

Neutrino Oscillations in Dark Matter

In collaboration with Ki-Young Choi (SKKU) & Jongkuk Kim (KIAS),
arXiv:1909.10478 & Work in progress

Eung Jin Chun



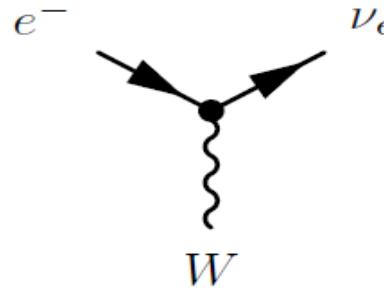
Outline

- Neutrino oscillations in vacuum
 - Flavors and masses; mixing and flavor change
- Neutrino oscillations in matter:
 - Wolfenstein potential; dispersion relations of neutrinos
- Neutrino oscillations in dark matter
 - Medium-induced mass and “Weyl” neutrino oscillations; Implications
- Discussion and conclusion

Neutrino Oscillations in Vacuum

Flavors and Masses

- Flavored neutrinos: Weak interaction eigenstates

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$$


$$\frac{g}{\sqrt{2}} \bar{l}_{\alpha L} \gamma^\mu \nu_{\alpha L} W_\mu^- + h.c.$$

- Neutrino masses: Majorana (LNV) Dirac (LNC)

$$\begin{matrix} \nu_1 & \nu_2 & \nu_3 \\ m_1 & m_2 & m_3 \end{matrix}$$

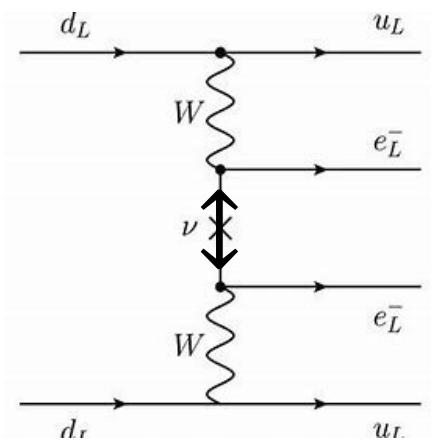
$$\nu^c = \nu \ (\nu_R \sim \nu_L^*)$$

$$\nu^c = C \bar{\nu}^T, \quad C = i\gamma_0\gamma_2$$

$$\frac{1}{2} m_i \left(\overline{\nu_{iR}^c} \nu_{iL} + \overline{\nu_{iL}} \nu_{iR}^c \right) \quad m_i \left(\overline{\nu_{iR}} \nu_{iL} + \overline{\nu_{iL}} \nu_{iR} \right)$$

$$= \frac{1}{2} m_i \left(\nu_{iL}^T C \nu_{iL} + \nu_{iL}^{*T} C^+ \nu_{iL}^* \right)$$

Majorana $\rightarrow 0\nu\beta\beta$



Mixing and flavor change

- Weak eigenstates \neq Mass eigenstates

$$\nu_\alpha = U_{\alpha i} \nu_i \quad U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & -s_{23} \\ 0 & s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & -s_{12} & 0 \\ s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot P_M$$

$$P_M = \text{Diag}[1, e^{i\varphi_2}, e^{i\varphi_2}]$$

- Two-flavor neutrino propagation in vacuum

$\nu_e \rightarrow \nu_\mu$ $U = \begin{bmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{bmatrix}$	$ \nu_e(0)\rangle = c_\theta \nu_1\rangle + s_\theta \nu_2\rangle$ $ \nu_e(t)\rangle = c_\theta e^{i\phi_1} \nu_1\rangle + s_\theta e^{i\phi_2} \nu_2\rangle$	Ultra-relativistic limit: $t \approx L$ $\phi_i = E_i t - p \cdot L$ $P_{e\mu} = \langle \nu_\mu \nu_e(t) \rangle ^2 = \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2 L}{4E} \right)$
----------------------------------------------------------------------------------------------------------------	----------------------------------------------------------------------------------------------------------------------------------------------------------------------	-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------

Neutrino evolution equation

- Propagation Hamiltonian:

$$i \frac{d}{dt} \psi = H \psi$$

$$\psi = (\nu_1, \nu_2, \nu_3)^T$$

$$H = \frac{m^2}{2E}$$

$$\psi = (\nu_e, \nu_\mu, \nu_\tau)^T$$

$$H_\nu = \frac{M^+ M}{2E} = \frac{U^+ m^2 U}{2E}$$

$$H_{\bar{\nu}} = \frac{M M^+}{2E} = \frac{U m^2 U^+}{2E} \left(\frac{V m^2 V^+}{2E} \right)$$

