$\Xi(1690)$ as a $\bar{K}\Sigma$ molecular state

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1. Introduction
2. Formulation
3. Results and discussions
4. Summary
5. Furthermore on exotic hadrons

Key words: $\Xi(1690)$, Hadronic molecules, Compositeness, Chiral unitary approach.

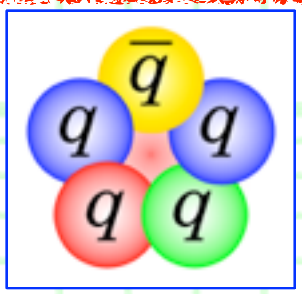
[1] T. S. , *Prog. Theor. Exp. Phys.*, 2015, 091D01.

1. Introduction

1. Introduction

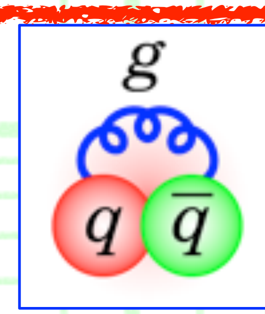
++ Exotic hadrons and their structure ++

- **Exotic hadrons** --- not same quark component as ordinary hadrons
  = not qqq nor $q\bar{q}$.

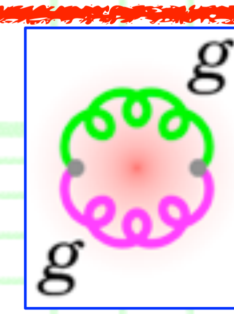


Penta-quarks

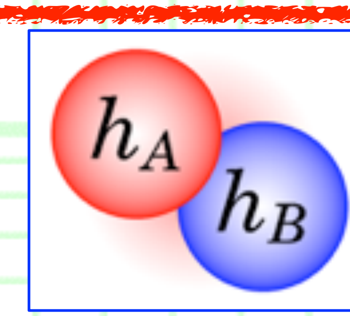
Tetra-quarks



Hybrids



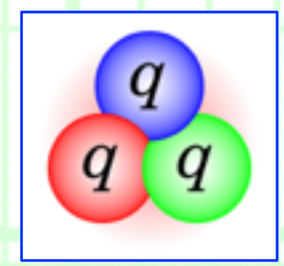
Glueballs

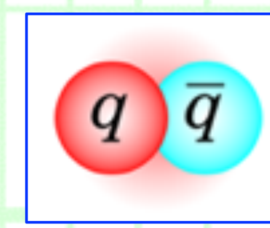


Hadronic molecules

...

--- Actually some hadrons cannot be described by the quark model.





Ordinary hadrons

- □ Do exotic hadrons really exist ?
- □ If they do exist, how are their properties ?
  --- Re-confirmation of quark models.
  --- Constituent quarks in multi-quarks ? "Constituent" gluons ?
- □ If they do not exist, what mechanism forbids their existence ?

<-- We know very few about hadrons (and dynamics of QCD).

1. Introduction

++ The Ξ(1690) resonance ++

- The Ξ(1690) resonance may be an exotic hadron.

--- Status: *** = existence ranges from very likely to certain, but further confirmation is desirable and/or quantum numbers, branching fractions, etc. are not well determined.

Citation: K.A. Olive et al. (Particle Data Group), Chin. Phys. C38, 090001 (2014) (URL: http://pdg.lbl.gov)

Ξ(1690) $I(J^P) = \frac{1}{2}(?^?)$ Status: ***

AUBERT 08AK, in a study of $\Lambda_c^+ \to \Xi^- \pi^+ K^+$, finds some evidence that the Ξ(1690) has $J^P = 1/2^-$.

DIONISI 78 sees a threshold enhancement in both the neutral and negatively charged $\Sigma\overline{K}$ mass spectra in $K^- p \to (\Sigma\overline{K}) K\pi$ at 4.2 GeV/c. The data from the $\Sigma\overline{K}$ channels alone cannot distinguish between a resonance and a large scattering length. Weaker evidence at the same mass is seen in the corresponding $\Lambda\overline{K}$ channels, and a coupled-channel analysis yields results consistent with a new Ξ.

BIAGI 81 sees an enhancement at 1700 MeV in the diffractively produced ΛK^- system. A peak is also observed in the $\Lambda\overline{K}^0$ mass spectrum at 1660 MeV that is consistent with a 1720 MeV resonance decaying to $\Sigma^0\overline{K}^0$, with the γ from the Σ^0 decay not detected.

BIAGI 87 provides further confirmation of this state in diffractive dissociation of Ξ^- into ΛK^-. The significance claimed is 6.7 standard deviations.

ADAMOVICH 98 sees a peak of 1400 ± 300 events in the $\Xi^-\pi^+$ spectrum produced by 345 GeV/c Σ^--nucleus interactions.

Ξ(1690) MASSES

MIXED CHARGES

VALUE (MeV)	DOCUMENT ID	
1690±10 OUR ESTIMATE		This is only an educated guess; the error given is larger than the error on the average of the published values.

Ξ(1690) WIDTHS

MIXED CHARGES

VALUE (MeV)	DOCUMENT ID
<30 OUR ESTIMATE	

Particle Data Group.

1. Introduction

++ Experiments of the Ξ(1690) resonance ++

- Historically Ξ(1690) was discovered as **a threshold enhancement in both the neutral and charged $\bar{K}\Sigma$ mass spectra** in the $K^- p \to (\bar{K}\Sigma)\, K\, \pi$ reaction at 4.2 GeV/c.

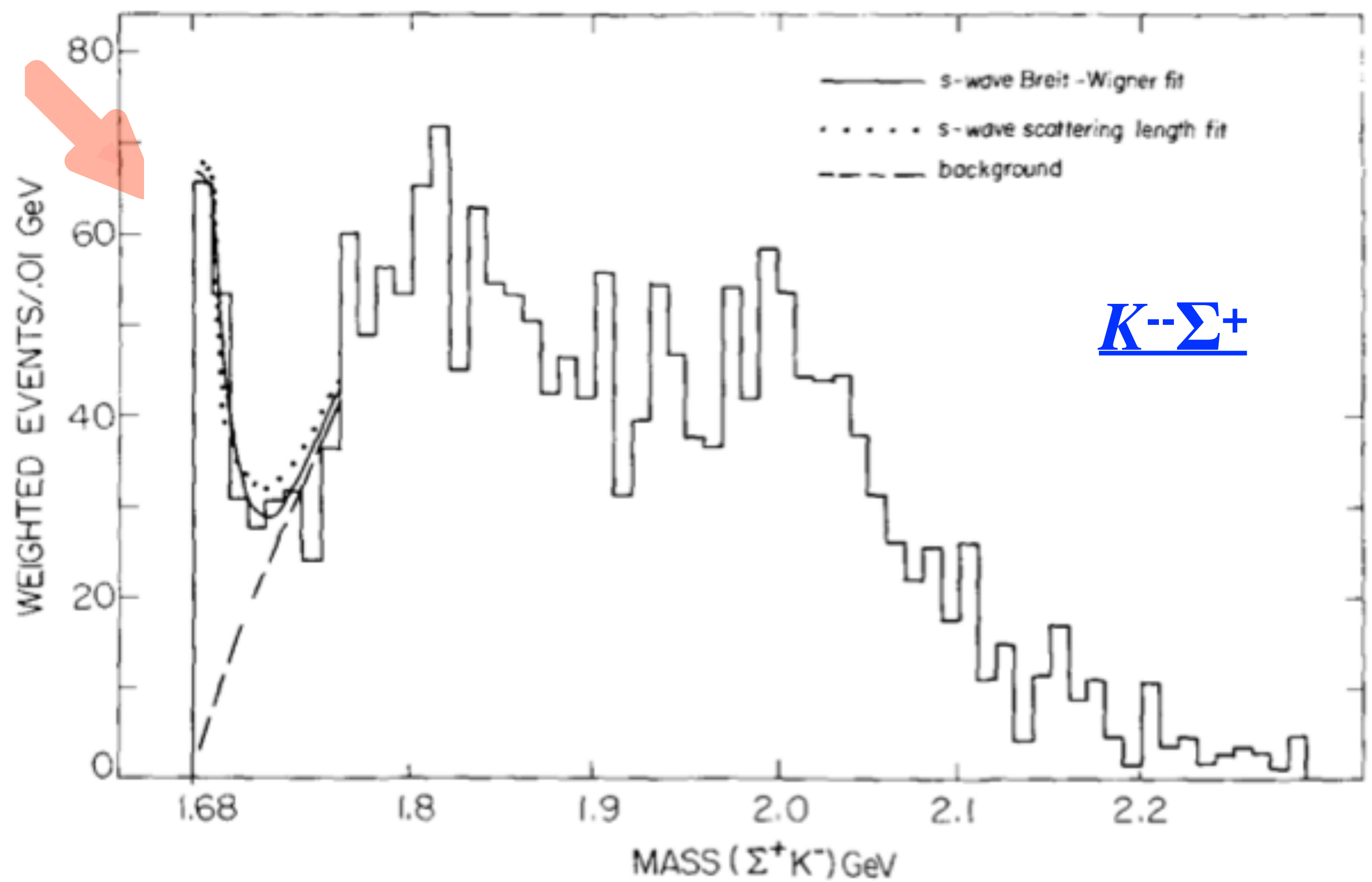


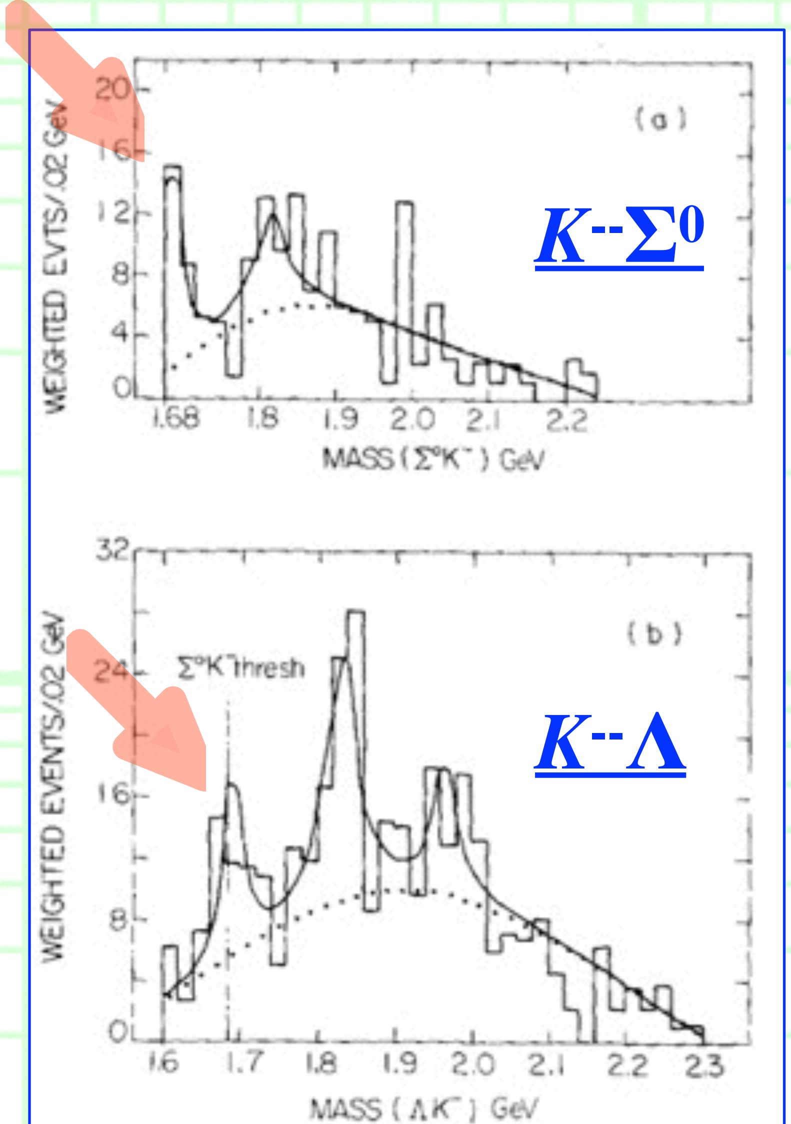
Fig. 1. The Σ^+K^- mass spectrum for the reaction $K^-p \to \Sigma^+K^-K^+\pi^-$ after elimination of ϕ events mass (K^+K^- less than 1.03 GeV). The origin of the curves is indicated.

