$$
\begin{aligned}
& -\mathcal{L}_1 \supset \kappa_e\, \overline{e_R} L_L\, H + \kappa_\mu\, \overline{L_R} \mu_L\, \phi + M_V \overline{L_L} L_R \ + \ \text{h. c.}, \\
\text{or} \quad & -\mathcal{L}_2 \supset \kappa_e\, \overline{e_R} E_L\, \phi + \kappa_\mu\, \overline{E_R} \mu_L\, H + M_V \overline{E_L} E_R \ + \ \text{h. c.}, \\
\text{and} \quad & -\mathcal{L}_3 \supset \kappa_\tau\, \overline{\tau_R} E_L\, \tilde{\phi} + \kappa'_\tau\, \overline{L_R} \tau_L\, \tilde{\phi} \ + \ \text{h. c.},
\end{aligned}
$$

a non-zero VEV of ϕ should be developed, breaking the $U(1)_X$. Let us discuss the following

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$$
-\mathcal{L} \supset \kappa_{12} \frac{\phi}{\Lambda_{\text{cut}}} \overline{e_R} \mu_L H \ + \ \kappa_{21} \frac{\phi^*}{\Lambda_{\text{cut}}} \overline{\mu_R} e_L H \ + \ \text{h. c.} \ ,
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•

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$$

Fields	$(\nu_e, e)_L$, ν_{eR}, e_R; L, E^c	$(\nu_\mu, \mu)_L$, $\nu_{\mu R}$, μ_R; L^c, E	$(\nu_\tau, \tau)_L$, $\nu_{\tau R}$, τ_R, H, quarks	ϕ	$\tilde{\phi}$
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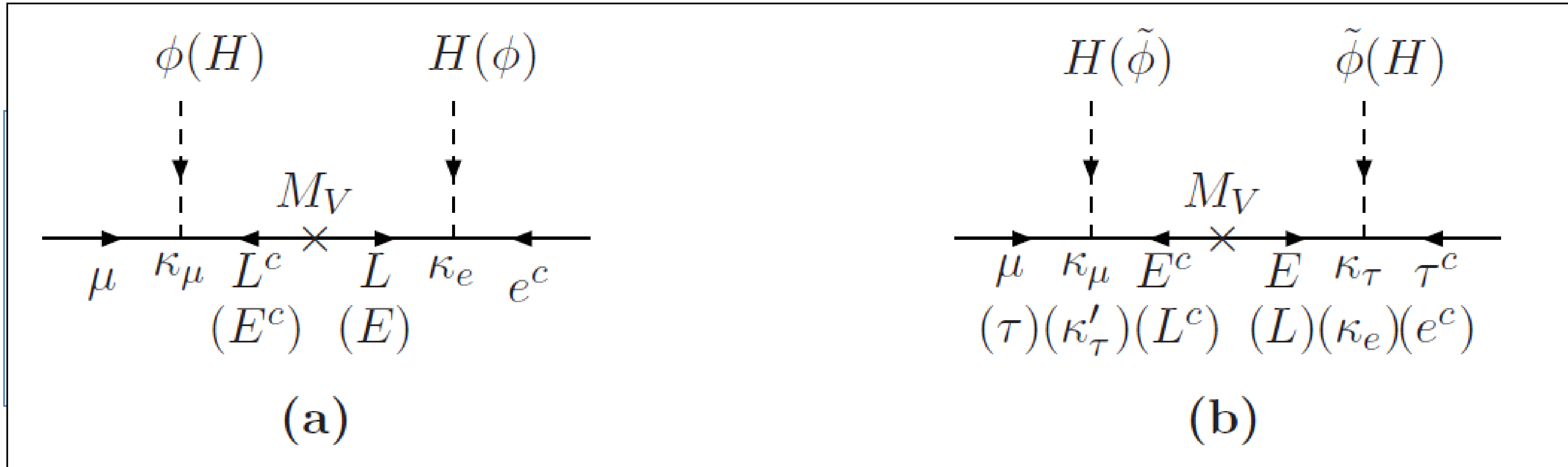

for ϕ ϕ^*

$$\kappa_{12} = \kappa_e \kappa_\mu \ , \quad \kappa_{13} = \kappa_e \kappa'_\tau \ , \quad \kappa_{32} = \kappa_\tau \kappa_\mu, \quad \text{and} \quad \Lambda_{\text{cut}} = M_V .$$