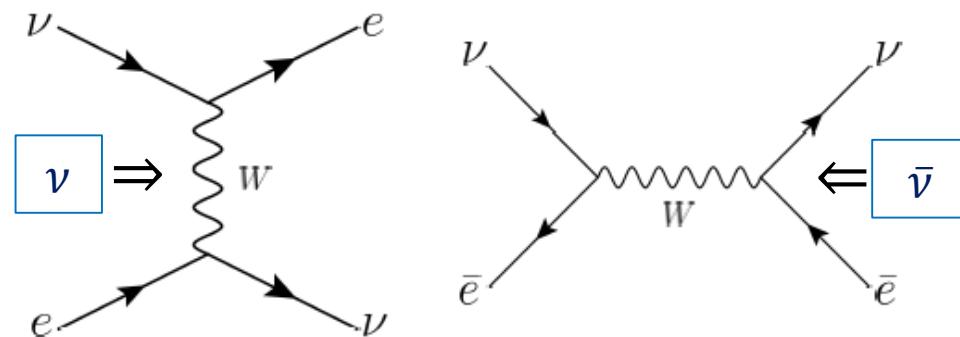
- Two-flavor evolution:

$$H = \frac{\Delta m^2}{4E} \begin{bmatrix} -c_{2\theta} & s_{2\theta} \\ s_{2\theta} & c_{2\theta} \end{bmatrix} \rightarrow P_{e\mu} = \left| \langle \nu_\mu | e^{iHt} | \nu_e \rangle \right|^2$$

Neutrino Oscillations in Matter

Wolfenstein potential

- Wolfenstein 1978: "Coherent forward scattering of neutrinos leaving the medium unchanged must be taken into account."



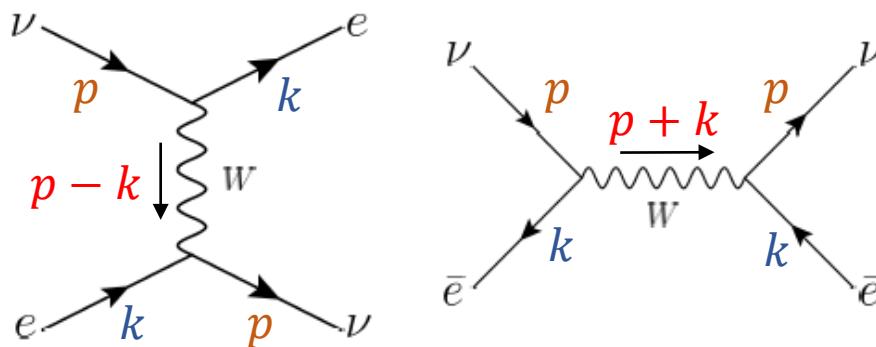
$$\mathcal{H}_{eff} = 2\sqrt{2} G_F \overline{\nu_{eL}} \gamma^\mu e_L \overline{e_L} \gamma_\mu \nu_{eL} \Rightarrow V_W \overline{\nu_{eL}} \gamma^0 \nu_{eL} \quad V_W = \sqrt{2} G_F N_e$$

- Neutrino evolutions in matter:

$$H_{\nu/\bar{\nu}} = \frac{M^2}{2E} \pm V_W$$

Generalized Wolfenstein potential

- In a medium with arbitrary N_e and $N_{\bar{e}}$



$$\mathcal{H}_{eff} = 2\sqrt{2} G_F m_W^2 \frac{\overline{\nu}_{eL} \gamma^\mu e_L \overline{e}_L \gamma_\mu \nu_{eL}}{m_W^2 - q^2}$$

$$\langle \mathcal{H}_\nu \rangle = \not{k} \sqrt{2} G_F m_W^2 \left[\frac{N_e/m_e}{m_W^2 - (p - k)^2} - \frac{N_{\bar{e}}/m_e}{m_W^2 - (p + k)^2} \right]$$

$$\langle \mathcal{H}_{\bar{\nu}} \rangle = \not{k} \sqrt{2} G_F m_W^2 \left[\frac{N_{\bar{e}}/m_e}{m_W^2 - (p - k)^2} - \frac{N_e/m_e}{m_W^2 - (p + k)^2} \right]$$

$$\begin{aligned} \langle N_e, N_{\bar{e}} | e \bar{e} | N_e, N_{\bar{e}} \rangle &= \\ -\frac{1}{2} \sum_s u_s(k) \bar{u}_s(k) \frac{N_e}{2k^0} \\ + \frac{1}{2} \sum_s v_s(k) \bar{v}_s(k) \frac{N_{\bar{e}}}{2k^0} \end{aligned}$$