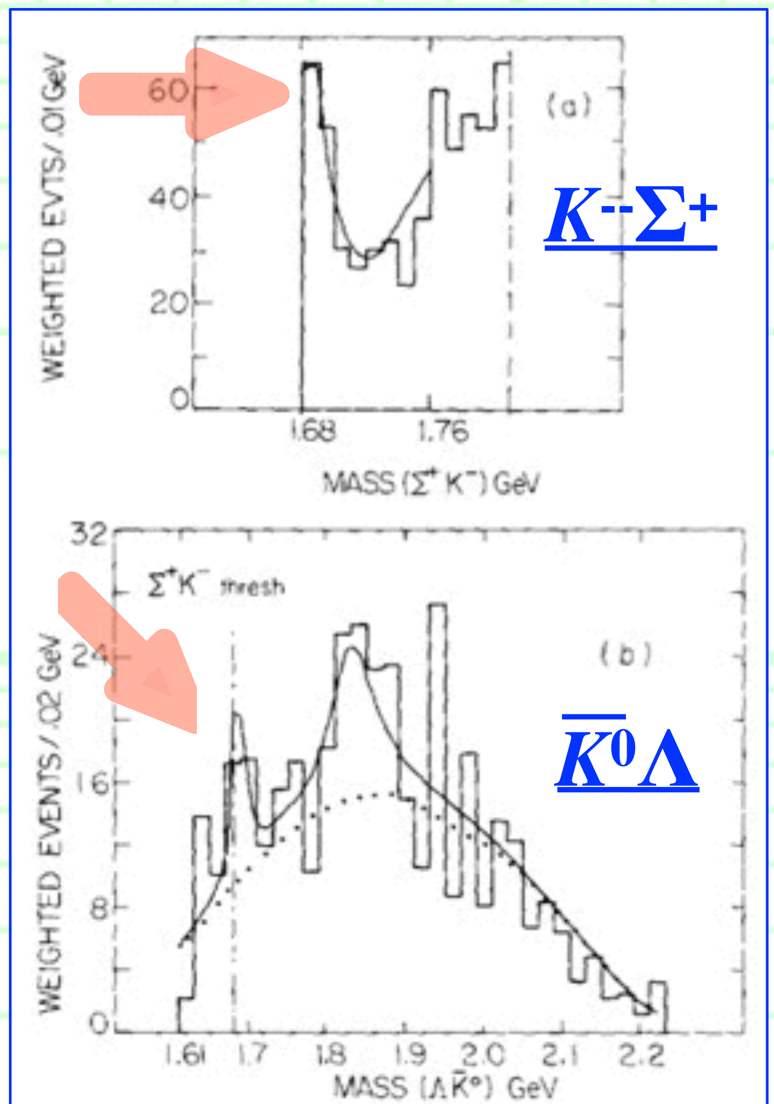
C. Dionisi *et al.*, *Phys. Lett.* B80 (1978) 145.

1. Introduction

++ Experiments of the $\Xi(1690)$ resonance ++

- Historically $\Xi(1690)$ was discovered as **a threshold enhancement in both the neutral and charged $\bar{K}\Sigma$ mass spectra** in the $K^{-} p \to (\bar{K}\Sigma) K \pi$ reaction at 4.2 GeV/c.





--- Rapid enhancement at the threshold of the $\bar{K}\Sigma$ mass spectra implies that this **couples to the $\bar{K}\Sigma$ channel in s wave**. <=> $J^P = 1/2^{-}$.

C. Dionisi *et al.*, *Phys. Lett.* B80 (1978) 145.

1. Introduction

++ Experiments of the Ξ(1690) resonance ++

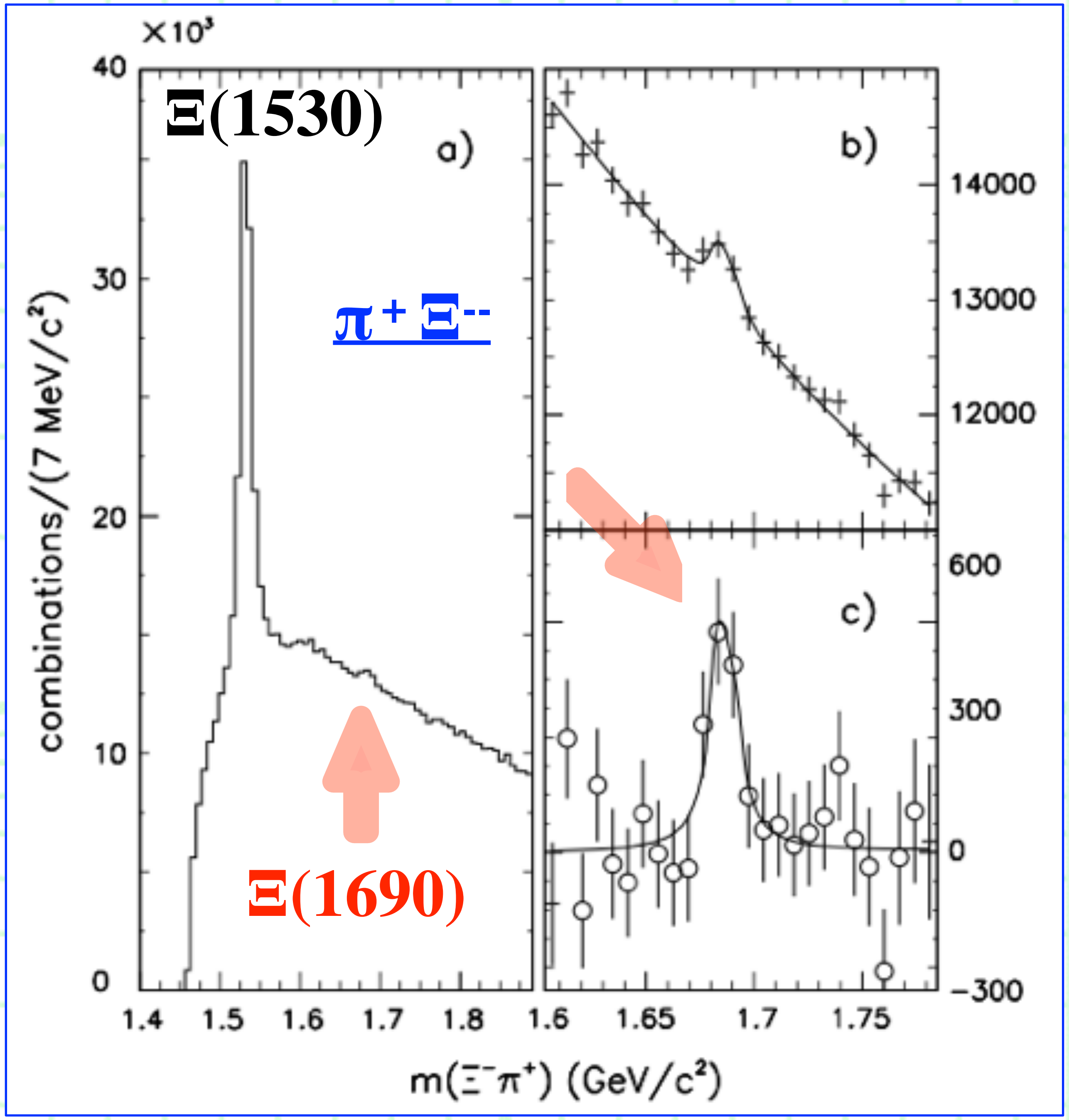
- Ξ(1690) has been observed and investigated in several experiments, for instance:
  - Small total decay width and tiny branching fraction to the πΞ state.

$$M = 1686 \pm 4\,\mathrm{MeV}/c^2\,,\Gamma = 10 \pm 6\,\mathrm{MeV}/c^2\,.$$

$$\frac{\sigma \cdot BR(\Xi^0(1690) \to \Xi^-\pi^+)}{\sigma \cdot BR(\Xi^0(1530) \to \Xi^-\pi^+)} = 0.022 \pm 0.005.$$

--- Using a Σ-- beam on nucleus.

M. I. Adamovich *et al.* [WA89 Collab.], *Eur. Phys. J.* C5 (1998) 621.



1. Introduction

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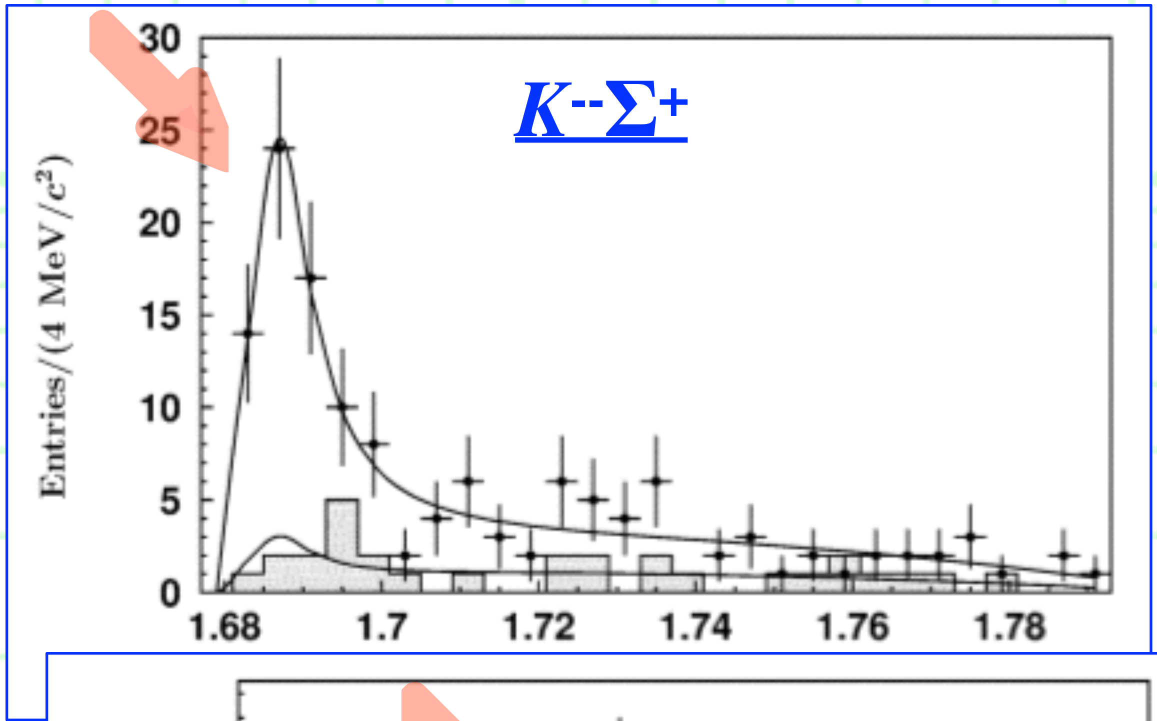
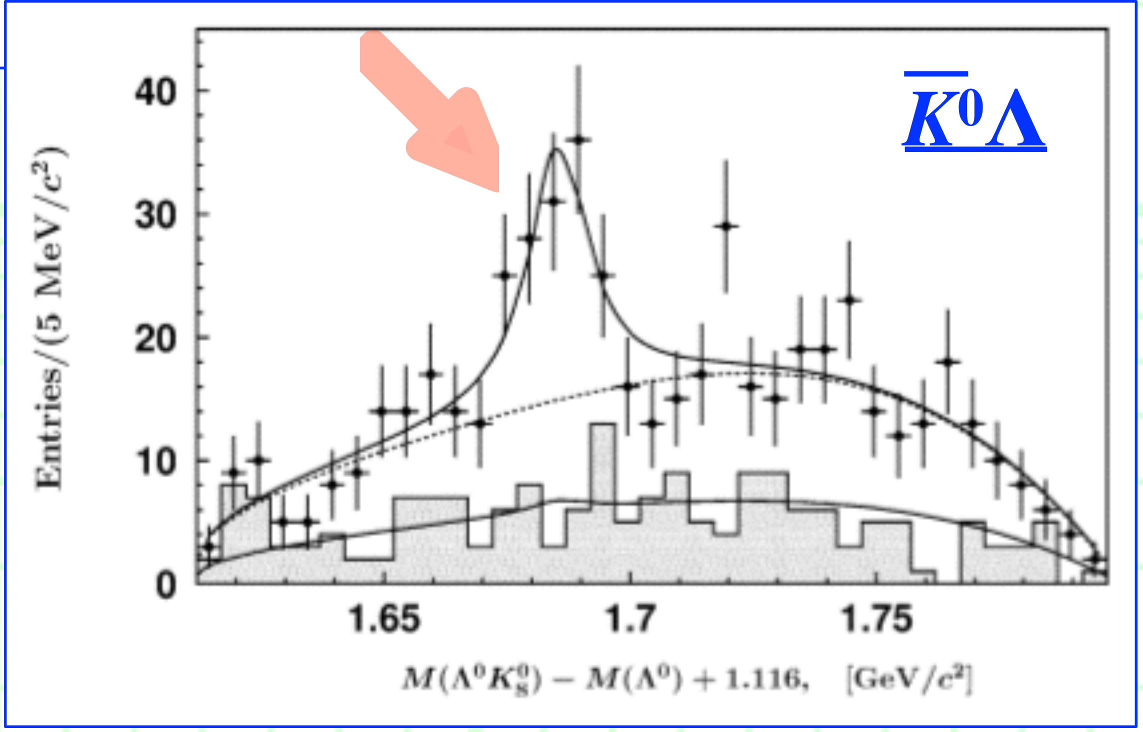
- Ξ(1690) has been observed and investigated in several experiments, for instance:
  - Small total decay width and tiny branching fraction to the πΞ state.
  - Ξ(1690) can be observed in decay of heavy hadrons as well, giving mass spectra, branching fractions, and their ratios involving Ξ(1690).

$$\frac{\mathcal{B}(\Xi(1690)^0 \to \Sigma^+ K^-)}{\mathcal{B}(\Xi(1690)^0 \to \Lambda^0 \overline{K}^0)} = 0.50 \pm 0.26.$$

--- Using $e^+ e^-$ colliders.





K. Abe *et al.* [Belle Collab.], *Phys. Lett.* B524 (2002) 33;
M. Ablikim *et al.* [BES III], *Phys.Rev.* D91 (2015) 092006.

1. Introduction

++ Experiments of the Ξ(1690) resonance ++

- Ξ(1690) has been observed and investigated in several experiments, for instance:
  - □ **Small total decay width** and **tiny branching fraction to the πΞ state**.
  - □ Ξ(1690) can be **observed in decay of heavy hadrons** as well, giving mass spectra, branching fractions, and their ratios involving Ξ(1690).
  - □ **A dip in the $P_0(\cos\theta)$ moment of the $\pi^+\Xi^{--}$ mass spectrum** appears in the vicinity of Ξ(1690), which **implies that Ξ(1690) has $J^P = 1/2^{--}$**.