$\tilde{\phi}$ $\tilde{\phi}^*$ $\tilde{\phi}^*$ $\tilde{\phi}$

From the heavy charged lepton search experiments,

$$M_V \ > \ 100.8 \text{ GeV}.$$

$\mathrm{U}(1)_{L_1-L_2}$	$+q$	$-q$

$U(1)_{L1-L2}$ and FCNC

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$$V_\phi = -m_\phi^2|\phi|^2 - m_{\tilde\phi}^2|\tilde\phi|^2 + \frac{\lambda}{2}|\phi|^4 + \frac{\tilde\lambda}{2}|\tilde\phi|^4 + \cdots$$

$$-\mathcal{L} \supset \kappa_{12}\frac{\phi}{\Lambda_{\rm cut}}\overline{e_R}\mu_L H \; + \; \kappa_{21}\frac{\phi^*}{\Lambda_{\rm cut}}\overline{\mu_R}e_L H \; + \; {\rm h.\,c.}\,,$$

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$$\frac{\kappa_{12}\langle\phi\rangle}{\Lambda_{\rm cut}} \sim \mathcal{O}(10^{-3})$$

$$\frac{\kappa_{32}\langle\tilde\phi\rangle}{\Lambda_{\rm cut}} \sim \mathcal{O}(10^{-2})$$

U(1)$_{L1\text{-}L2}$ and FCNC

Once ϕ gets a non-zero VEV,

$$\left[\frac{\kappa_{12}\langle\phi\rangle}{\Lambda_{\rm cut}}\right]\times\overline{e_R}\,\mu_L H \quad + \quad \left[\frac{\kappa_{12}H}{\Lambda_{\rm cut}}\right]\times\delta\phi\,\overline{e_R}\,\mu_L$$

After diagonalization of the mass matrices,

$$-\mathcal{L}_{\rm int.}\supset\kappa'_{ii}\left[\frac{H}{\Lambda_{\rm cut}}\right]\delta\phi\overline{e'_{iR}}e'_{iL}+\kappa'_{ij}\left[\frac{H}{\Lambda_{\rm cut}}\right]\delta\phi\overline{e'_{iR}}e'_{jL},$$

which play the role of the sources of FCNC.   For small enough κ's,

$$\boxed{(1)\ |y_{ii}|\ll|\kappa_{ij}|\ \text{or}\ (2)\ y_{11}\approx y_{22}\approx y_{33}}$$

$U(1)_{L1\text{-}L2}$ and FCNC

Once ϕ gets a non-zero VEV,

$$\left[\frac{\kappa_{12}\langle\phi\rangle}{\Lambda_{\rm cut}}\right] \times \overline{e_R}\,\mu_L H \quad + \quad \left[\frac{\kappa_{12} H}{\Lambda_{\rm cut}}\right] \times \delta\phi\,\overline{e_R}\,\mu_L$$

contributes to diagonalization of the mass matrix together with the diagonal components

Not completely diagonalized in the diagonal basis !

After diagonalization of the mass matrices,

$$-\mathcal{L}_{\rm int.} \supset \kappa'_{ii}\left[\frac{H}{\Lambda_{\rm cut}}\right]\delta\phi\overline{e'_{iR}}e'_{iL} + \kappa'_{ij}\left[\frac{H}{\Lambda_{\rm cut}}\right]\delta\phi\overline{e'_{iR}}e'_{jL},$$

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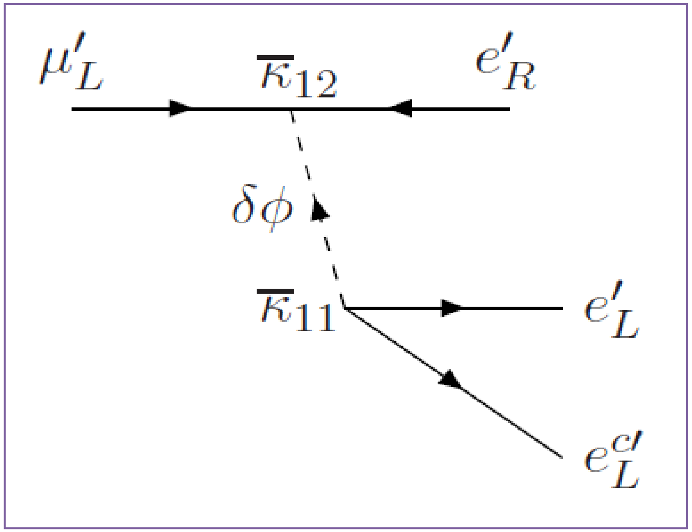
$\mu^- \to e^- e^- e^+$