Medium-dressed spinors

- Self-energy correction to neutrinos

$$\mathcal{L}_{kin} \Rightarrow \overline{\nu_L} \left(p^\mu - g_\nu^2 \frac{\langle \bar{e}_L \gamma^\mu e_L \rangle}{m_W^2 - q^2} \right) \gamma_\mu \nu_L \equiv \overline{\nu_L} (p^\mu - k^\mu \Sigma_W) \gamma_\mu \nu_L$$

$$\Sigma_W = g_\nu^2 \frac{N_{e+\bar{e}}}{m_e} \frac{-2m_e E + \epsilon(m_W^2 - p^2 - m_e^2)}{(m_W^2 - p^2 - m_e^2)^2 - 4m_e^2 E^2} \quad \epsilon \equiv \frac{N_e - N_{\bar{e}}}{N_e + N_{\bar{e}}} \quad p^\mu = (E, \vec{p}) \\ k^\mu = (m_e, \vec{0})$$

- Massless spinors satisfy

$$(p - k\Sigma_W)^2 = (E - m_e \Sigma_W)^2 - \mathbf{p}^2 = 0 \xrightarrow{\pm \epsilon \text{ for } \nu/\bar{\nu}} E = m_e \Sigma_W \pm \mathbf{p}$$

Dispersion relations

- Wolfenstein potential for $m_W^2 \gg p^2, N_{e+\bar{e}}/m_e$,

$$E_{\nu/\bar{\nu}} = p \pm \epsilon g_\nu^2 \frac{N_{e+\bar{e}}}{m_W^2} + \epsilon^2 g_\nu^2 \frac{N_{e+\bar{e}}^2/m_W^4}{p} + \dots$$

- Medium-induced neutrino oscillation for $2m_e p \gg m_W^2$,

$$E_{\nu/\bar{\nu}} = p + g_\nu^2 \frac{N_{e+\bar{e}}/m_e}{2p} \mp \epsilon g_\nu^2 \frac{(N_{e+\bar{e}}/m_e^2)m_W^2}{4p^2} + \dots$$

Neutrino Oscillations in Dark Matter

Generalized medium

- Variant models of dark matter and mediator

$$\mathcal{L}' = g_{\alpha i} \overline{f_{iL}} \gamma^\mu \nu_{\alpha L} X_\mu + h.c. \quad (1)$$

$$g_{\alpha i} \overline{f_R} \nu_{\alpha L} \phi_i + h.c. \quad (2)$$

$$g_{\alpha i} \overline{f_{iR}} \nu_{\alpha L} \phi + h.c. \quad (3)$$

$$g_{\alpha\beta} \overline{\nu_{\beta R}^c} \nu_{\alpha L} \phi + h.c. \quad (4)$$

$$g_{\alpha\beta} \overline{\nu_{\beta R}^c} \nu_{\alpha L} \phi + y \phi \bar{f}_R f_L + h.c. \quad (5)$$

$$\begin{aligned} \mathcal{L}_0 = & \frac{1}{2} \overline{\nu_L} i\partial^\mu \nu_L + \frac{1}{2} \overline{\nu_R^c} i\partial^\mu \nu_R^c \\ & - \frac{1}{2} \overline{\nu_R^c} M \nu_L - \frac{1}{2} \overline{\nu_L} M^+ \nu_R^c \end{aligned}$$

(u_L, v_L) or $(u_L, u_R = (v_L)^c)$

General formulation

- In a Lorenz invariant medium:

$$\mathcal{L} = \bar{u}_L(p)(\not{p} - \not{p}\Sigma_1 - \not{k}\Sigma_2)u_L(p) + \bar{u}_R(p)(\not{p} - \not{p}\bar{\Sigma}_1 - \not{k}\bar{\Sigma}_2)u_R(p)$$
$$- \bar{u}_R(p)(M + \Sigma_0)u_L(p) - \bar{u}_L(p)(M^+ + \bar{\Sigma}_0)u_R(p)$$

(3,4)