B. Aubert *et al.* [BaBar Collab.], *Phys. Rev.* D78 (2008) 034008.

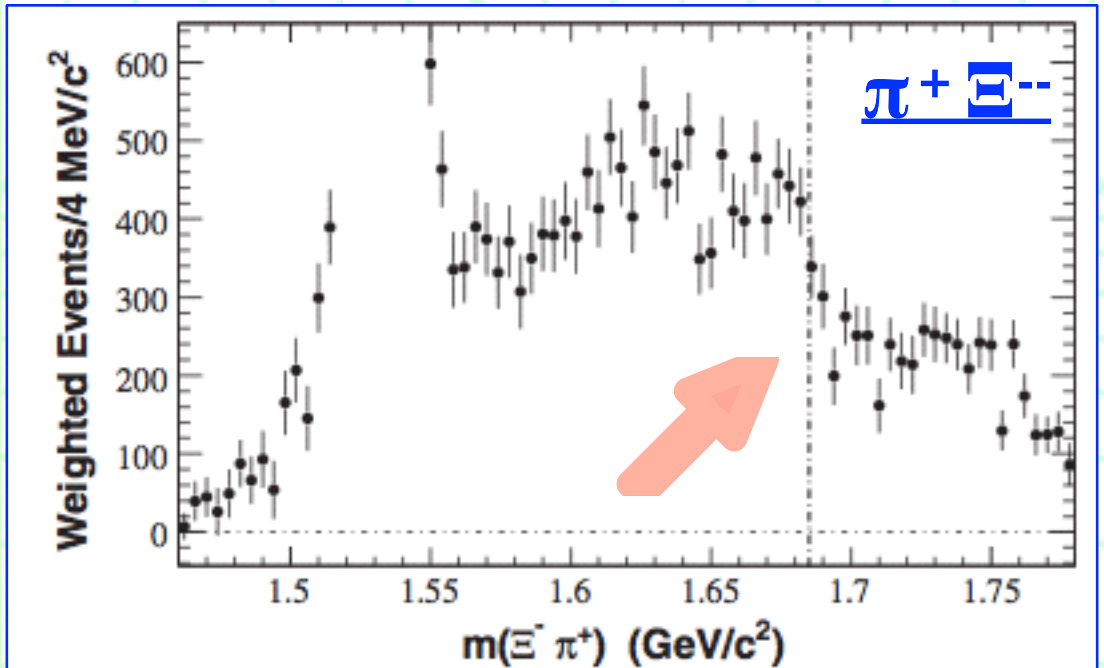
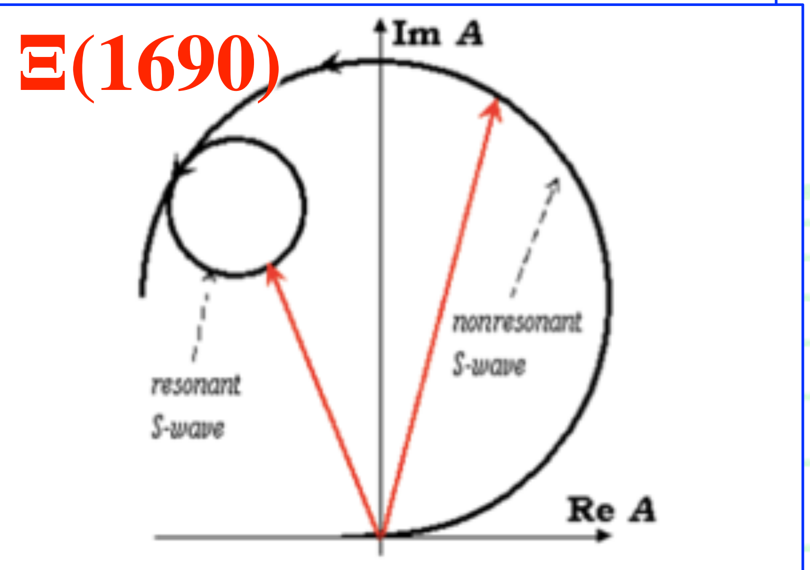


FIG. 10. The subtracted P_0(cos mass distribution Fig. 3(a) with the dashed line indic



s-wave Argand diagram

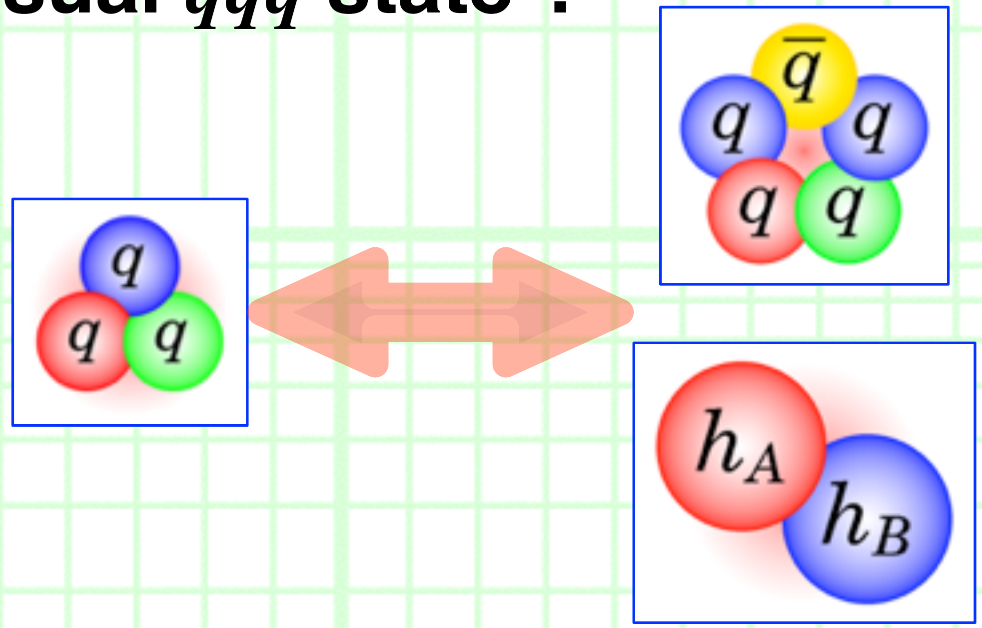
1. Introduction

++ Experiments of the Ξ(1690) resonance ++

- Ξ(1690) has been observed and investigated in several experiments, for instance:
  - □ Small total decay width and tiny branching fraction to the πΞ state.
  - □ Ξ(1690) can be observed in decay of heavy hadrons as well, giving mass spectra, branching fractions, and their ratios involving Ξ(1690).
  - □ A dip in the $P_0(\cos\theta)$ moment of the $\pi^+\Xi^-$ mass spectrum appears in the vicinity of Ξ(1690), which implies that Ξ(1690) has $J^P = 1/2^-$.

- The small decay width and tiny branching fraction to the πΞ state are un-natural.

--> Ξ(1690) might have a some non-trivial structure than usual qqq state ?



-- But its properties and structure are still unclear.

1. Introduction

++ Theories of the Ξ(1690) resonance ++

- Ξ(1690) and other Ξ* resonances has been investigated in several theoretical frameworks as well, for instance:

  - Quark models.

    K. T. Chao, N. Isgur and G. Karl, *Phys. Rev.* D23 (1981) 155;
    S. Capstick and N. Isgur, *Phys. Rev.* D34 (1986) 2809;
    M. Pervin and W. Roberts, *Phys. Rev.* C77 (2008) 025202;
    L. Y. Xiao and X. H. Zhong, *Phys. Rev.* D87 (2013) 094002;
    N. Sharma, A. Martinez Torres, K. P. Khemchandani and H. Dahiya, *Eur. Phys. J.* A49 (2013) 11;
    ...

  - Skyrme model.

    Y. Oh, *Phys. Rev.* D75 (2007) 074002.

  - Chiral unitary approach.

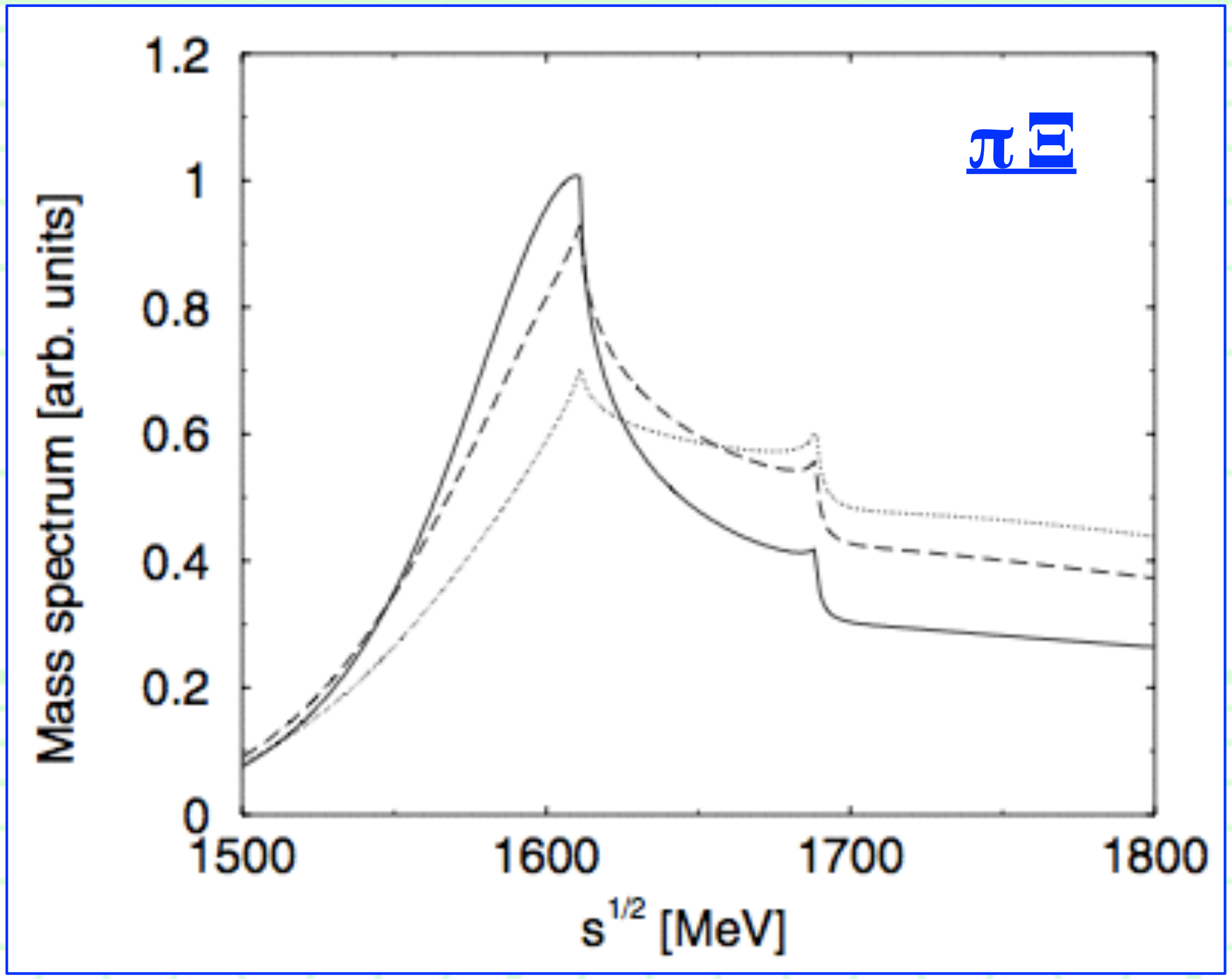
    A. Ramos, E. Oset and C. Bennhold, *Phys. Rev. Lett.* 89 (2002) 252001;
    C. Garcia-Recio, M. F. M. Lutz and J. Nieves, *Phys. Lett.* B582 (2004) 49;
    D. Gamermann, C. Garcia-Recio, J. Nieves and L. L. Salcedo, *Phys. Rev.* D84 (2011) 056017.

1. Introduction

++ Ξ* resonances in chiral unitary approach ++

- **Ξ* resonances in chiral unitary approach.**

--- Based on the combination of the chiral perturbation theory and the unitarization of the scattering amplitude.



□ First, another Ξ* resonance, Ξ(1620), was studied in the *s*-wave $\pi\Xi$-$\bar{K}\Lambda$-$\bar{K}\Sigma$-$\eta\Xi$ coupled-channels scattering in the chiral unitary approach.

--- Ξ(1620) status: *

$J^P = 1/2^{-}$?

A. Ramos, E. Oset and C. Bennhold, *Phys. Rev. Lett.* 89 (2002) 252001.

1. Introduction

++ Ξ* resonances in chiral unitary approach ++

- **Ξ* resonances in chiral unitary approach.**

--- Based on the combination of the chiral perturbation theory and the unitarization of the scattering amplitude.

$(\frac{1}{2}, -2)$		$[\pi\Xi]$ 7.5	5.6	seen	2.6
$\Xi(1620)^{*}$		$[\bar{K}\Lambda]$ 5.2	2.8	seen	−1.5
$M \approx 1620$	1565	$[\bar{K}\Sigma]$ 0.7	2.6	0	−0.8
$\Gamma = 23$	247	$[\eta\Xi]$ 0.3	4.9	0	0.3
$(\frac{1}{2}, -2)$		$[\pi\Xi]$ 0.02	0.1	seen	−0.1
$\Xi(1690)^{***}$		$[\bar{K}\Lambda]$ 0.16	6.0	seen	0.9
$M = 1690 \pm 10$	1663	$[\bar{K}\Sigma]$ 5.15	3.1	seen	−2.5
$\Gamma = 10 \pm 6$	4	$[\eta\Xi]$ 2.28	3.2	0	−1.7

- Then, **systematic studies were done for several Ξ* states** together with many other resonances.