$$-\mathcal{L}_{\text{eff}} = \frac{\overline{\kappa}_{11}\overline{\kappa}_{12}}{m_\phi^2} \left(\overline{e}P_R e\right)\left(\overline{e}P_L\mu\right)$$

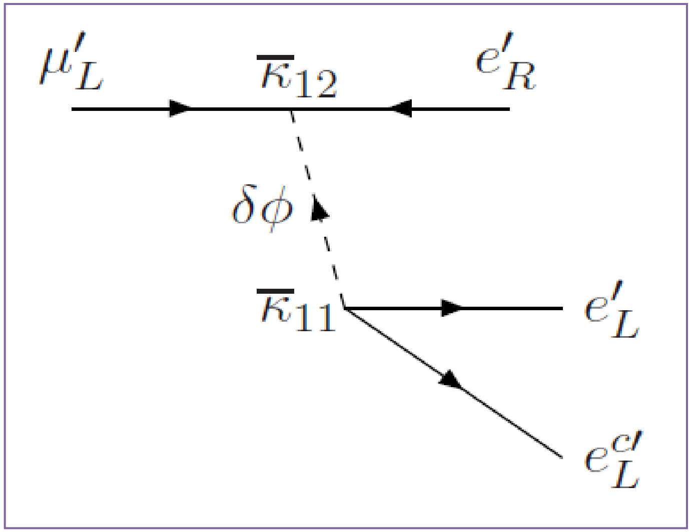
where $\overline{\kappa}_{ij} \equiv \kappa'_{ij} \times \dfrac{\langle H\rangle}{\Lambda_{\text{cut}}}$

$$\frac{\Gamma_{\mu\to ee\overline{e}}}{\Gamma_{\text{tot}}} \approx \frac{2\left|\overline{\kappa}_{11}\overline{\kappa}_{12}\right|^2}{32 m_\phi^4 G_F^2} \approx \frac{\left|\overline{\kappa}_{11}\overline{\kappa}_{12}\right|^2}{8\pi^2\alpha_2^2} \times \frac{M_W^4}{m_\phi^4} \approx \frac{\left|\kappa'_{11}\kappa'_{12}\right|^2}{8\pi^2\alpha_2^2} \times \left[\frac{\langle H\rangle}{\Lambda_{\text{cut}}}\right]^4 \left[\frac{M_W}{m_\phi}\right]^4$$

$$\approx 0.5 \times 10^{-12} \times \frac{\left|\kappa'_{11}\kappa'_{12}\right|^2 \lambda^2}{\kappa_{12}^4} \times \left[\frac{\langle H\rangle}{m_\phi}\right]^8 ,$$

which should be smaller than the Exp. Bound, 1.0×10^{-12}.

$\mu^- \to e^- e^- e^+$



$$m_\phi \gtrsim 0.9 \times \frac{|\kappa'_{11}\kappa'_{12}\lambda|^{1/4}}{|\kappa_{12}|^{1/2}} \times \langle H \rangle,$$

$$M_g \gtrsim Q_\phi g_X \langle \phi \rangle = \frac{Q_\phi g_X}{\sqrt{\lambda}} \times m_\phi$$

$$\frac{\Gamma_{\mu\to ee\bar{e}}}{\Gamma_{\mathrm{tot}}} \approx \frac{2\left|\overline{\kappa}_{11}\overline{\kappa}_{12}\right|^2}{32 m_\phi^4 G_F^2} \approx \frac{\left|\overline{\kappa}_{11}\overline{\kappa}_{12}\right|^2}{8\pi^2\alpha_2^2} \times \frac{M_W^4}{m_\phi^4} \approx \frac{\left|\kappa'_{11}\kappa'_{12}\right|^2}{8\pi^2\alpha_2^2} \times \left[\frac{\langle H \rangle}{\Lambda_{\mathrm{cut}}}\right]^4 \left[\frac{M_W}{m_\phi}\right]^4$$