(1,2,3,4)

$$\Sigma_{1,2}^+ = \Sigma_{1,2}$$

(5)

- Rest mass correction:

$$\Sigma_0 = gy \rho_{DM}/m_\phi^3$$

- Kinetic correction → Dispersion relation changed → “Effective mass” (refraction index) & DM potential induced.
- Denser medium → Heavier mass (larger refraction)

Neutrinos in a scalar medium

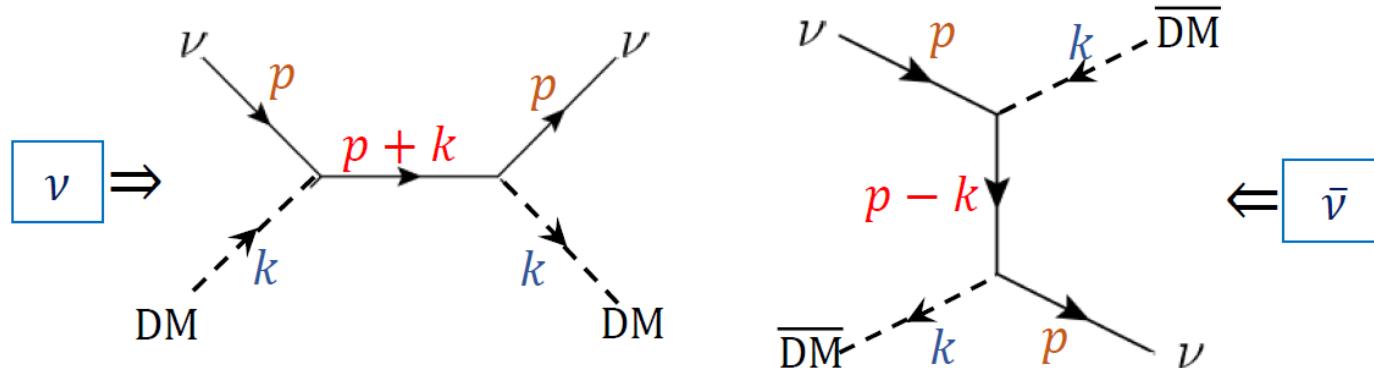
- Neutrino-DM interaction

$$\mathcal{L}' = g_{\alpha i} \overline{\textcolor{green}{f}_{iR}} \nu_{\alpha L} \phi + h.c. = g_{\alpha i}^* \overline{\textcolor{green}{f}_{iL}^c} \nu_{\alpha R}^c \phi^* + h.c.$$

- Self-energy corrections by the medium: $\Sigma_{1,2}$

$$\mathcal{L}_{kin} = \bar{u}_L(p) (\not{p} - \not{p}\Sigma_1 - \not{k}\Sigma_2) u_L(p) + \bar{u}_R(p) (\not{p} - \not{p}\bar{\Sigma}_1 - \not{k}\bar{\Sigma}_2) u_R(p)$$

Diagrammatic calculation



$$\mathcal{L}' = g_{\alpha i} \overline{f_{iR}} \nu_{\alpha L} \phi + h.c. = g_{\alpha i}^* \overline{f_{iL}^c} \nu_{\alpha R}^c \phi^* + h.c.$$

$$\begin{aligned} \mathcal{L}_{\text{eff}} = & \bar{u}_L(p) \left(\not{p} - \lambda \left[\frac{\not{p} + \not{k}}{(p+k)^2 - m_X^2} \frac{N_{\text{DM}}}{2k^0} + \frac{\not{p} - \not{k}}{(p-k)^2 - m_X^2} \frac{N_{\text{DM}}}{2k^0} \right] \right) u_L(p) \\ & + \bar{u}_R(p) \left(\not{p} - \lambda^* \left[\frac{\not{p} + \not{k}}{(p+k)^2 - m_X^2} \frac{N_{\text{DM}}}{2k^0} + \frac{\not{p} - \not{k}}{(p-k)^2 - m_X^2} \frac{N_{\text{DM}}}{2k^0} \right] \right) u_R(p) \\ & - \bar{u}_R(p) M u_L(p) - \bar{u}_L(p) M^+ u_R(p) \end{aligned}$$