C. Garcia-Recio, M. F. M. Lutz and J. Nieves, *Phys. Lett.* B582 (2004) 49.

8 (1134)	2037–24i	0.6	0.6	0.3	0.2	0.3	↑0.5	1.5	0.6	**1.8**	**2.4**	1.1	0.2	1.0	**2.1**	
10 (70)	1729–46i	0.6	1.4	0.4	↑1.6	1.4	**2.1**	1.0	0.4	**3.3**	1.5	0.4	0.2	1.6	1.0	Ξ(1950) ★★★
8 (70)	1651–2i	0.2	0.3	↑**2.2**	1.3	1.0	**2.6**	0.2	0.6	0.9	0.4	0.2	**1.7**	0.4	0.2	Ξ(1690) ★★★
8 (56)	1577–139i	**2.6**	↑**1.7**	0.5	0.1	0.8	1.0	0.7	0.1	0.6	1.3	0.3	0.1	0.2	1.2	Ξ(1620) ★

--- Narrow width for Ξ(1690) ! But its mass is lower than Exp. value.

D. Gamermann, C. Garcia-Recio, J. Nieves and L. L. Salcedo, *Phys. Rev.* D84 (2011) 056017.

1. Introduction

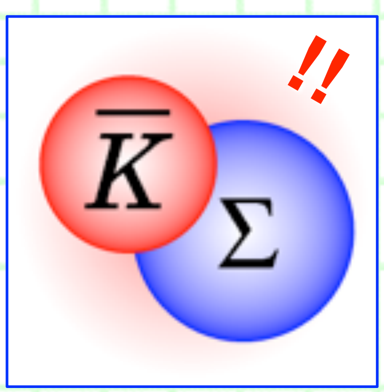
++ In this study ... ++

- **In this study we concentrate on the phenomena near the $\bar{K}\Sigma$ threshold and on the $\Xi(1690)$ resonance.**

- **By using the chiral unitary approach and adjusting parameters, we show the narrow $\Xi(1690)$ state, which was studied in the previous studies, can exist near the $\bar{K}\Sigma$ threshold with $J^P = 1/2^-$, and it reproduces experimental mass spectra qualitatively well.**

- **We investigate and clarify properties of the $\Xi(1690)$ state, including its small decay width, molecular structure, etc.**

--- We especially show that the $\Xi(1690)$ resonance can be indeed an s-wave $\bar{K}\Sigma$ molecular state in terms of the compositeness.



Hyodo-Jido-Hosaka (2012), Aceti-Oset (2012), Nagahiro-Hosaka (2014), ...
See Hyodo, *Int. J. Mod. Phys.* A28 (2013) 1330045;
T. S. , Hyodo and Jido, *PTEP* (2015) 063D04.

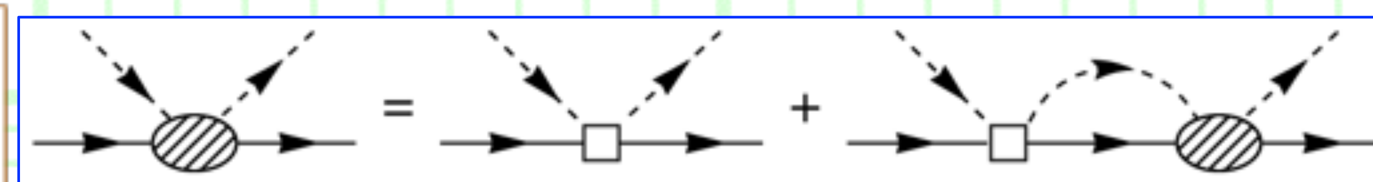
2. Formulation

2. Formulation

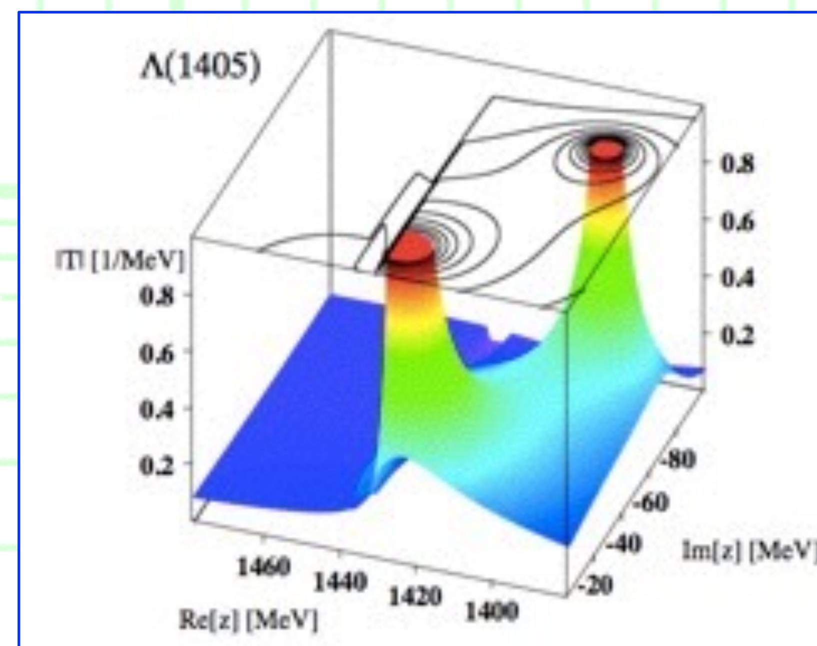
++ Chiral unitary approach ++

- We employ **the chiral unitary approach** for the *s*-wave $\bar{K}\Sigma$-$\bar{K}\Lambda$-$\pi\Xi$-$\eta\Xi$ coupled-channels scattering.

$$T_{jk}(w) = V_{jk}(w) + \sum_{l} V_{jl}(w) G_l(w) T_{lk}(w)$$



- T is the scattering amplitude which we want to obtain.
- V is the interaction kernel taken from the chiral perturbation theory projected to *s*-wave.
- G is the loop function for the meson-baryon two-body system.



Hyodo and Jido (2012).

- The chiral unitary approach is **most successful in the $\bar{K}N$ interaction and $\Lambda(1405)$**.

  Kaiser-Siegel-Weise (1995), Oset-Ramos (1998), Oller-Meissner (2001), Lutz-Kolomeitsev (2002), Jido *et al.* (2003), ...

2. Formulation

++ Interaction kernel ++

- In this study we use **the Weinberg-Tomozawa interaction** for V.

--- The leading order term in s wave:

$$V_{jk}(w) = -\frac{C_{jk}}{4f_j f_k}(2w - M_j - M_k)\sqrt{\frac{E_j + M_j}{2M_j}}\sqrt{\frac{E_k + M_k}{2M_k}}$$

- The meson decay constant f_j is chosen at their physical values:

$$f_\pi = 92.2 \text{ MeV}, \quad f_K = 1.2 f_\pi, \quad f_\eta = 1.3 f_\pi$$

Particle Data Group.

- The Clebsch-Gordan coefficient C_{jk} is determined from the group structure of the flavor $SU(3)$ symmetry:

	$K^-\Sigma^+$	$\bar{K}^0\Sigma^0$	$\bar{K}^0\Lambda$	$\pi^+\Xi^-$	$\pi^0\Xi^0$	$\eta\Xi^0$
$K^-\Sigma^+$	1	$-\sqrt{2}$	0	0	$-1/\sqrt{2}$	$-\sqrt{3/2}$
$\bar{K}^0\Sigma^0$	$-\sqrt{2}$	0	0	$-1/\sqrt{2}$	$-1/2$	$\sqrt{3/4}$
$\bar{K}^0\Lambda$	0	0	0	$-\sqrt{3/2}$	$\sqrt{3/4}$	$-3/2$
$\pi^+\Xi^-$	0	$-1/\sqrt{2}$	$-\sqrt{3/2}$	1	$-\sqrt{2}$	0
$\pi^0\Xi^0$	$-1/\sqrt{2}$	$-1/2$	$\sqrt{3/4}$	$-\sqrt{2}$	0	0
$\eta\Xi^0$	$-\sqrt{3/2}$	$\sqrt{3/4}$	$-3/2$	0	0	0

--- We have **no free parameters in the interaction kernel**.

2. Formulation

++ Loop function ++

- For the loop function we take **a covariant expression**:

$$G_j(w) = i \int \frac{d^4 q}{(2\pi)^4} \frac{1}{(P/2+q)^2 - m_j^2 + i0} \frac{2M_j}{(P/2-q)^2 - M_j^2 + i0}$$

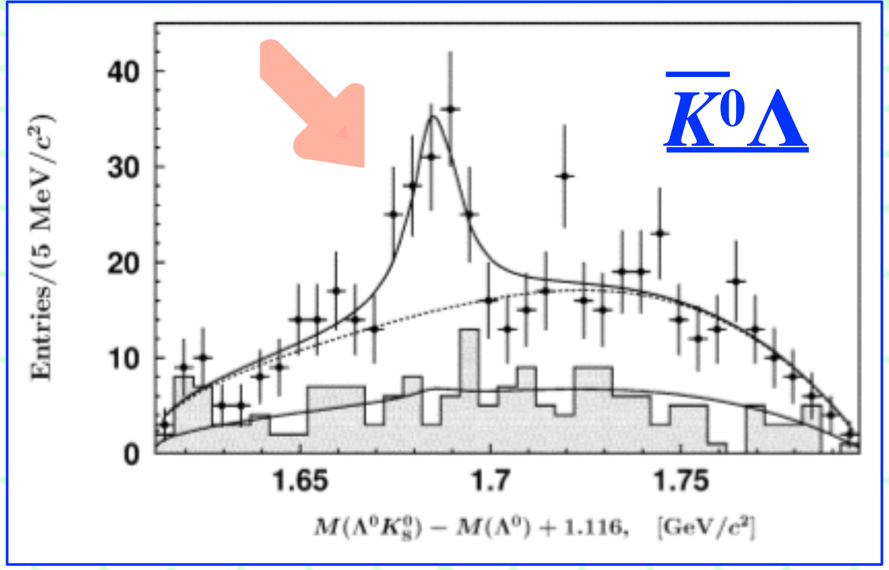
  - The integral is calculated with the dimensional regularization, and an infinite constant is replaced with a subtraction constant in each channel.

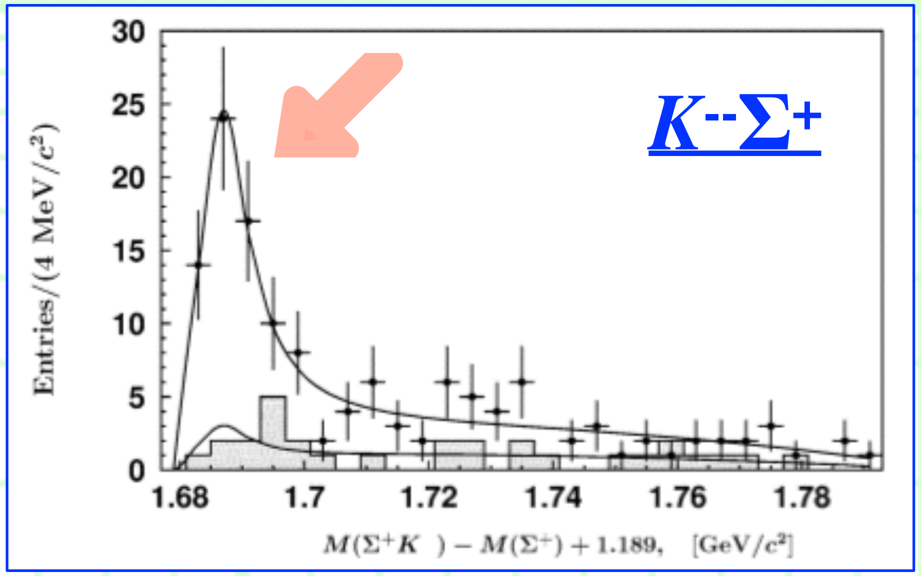
--> **Subtraction constants are free parameters.**

  - We assume the isospin symmetry for the subtraction constants, so we have **4 free parameters** ($a_{K\Sigma}$, $a_{K\Lambda}$, $a_{\pi\Xi}$, and $a_{\eta\Xi}$), which are fixed so as to reproduce the mass spectra by Belle.

--- **Neutral $\Xi(1690)$**.