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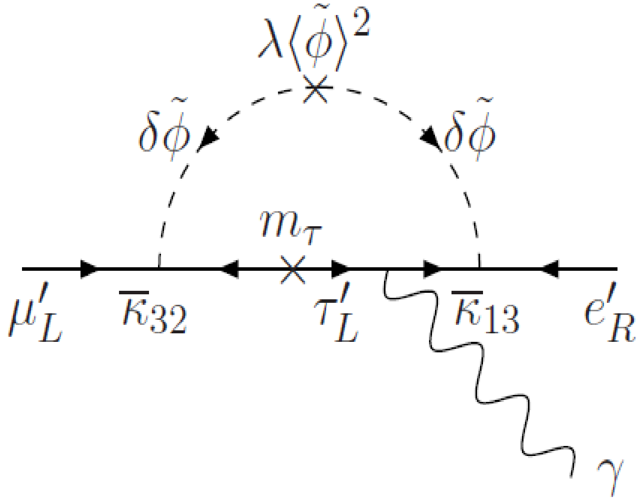
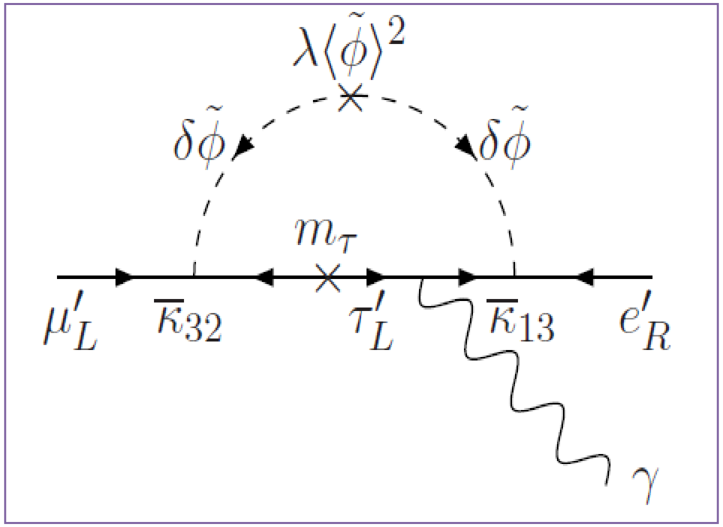
$\mu \to e\,\gamma$



$$\Gamma_{\mu\to e\gamma} \approx \frac{\alpha_2 \sin^2\theta_W}{128\pi^4}\left(\overline{\kappa}_{13}\overline{\kappa}_{32}\right)^2 \times \frac{m_\mu^3 m_\tau^2}{m_{\tilde{\phi}}^4}\left[\frac{\lambda\langle\tilde{\phi}\rangle^2}{m_{\tilde{\phi}}^2}\right]^2$$

where $\overline{\kappa}_{ij} \equiv \kappa'_{ij} \times \dfrac{\langle H\rangle}{\Lambda_{\mathrm{cut}}}$

μ → e γ



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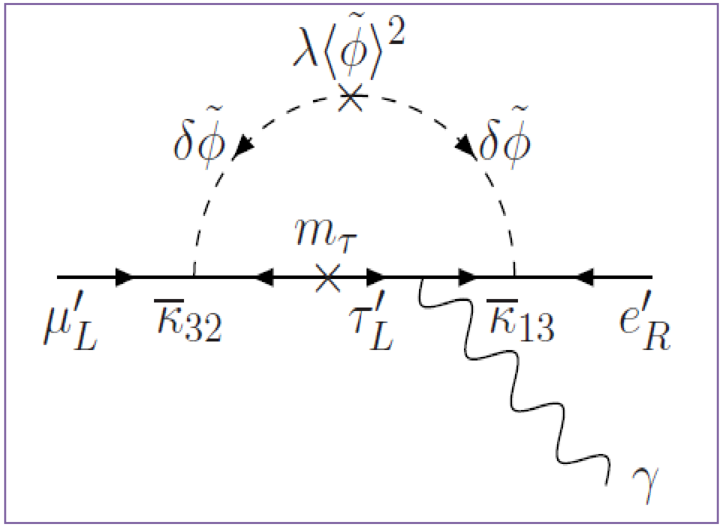
where $\overline{\kappa}_{ij} \equiv \kappa'_{ij} \times \frac{\langle H\rangle}{\Lambda_{\text{cut}}}$