$$\lambda = \underset{\text{diag.}}{g g^\dagger} \longrightarrow |g|^2$$

Finite temperature/density calculation

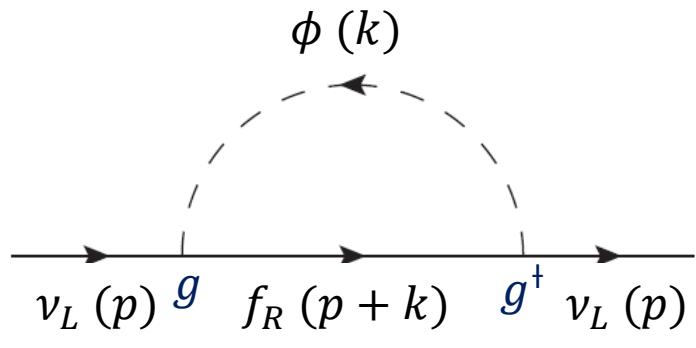
- Modified neutrino propagator in a medium:

$$S_\nu^{-1}(p) = (\not{p} - \Sigma) = (\not{p} - \not{p}\Sigma_1 - \not{k}\Sigma_2)$$

$$\Sigma = i g g^\dagger \int \frac{d^4 k}{(2\pi)^4} \Delta_\phi(k) S_f(p+k)$$

$$\Delta_\phi(k) = \frac{1}{k^2} - 2\pi i \delta(k^2 - m_\phi^2) f_\phi(k)$$

$$S_f(q) = (\not{q} + m_f) \left(\frac{1}{q^2 - m_f^2} + 2\pi i \delta(q^2 - m_f^2) f_f(q) \right)$$



Dark matter 4-momentum: $k = (k^0, \vec{k}) \simeq (m_\phi, 0)$

Distribution functions:

$$f_f(q) \equiv 0$$

$$f_\phi(k) = (\theta(k_0)n_\phi + \theta(-k^0)n_{\bar{\phi}})(2\pi)^3 \delta^3(\vec{k})$$

Self-energy corrections

- For ν and $\bar{\nu}$, $(p - \Sigma) = (p - p\Sigma_1 - k\Sigma_2)$

$$(p - \bar{\Sigma}) = (p - p\bar{\Sigma}_1 - k\bar{\Sigma}_2)$$

$$\Sigma_1(\bar{\Sigma}_1) \Rightarrow \delta m^2^{(*)} \frac{(p^2 + m_\phi^2 - m_f^2) \pm \epsilon(2m_\phi E)}{(p^2 + m_\phi^2 - m_f^2)^2 - 4m_\phi^2 E^2}$$

$$\Sigma_2(\bar{\Sigma}_2) = \delta m^2^{(*)} \frac{\pm \epsilon(p^2 + m_\phi^2 - m_f^2) - (2m_\phi E)}{(p^2 + m_\phi^2 - m_f^2)^2 - 4m_\phi^2 E^2}$$

$$p \equiv (E, \vec{p}), p \equiv |\vec{p}|$$

$$\delta m^2 \equiv \frac{gg^+}{2} \frac{\rho_{\text{DM}}}{m_{\text{DM}}^2}$$

$$\rho_{\text{DM}} \equiv m_{\text{DM}}(N_{\text{DM}} + N_{\overline{\text{DM}}})$$

$$\epsilon \equiv \frac{N_{\text{DM}} - N_{\overline{\text{DM}}}}{N_{\text{DM}} + N_{\overline{\text{DM}}}}$$

Free spinors in a scalar medium

- Eqs of motion

$$\begin{aligned}(p - \Sigma)_\mu \gamma^\mu u_L(p) &= M^+ u_R(p) & \Sigma_\mu &\equiv p_\mu \Sigma_1 + k_\mu \Sigma_2 \\(p - \bar{\Sigma})_\mu \gamma^\mu u_R(p) &= M^- u_L(p) & \bar{\Sigma}_\mu &\equiv p_\mu \bar{\Sigma}_1 + k_\mu \bar{\Sigma}_2\end{aligned}$$

- Dispersion relations for “massless” (Weyl) neutrinos: $M \equiv 0$

$$(p - \Sigma)^2 = 0$$

$$(p - \bar{\Sigma})^2 = 0$$



$$\begin{aligned}(E(1 - \Sigma_1) - m_{DM} \Sigma_2)^2 - p^2(1 - \Sigma_1)^2 &= 0 & \text{for } \nu \\(E(1 - \bar{\Sigma}_1) - m_{DM} \bar{\Sigma}_2)^2 - p^2(1 - \bar{\Sigma}_1)^2 &= 0 & \text{for } \bar{\nu}\end{aligned}$$

*) Different dispersion relations for Majorana and Dirac neutrinos ($M \neq 0$).