K. Abe *et al.* [Belle Collab.], *Phys. Lett.* B524 (2002) 33.





3. Results and discussions

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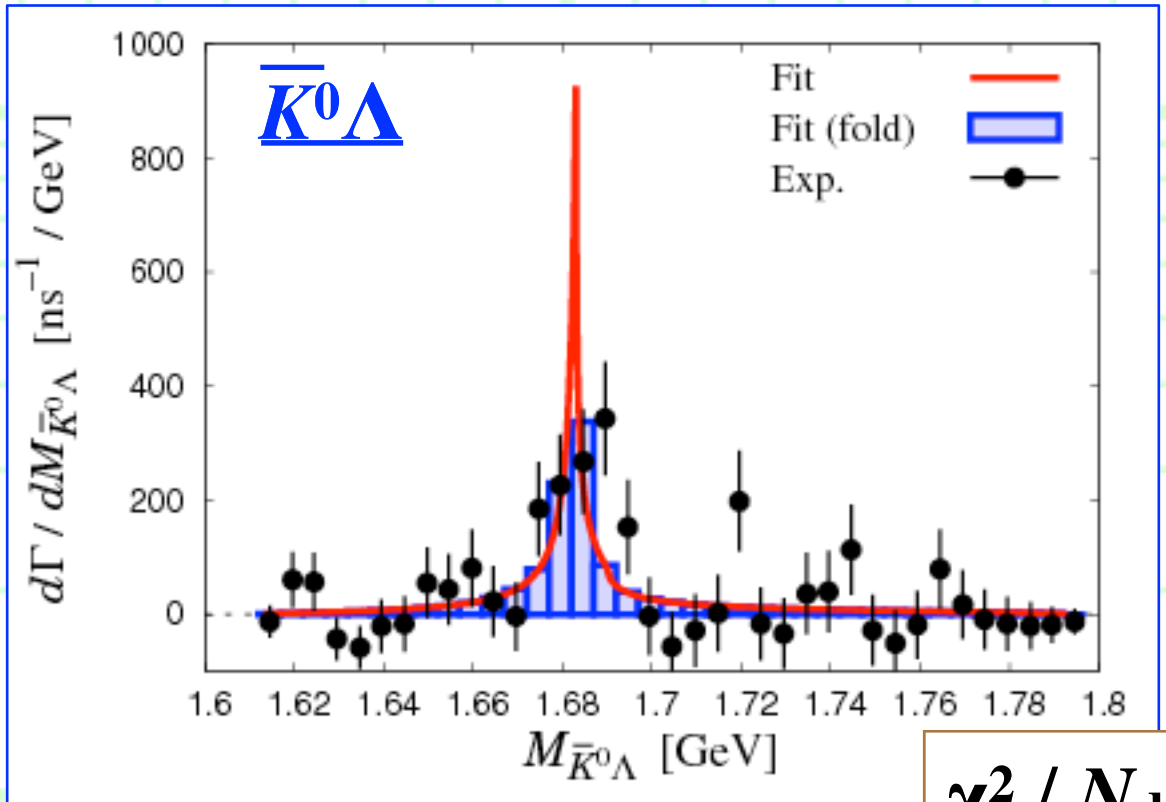
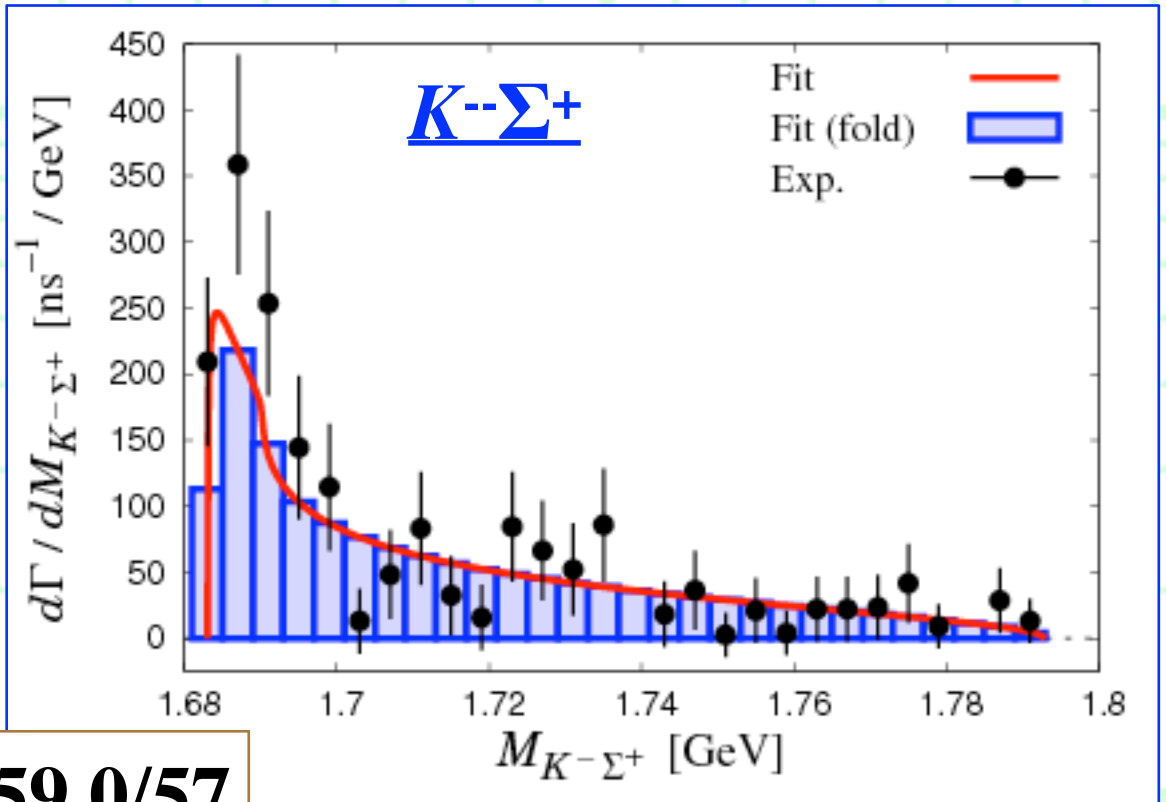
++ Fitting to the Belle data ++

- We fix **4 free parameters** ($a_{K\Sigma}$, $a_{K\Lambda}$, $a_{\pi\Xi}$, and $a_{\eta\Xi}$) **so as to reproduce the mass spectra by Belle**. The result of the best fit is:





$\chi^2 / N_{d.o.f} = 59.0/57$

- Background of the Belle data is subtracted.
- Relative scale between $\bar{K}^0\Lambda$ and $K^-\Sigma^+$ is fixed with the branching fractions:

$$\mathcal{B}[\Lambda_c^+ \to \Xi(1690)^0 K^+ \to (K^-\Sigma^+)K^+] = (1.3 \pm 0.5) \times 10^{-3}$$

$$\mathcal{B}[\Lambda_c^+ \to \Xi(1690)^0 K^+ \to (\bar{K}^0\Lambda)K^+] = (8.1 \pm 3.0) \times 10^{-4}$$

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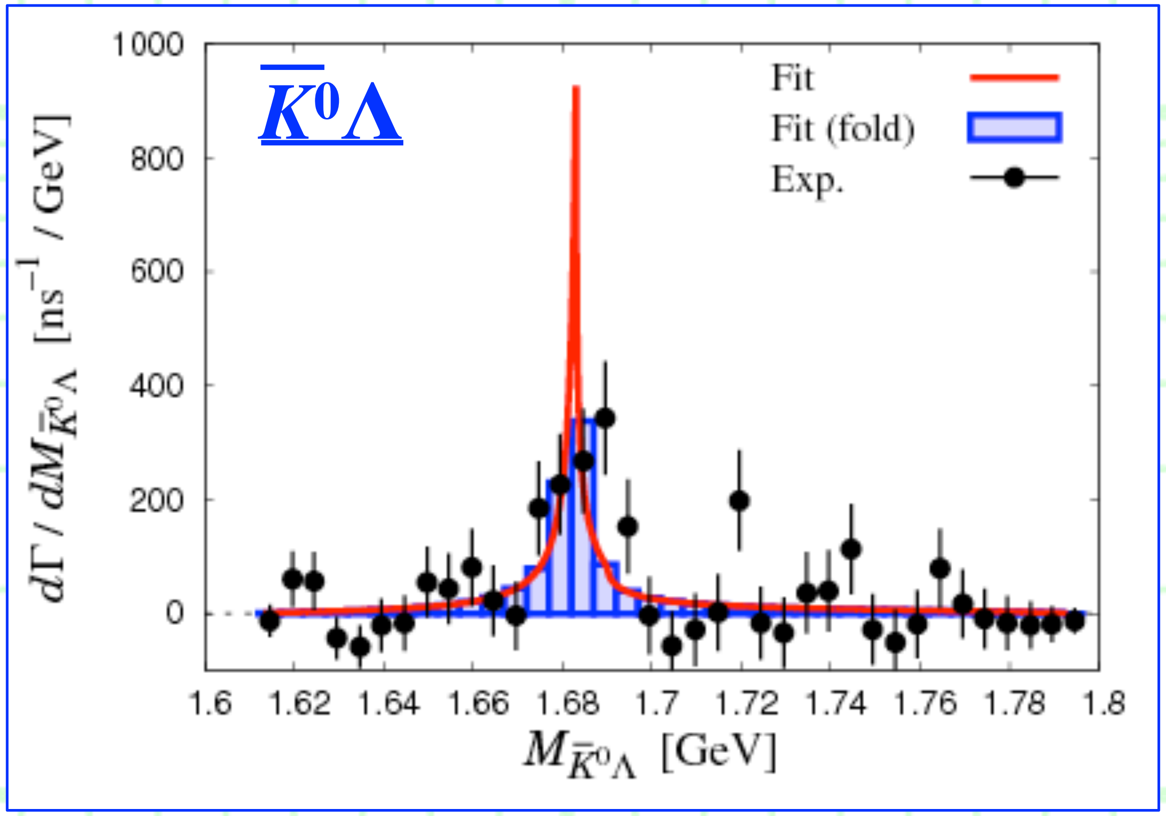

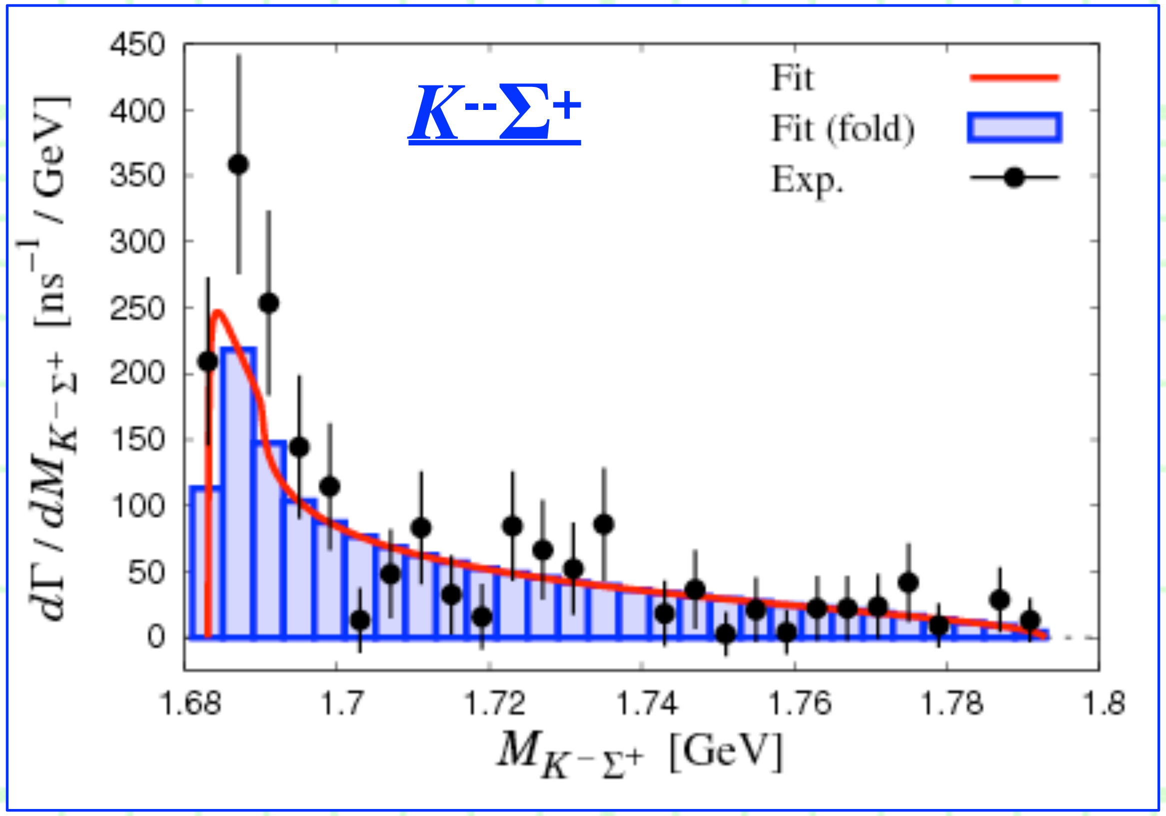

1. The Belle data on $\Xi(1690)$ are **reproduced qualitatively well** with **very small width ~ 1 MeV**.

--- We can calculate the ratio

$$R \equiv \frac{\mathcal{B}[\Lambda_c^+ \to \Xi(1690)^0 K^+ \to (K^-\Sigma^+)K^+]}{\mathcal{B}[\Lambda_c^+ \to \Xi(1690)^0 K^+ \to (\bar{K}^0\Lambda)K^+]}$$

$R_{\mathrm{th}} = 1.06$ <=> $R_{\mathrm{exp}} = 0.62 \pm 0.33$. --- In 2σ errors.