It is compared with the total decay rate,

$$\Gamma_{\text{tot}} \approx \Gamma_{\mu\to e\nu\bar{\nu}} = \frac{G_F^2 m_\mu^5}{192\pi^3} = \frac{\alpha_2^2}{384\pi}\frac{m_\mu^5}{M_W^4}.$$

μ → e γ



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where $$\overline{\kappa}_{ij} \equiv \kappa'_{ij} \times \frac{\langle H\rangle}{\Lambda_{\text{cut}}}$$

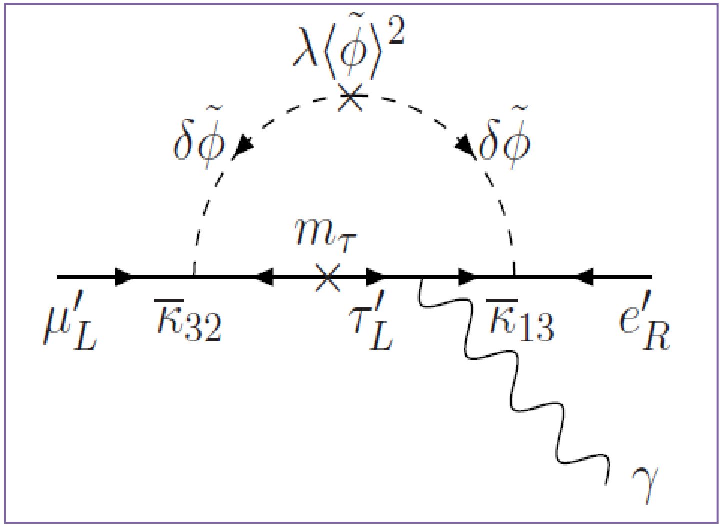
$$\mathrm{Br}(\mu^- \to e^-\gamma) \approx 2.7 \times (\kappa'_{13}\kappa'_{32})^2 \left[\frac{\langle H\rangle}{\Lambda_{\text{cut}}}\right]^4 \left[\frac{m_\tau}{m_\mu}\right]^2 \left[\frac{M_W^2}{m_{\tilde{\phi}}^2}\right]^2 \left[\frac{\lambda\langle\tilde{\phi}\rangle^2}{m_{\tilde{\phi}}^2}\right]^2$$

$$\approx 3.4 \times 10^{-7} \times \frac{(\kappa'_{13}\kappa'_{32})^2\,\lambda^2}{\kappa_{32}^4} \left[\frac{\langle H\rangle^2}{m_{\tilde{\phi}}^2}\right]^4 < \; 4.2 \times 10^{-13},$$

(experimental bound)

$$\text{or} \qquad m_{\tilde{\phi}} \; > \; 5.5 \times \frac{|\kappa'_{13}\kappa'_{32}\lambda|^{1/4}}{|\kappa_{32}|^{1/2}} \times \langle H\rangle$$

$\mu \to e\,\gamma$



$$M_g \gtrsim \frac{Q_{\tilde{\phi}}\; g_X}{\sqrt{\lambda}} \times \mathcal{O}(1\ \mathrm{TeV}).$$

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$$\text{or} \qquad m_{\tilde{\phi}} \ > \ 5.5 \times \frac{|\kappa'_{13}\kappa'_{32}\lambda|^{1/4}}{|\kappa_{32}|^{1/2}} \times \langle H\rangle$$

Conclusion

- Family-dependent extra U(1) (gauge) symmetries constrain the forms of Yukawa textures.

- For mixing effects observed in weak interactions. such U(1) symmetries need to be broken at the weak or higher energy scales.

- Otherwise, unwanted phenomena such as FCNC could arise.

- The extra U(1) gauge bosons should typically be heavier than 1TeV , but can be lighter by some tunings.

Thank you!