Dispersion relations

$$\delta m^2 \equiv \frac{gg^+}{2} \frac{\rho_{\text{DM}}}{m_{\text{DM}}^2}$$

- $m_f^2 \gg \delta m^2, 2m_\phi p \gg m_\phi^2$

$$E^2 = p^2 - 2\epsilon m_\phi p \frac{\delta m^2}{m_f^2} - 4p^2 \frac{m_\phi^2 \delta m^2}{m_f^4} + \epsilon^2 m_\phi^2 \left(\frac{\delta m^2}{m_f^2} \right)^2 \dots$$

$$E \xrightarrow{\text{High } p} p - \epsilon m_\phi \frac{\delta m^2}{m_f^2} - 2p \frac{m_\phi^2 \delta m^2}{m_f^4} + \dots$$

- $2m_\phi p \gg \delta m^2, m_f^2 \gg m_\phi^2$

$$E^2 = p^2 + \delta m^2 - \epsilon \delta m^2 \frac{2\delta m^2 - m_f^2}{2m_\phi p} + \dots$$

$$E \xrightarrow{\text{High } p} p + \frac{\delta m^2}{2p} - \epsilon \frac{\delta m^2}{2p} \frac{2\delta m^2 - m_f^2}{2m_\phi p} + \dots$$

- $\delta m^2 \gg 2m_\phi p, m_f^2 \gg m_\phi^2$

$$E^2 = p^2 + \delta m^2 + m_f^2 + 2\epsilon m_\phi p + \dots$$

$$E \xrightarrow{\text{High } p} p + \frac{\delta m^2}{2p} + \epsilon m_\phi + \dots$$

$$E \xrightarrow{\text{Low } p} \sqrt{\delta m^2} \gg p \quad (?)$$

Effective mass in a medium

- For the galactic DM density; $\rho_{\text{DM}}^{\text{gal}} \approx 0.3 \text{ GeV/cm}^3 \approx 2.3 \times 10^{-6} \text{ eV}^4$

$$\delta m_{\text{gal}}^2 \equiv -\frac{1}{2} \frac{\rho_{\text{DM}}^{\text{gal}}}{m_{\text{DM}}^2} \approx 2.5 \times 10^{-3} \text{ eV}^2 (gg^+)_{{\alpha}{\beta}} \left(\frac{0.02 \text{ eV}}{m_{\text{DM}}} \right)^2$$

- It is an L-conserving mass-squared: $E^2 \approx p^2 + \delta m^2$

- Medium-induced neutrino oscillation:

$$E \approx p + \frac{\delta m^2}{2p}$$

- Low-p limit: $E \approx \sqrt{\delta m^2} ?$

Weyl neutrino oscillations

- Solar and Atmospheric neutrinos: $E_\nu \approx p$

$$p_{\text{sol}} = (0.1 - 10) \text{ MeV}, \quad p_{\text{atm}} = (0.1 - 100) \text{ GeV}$$

$$\Delta m_{\text{sol}}^2 = 7.5 \times 10^{-5} \text{ eV}^2, \quad \Delta m_{\text{atm}}^2 = 2.5 \times 10^{-3} \text{ eV}^2$$

$$\frac{\Delta m^2}{2p} = \left\{ \begin{array}{l} 3.8 \times 10^{-10:-12} \text{ eV} \\ 1.2 \times 10^{-11:-14} \text{ eV} \end{array} \right\}$$

- Neutrino oscillation by medium-induced $(\text{mass})^2$ for
$$p^2 \gg 2m_{\text{DM}}p \gg \delta m^2, m_f^2; \quad p^2 \gg \delta m^2 \gg 2m_{\text{DM}}p, m_f^2$$
- CP asymmetry may be observable for $m_{\text{DM}} \approx 10^{-10:-14} \text{ eV}$:

$$E_\nu \ni \mp \epsilon \frac{2\delta m^2 - m_f^2}{2m_{\text{DM}}p} \frac{\delta m^2}{2p}; \quad \pm \epsilon m_{\text{DM}}$$

Discussions

- Neutrino oscillation may be due to the medium effect.
- Observing $0\nu\beta\beta$ will exclude the Weyl neutrino oscillation.
- **Astrophysical & cosmological bounds on the neutrino-DM interaction.**
- Origin of DM density?
 Ultra-light cold DM, coherent oscillation, late decay, ...
- Sensible UV-completion?
 Dark sector coupling only to neutrinos...