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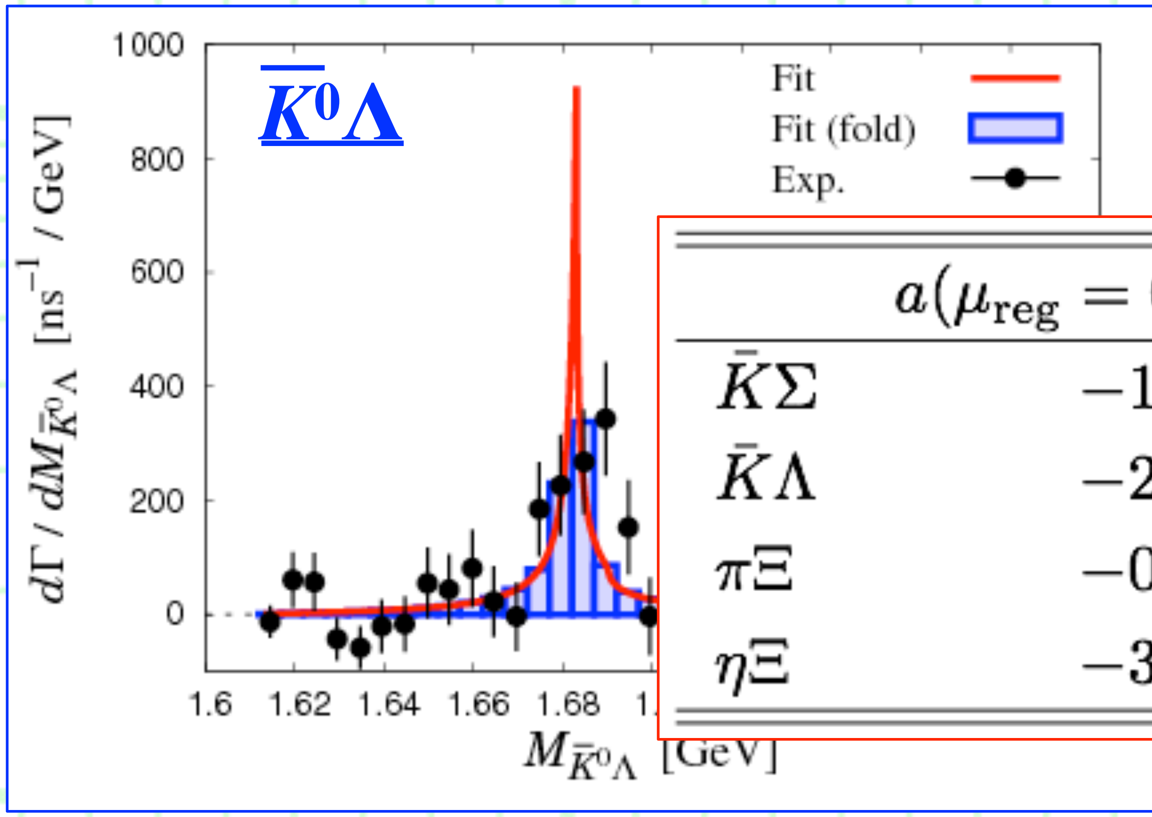
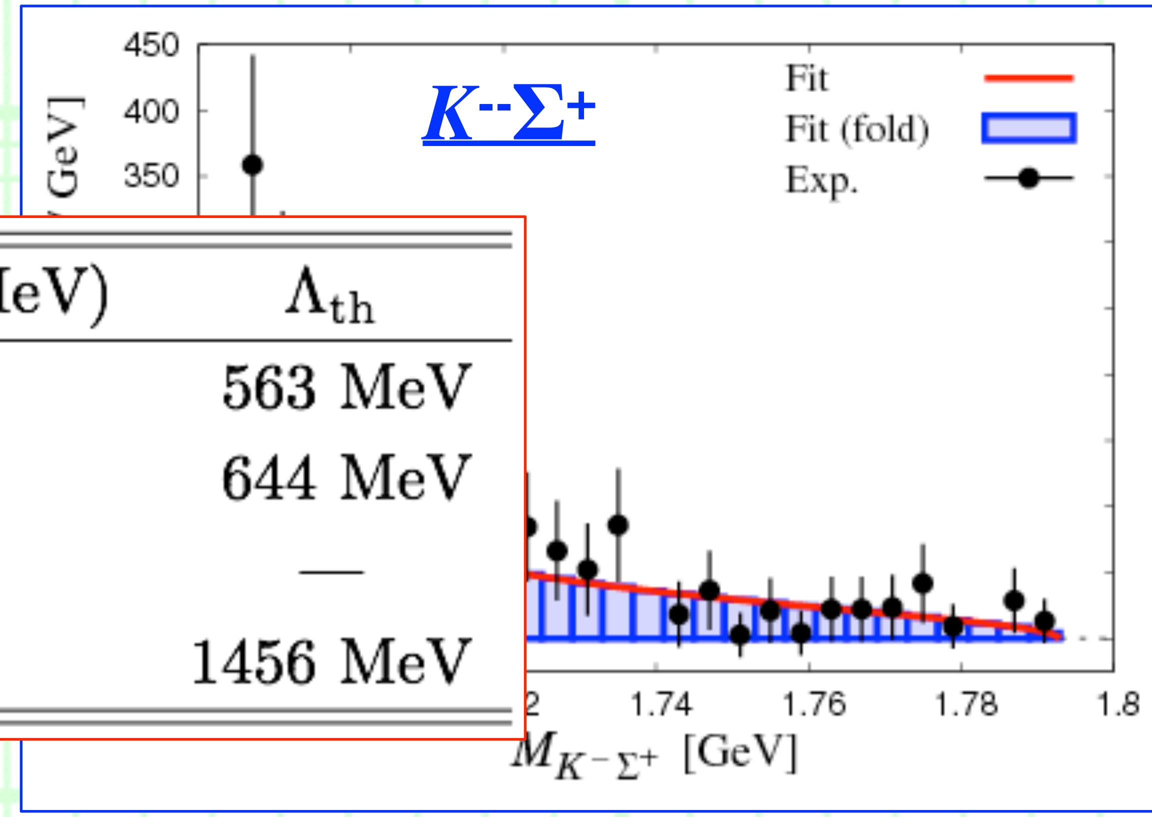




	$a(\mu_{\text{reg}} = 630\ \text{MeV})$	Λ_{th}
$\bar{K}\Sigma$	-1.98	563 MeV
$\bar{K}\Lambda$	-2.07	644 MeV
$\pi\Xi$	-0.75	—
$\eta\Xi$	-3.31	1456 MeV

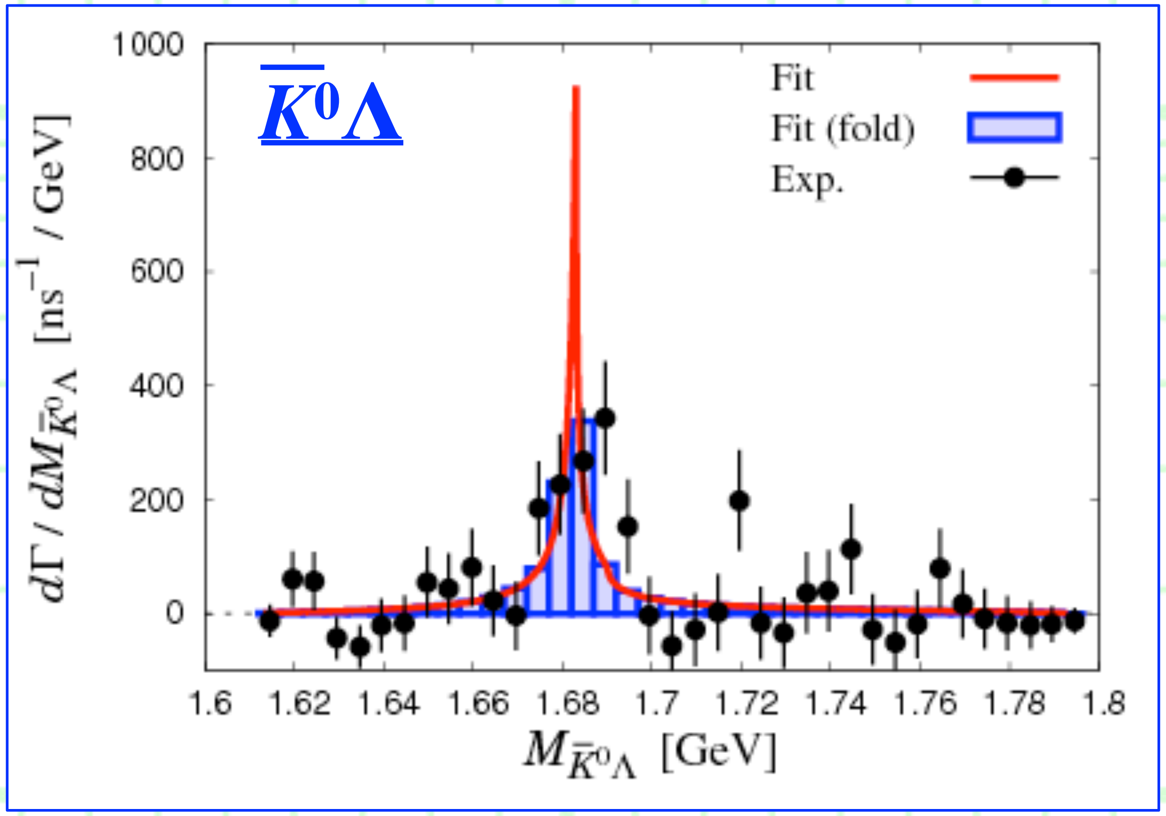
1. The Belle data on Ξ(1690) are **reproduced qualitatively well**.
2. Subtraction constants are **"natural"** (except for $a_{\pi\Xi}$), as the values of the corresponding three-dimensional cut-off at the threshold, Λ_{th}, is about 500 - 1500 MeV.

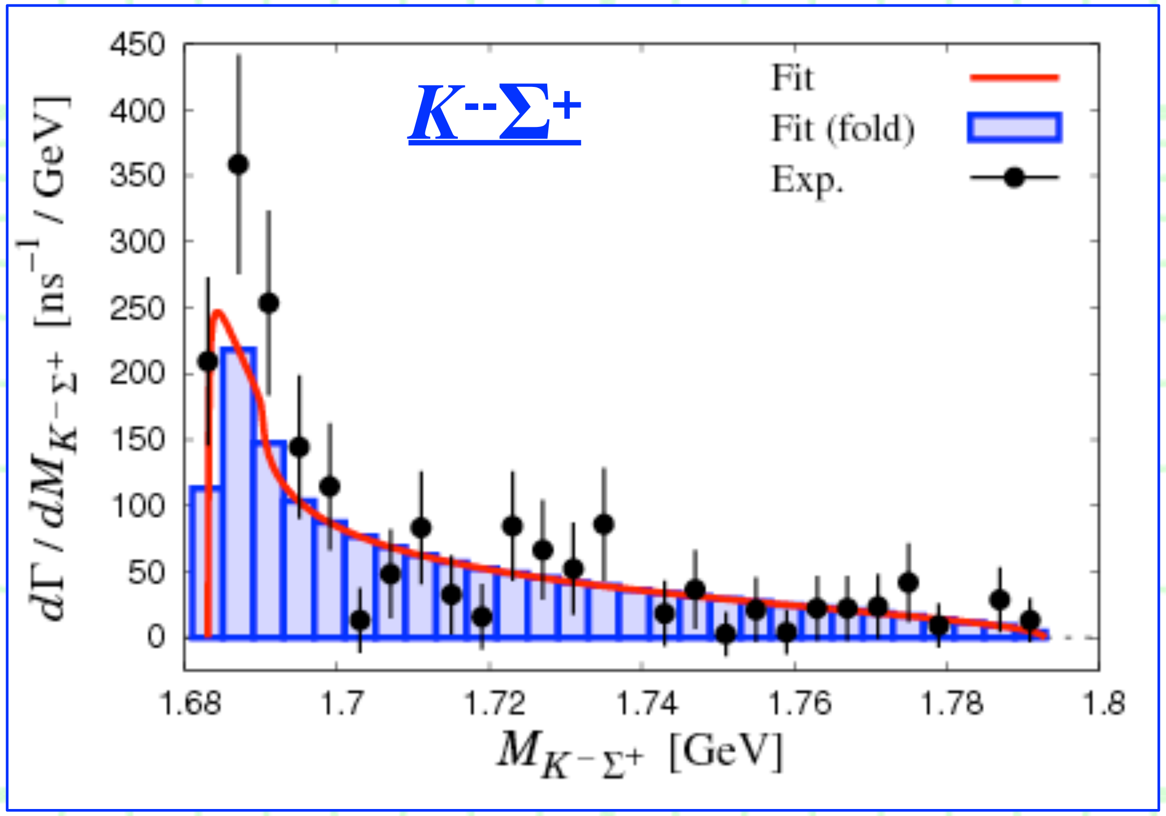
--- The πΞ channel negligibly contributes to Ξ(1690).

3. Results and discussions

++ Fitting to the Belle data ++

- We fix 4 free parameters ($a_{K\Sigma}$, $a_{K\Lambda}$, $a_{\pi\Xi}$, and $a_{\eta\Xi}$) so as to reproduce the mass spectra by Belle. The result of the best fit is:





1. The Belle data on $\Xi(1690)$ are reproduced qualitatively well.
2. Subtraction constants are "natural" (except for $a_{\pi\Xi}$).
3. The $\Xi(1690)$ pole is dynamically generated at $1684.3 - 0.5\,i$ MeV, whose real part is between the $K^- \Sigma^+$ and the $\bar{K}^0 \Sigma^0$ thresholds.

--- Pole in the first Riemann sheet of both $\bar{K}\Sigma$ channels. --> "Cusp".

3. Results and discussions

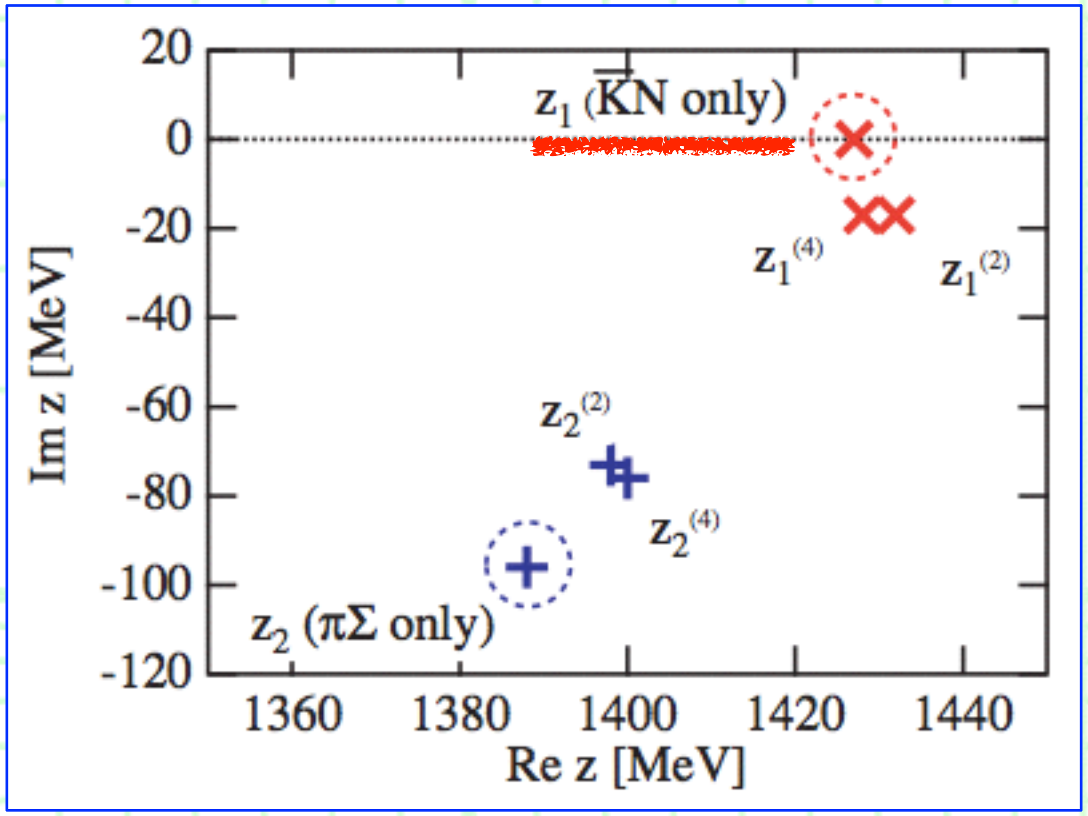
++ Origin of Ξ(1690) ++

- We naively expect that **the Ξ(1690)^0** (pole at 1684.3 -- 0.5 *i* MeV) **would originate from the $\bar{K}\Sigma(I=1/2)$ bound state** generated by the strongly attractive interaction between $\bar{K}\Sigma(I=1/2)$.

--- *cf*. The strongly attractive $\bar{K}N(I=0)$ interaction for Λ(1405).

- Thus we consider **a $\bar{K}\Sigma(I=1/2)$ single channel problem** (isospin basis), in which a bound state would appear at the energy of $V^{-1} = G$.

C_{jk}	$\bar{K}\Sigma$	$\bar{K}\Lambda$	$\pi\Xi$	$\eta\Xi$
$\bar{K}\Sigma$	2	0	−1/2	3/2
$\bar{K}\Lambda$	0	0	−3/2	−3/2
$\pi\Xi$	−1/2	−3/2	2	0
$\eta\Xi$	3/2	−3/2	0	0



For Λ(1405).

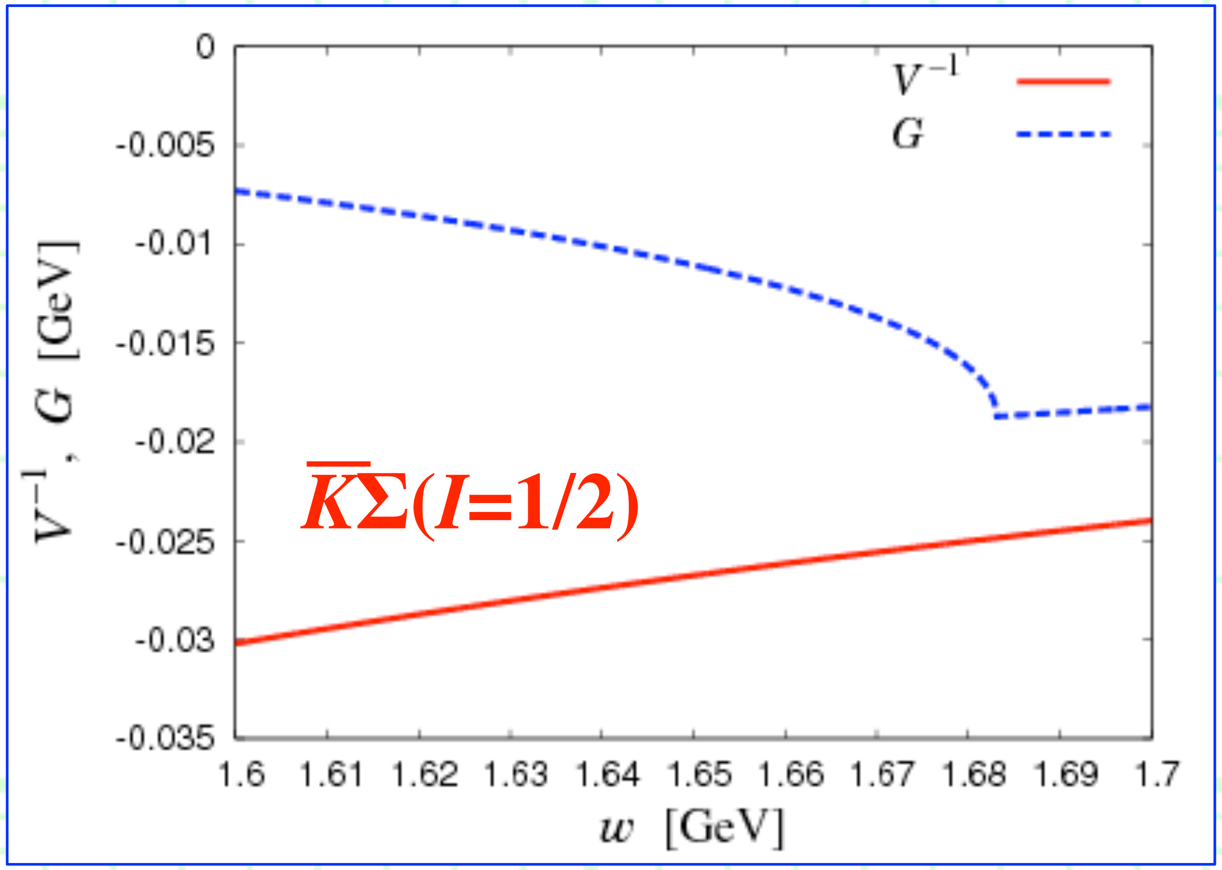
T. Hyodo and W. Weise, *Phys. Rev.* C77 (2008) 035204.

(isospin basis)

3. Results and discussions

++ Origin of Ξ(1690) ++

- We consider a $\bar{K}\Sigma(I=1/2)$ single channel problem (isospin basis), in which a bound state would appear at the energy of $V^{-1} = G$.



- □ V^{-1} is below G, which means that the chiral $\bar{K}\Sigma$ interaction is attractive but not strong enough to generate a bound state in a single channel case.

--- In contrast to the $\bar{K}N(I=0)$ Int. , which can solely generate a bound state for Λ(1405).

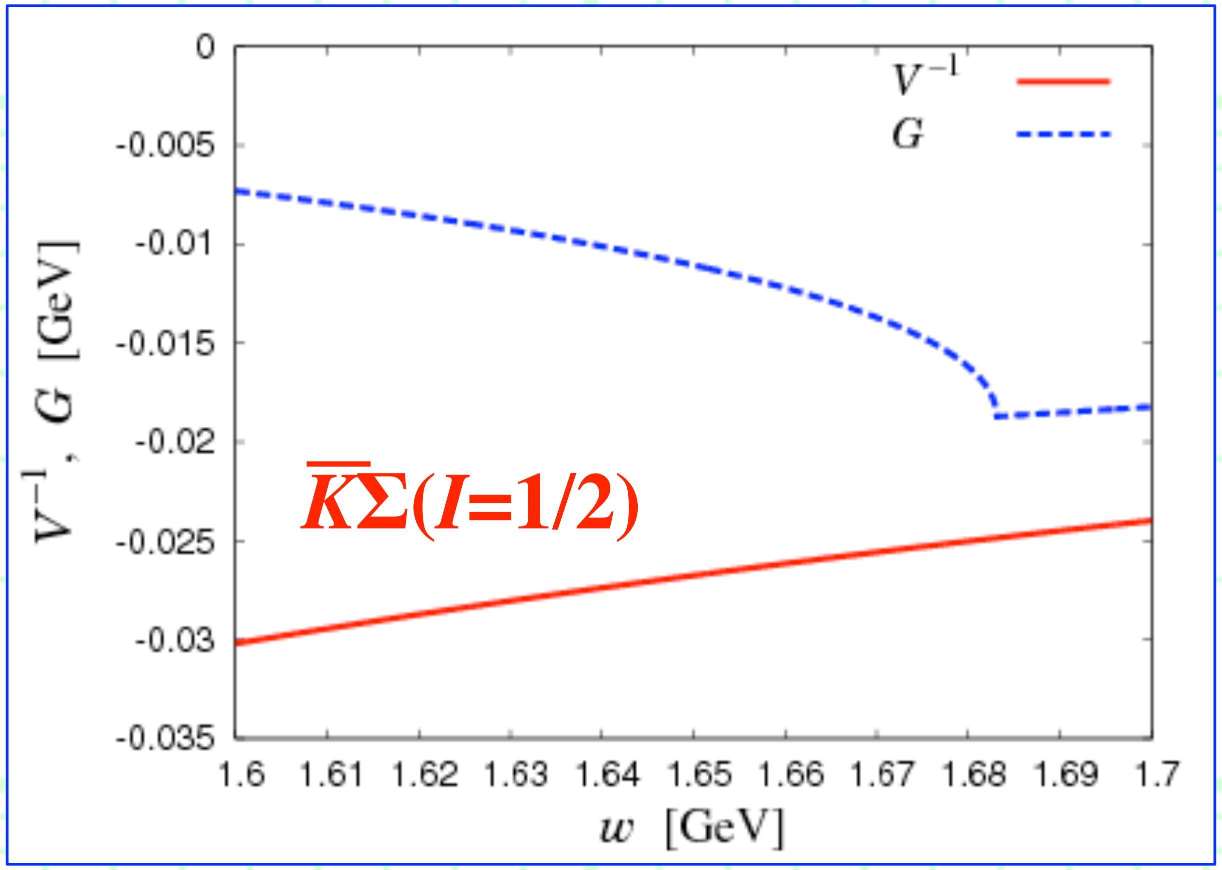
C_{jk}	$\bar{K}\Sigma$	$\bar{K}\Lambda$	$\pi\Xi$	$\eta\Xi$
$\bar{K}\Sigma$	2	0	$-1/2$	$3/2$
$\bar{K}\Lambda$	0	0	$-3/2$	$-3/2$
$\pi\Xi$	$-1/2$	$-3/2$	2	0
$\eta\Xi$	$3/2$	$-3/2$	0	0

C_{jk}	$\bar{K}N$	$\pi\Sigma$	$\eta\Lambda$	$K\Xi$
$\bar{K}N$	3	$-\sqrt{3/2}$	$3/\sqrt{2}$	0
$\pi\Sigma$	$-\sqrt{3/2}$	4	0	$\sqrt{3/2}$
$\eta\Lambda$	$3/\sqrt{2}$	0	0	$-3/\sqrt{2}$
$K\Xi$	0	$\sqrt{3/2}$	$-3/\sqrt{2}$	3

3. Results and discussions

++ Origin of $\Xi(1690)$ ++

- We consider **a $\bar{K}\Sigma(I=1/2)$ single channel problem** (isospin basis), in which a bound state would appear at the energy of $V^{-1} = G$.



□ V^{-1} is below G, which means that the chiral $\bar{K}\Sigma$ interaction is attractive but not strong enough to generate a bound state in a single channel case.

--- In contrast to the $\bar{K}N(I=0)$ Int. , which can solely generate a bound state for $\Lambda(1405)$.

- This fact implies that the multiple scatterings, such as $\bar{K}\Sigma$ --> $\eta\Xi$ --> $\bar{K}\Sigma$, assist the $\bar{K}\Sigma$ interaction in dynamically generating $\Xi(1690)$ as a $\bar{K}\Sigma$ quasi-bound state which is located very close to the $\bar{K}\Sigma$ threshold.

3. Results and discussions

++ Small decay width ++

- In addition, **the structure of the interaction strength** qualitatively explains **a remarkable property of Ξ(1690)^0, its very small width**: $\Gamma = -\,2\,\mathrm{Im}(\,w_{\rm pole}\,) \sim 1$ MeV.

C_{jk}	$\bar{K}\Sigma$	$\bar{K}\Lambda$	$\pi\Xi$	$\eta\Xi$
$\bar{K}\Sigma$	2	0	$-1/2$	$3/2$
$\bar{K}\Lambda$	0	0	$-3/2$	$-3/2$
$\pi\Xi$	$-1/2$	$-3/2$	2	0
$\eta\Xi$	$3/2$	$-3/2$	0	0

(I = 1/2, isospin basis)

1. Transition of $\bar{K}\Sigma$ <--> $\bar{K}\Lambda$ is forbidden at the leading order ($C_{jk} = 0$), so the $\bar{K}\Sigma$ --> $\bar{K}\Lambda$ --> $\bar{K}\Sigma$ multiple process gives **zero**.
2. $\bar{K}\Sigma$ <--> $\pi\Xi$ is not strong compared to, *e.g.*, $\bar{K}N(I=0)$ <--> $\pi\Sigma$.
   --- $C_{jk} = -\,0.5$ vs. $-\sqrt{1.5} = -\,1.22$ ... .
3. $\bar{K}\Sigma$ <--> $\eta\Xi$ is the strongest.

C_{jk}	$\bar{K}N$	$\pi\Sigma$	$\eta\Lambda$	$K\Xi$
$\bar{K}N$	3	$-\sqrt{3/2}$	$3/\sqrt{2}$	0
$\pi\Sigma$	$-\sqrt{3/2}$	4	0	$\sqrt{3/2}$
$\eta\Lambda$	$3/\sqrt{2}$	0	0	$-3/\sqrt{2}$
$K\Xi$	0	$\sqrt{3/2}$	$-3/\sqrt{2}$	3

--> As a consequence, the $\eta\Xi$ channel is most important in the multiple scatterings for $\bar{K}\Sigma$ to dynamically generate **Ξ(1690) which cannot couple strongly to $\bar{K}\Lambda$ nor $\pi\Xi$**.

--- This reproduces small decay width and tiny BR fraction to $\pi\Xi$.

3. Results and discussions

++ Comparison with previous ChUA calculations ++

- The discussion on <u>the $\bar{K}\Sigma$ interaction</u> can be further utilized for **comparison of our result on Ξ(1690)** (pole at 1684.3 -- 0.5 *i* MeV) **with previous ones in chiral unitary approach.**

$(\frac{1}{2}, -2)$		$[\pi\Xi]$ 7.5	5.6	seen	2.6
$\Xi(1620)^{*}$		$[\bar{K}\Lambda]$ 5.2	2.8	seen	−1.5
$M \approx 1620$	1565	$[\bar{K}\Sigma]$ 0.7	2.6	0	−0.8
$\Gamma = 23$	247	$[\eta\Xi]$ 0.3	4.9	0	0.3
$(\frac{1}{2}, -2)$		$[\pi\Xi]$ 0.02	0.1	seen	−0.1
$\Xi(1690)^{***}$		$[\bar{K}\Lambda]$ 0.16	6.0	seen	0.9
$M = 1690 \pm 10$	1663	$[\bar{K}\Sigma]$ 5.15	3.1	seen	−2.5
$\Gamma = 10 \pm 6$	4	$[\eta\Xi]$ 2.28	3.2	0	−1.7

<--> <u>Qualitatively similar</u>, but **the mass** (= real part of the pole position) **of our result is 20 - 30 MeV larger than others.**

C. Garcia-Recio, M. F. M. Lutz and J. Nieves, *Phys. Lett.* <u>B582</u> (2004) 49.

8 (1134)	2037–24i	0.6	0.6	0.3	0.2	0.3	↑0.5	1.5	0.6	**1.8**	**2.4**	1.1	0.2	1.0	**2.1**	
10 (70)	1729–46i	0.6	1.4	0.4	↑1.6	1.4	**2.1**	1.0	0.4	**3.3**	1.5	0.4	0.2	1.6	1.0	Ξ(1950) ★★★
8 (70)	1651–2i	0.2	0.3	↑**2.2**	1.3	1.0	**2.6**	0.2	0.6	0.9	0.4	0.2	**1.7**	0.4	0.2	Ξ(1690) ★★★
8 (56)	1577–139i	**2.6**	↑**1.7**	0.5	0.1	0.8	1.0	0.7	0.1	0.6	1.3	0.3	0.1	0.2	1.2	Ξ(1620) ★

D. Gamermann, C. Garcia-Recio, J. Nieves and L. L. Salcedo, *Phys. Rev.* <u>D84</u> (2011) 056017.

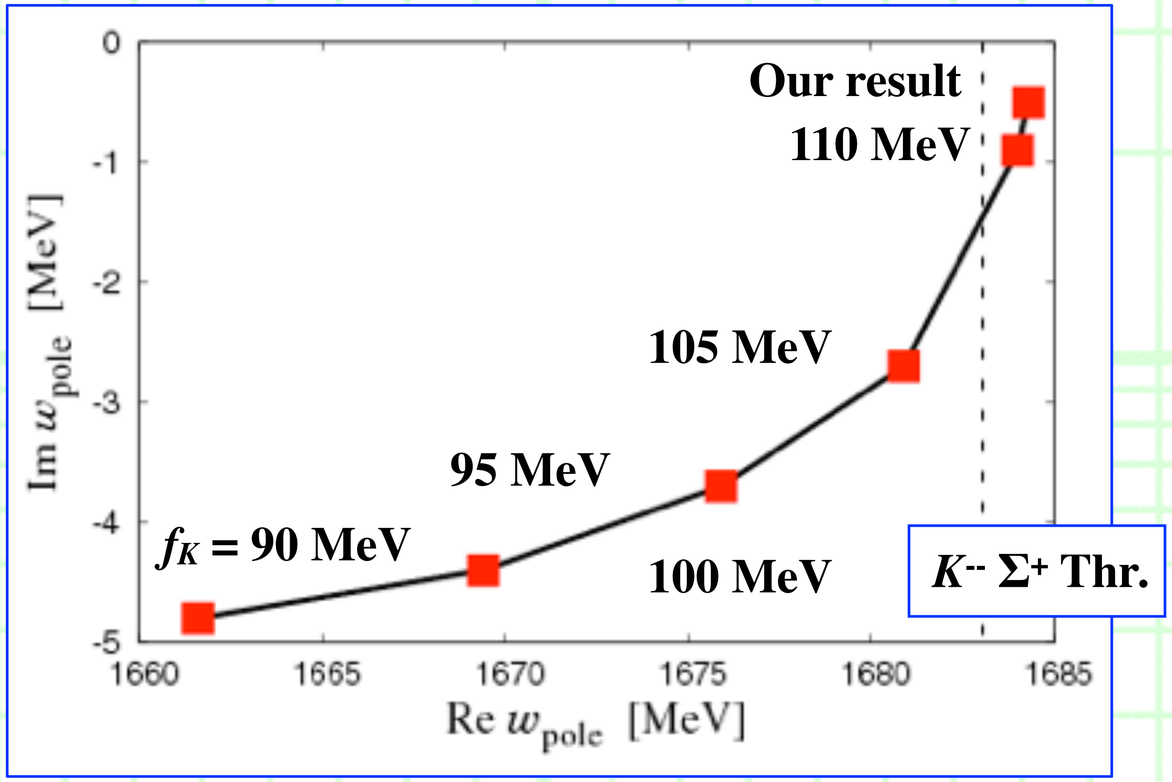
3. Results and discussions

++ Comparison with previous ChUA calculations ++

- **The discussion on the $\bar{K}\Sigma$ interaction can be further utilized for comparison of our result on Ξ(1690) (pole at 1684.3 -- 0.5 *i* MeV) with previous ones in chiral unitary approach.**

□ **In Ref. [1] they used the meson decay constant f = 90 MeV in all channels, while we use their physical values (f_K = 110.64 MeV).**

--> The Ξ(1690) pole moves as:



□ **In Ref. [2] they introduced channels with vector mesons, which would assist more the $\bar{K}\Sigma$ interaction, and hence the mass of Ξ(1690) shifted to lower energies.**

[1] C. Garcia-Recio, M. F. M. Lutz and J. Nieves, *Phys. Lett.* B582 (2004) 49.

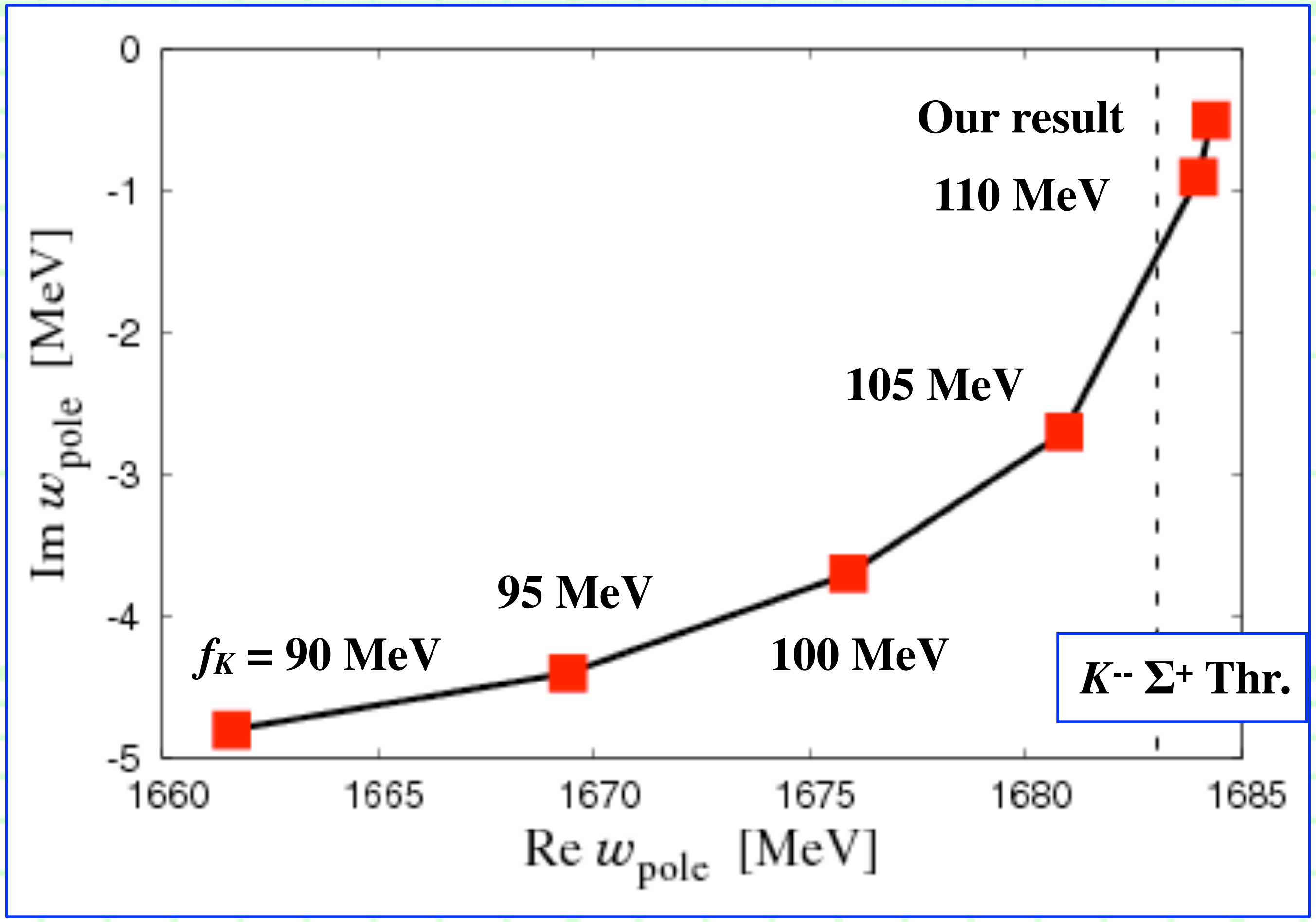
[2] D. Gamermann, C. Garcia-Recio, J. Nieves and L. L. Salcedo, *Phys. Rev.* D84 (2011) 056017.

3. Results and discussions

++ Compositeness for Ξ(1690) ++

- **Our Ξ(1690) pole** exists at 1684.3 -- 0.5 *i* MeV, whose real part is **very close to the $K^- \Sigma^+$ threshold** (= 1863.1 MeV).

--- The pole exists in the first Riemann sheet of the $K^- \Sigma^+$ channel.



□ **Our Ξ(1690) state should be genuinely $\bar{K}\Sigma$ composite !** (coupled-channels version)

□ **"Theorem" (single channel)**: The bound state with the field renormalization const. $Z \sim 0$ naturally appears when the state exists near the threshold, and especially Z vanishes in the limit B --> 0.

--> The state should be **genuinely composite**.

T. Hyodo, *Phys. Rev.* C90 (2014) 055208;
C. Hanhart, J. R. Pelaez and G. Rios, *Phys. Lett.* B739 (2014) 375.

3. Results and discussions

++ Compositeness for Ξ(1690) ++

- **Our Ξ(1690) pole** exists at 1684.3 -- 0.5 *i* MeV, whose real part is **very close to the $K^-\Sigma^+$ threshold** (= 1863.1 MeV).

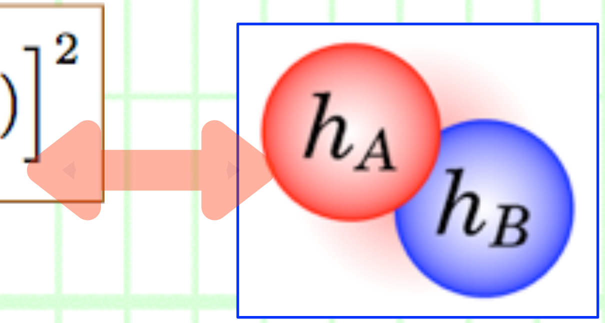
--- The pole exists in the first Riemann sheet of the $K^-\Sigma^+$ channel.

□ Its $\bar{K}\Sigma$ component can be measured in terms of the **compositeness**, which is defined as the contribution of the two-body component to **the normalization of the total wave function**.

Hyodo, *Int. J. Mod. Phys.* A28 (2013) 1330045; T. S. , Hyodo and Jido, *PTEP* (2015) 063D04.

$$\langle \tilde{\Psi} | \Psi \rangle = X + Z = 1$$

$$X = \int \frac{d^3 q}{(2\pi)^3} \langle \tilde{\Psi} | \mathbf{q} \rangle \langle \mathbf{q} | \Psi \rangle = \int \frac{d^3 q}{(2\pi)^3} \left[\tilde{\psi}(\mathbf{q}) \right]^2$$



$X_{K^-\Sigma^+}$	$0.83 - 0.31i$
$X_{\bar{K}^0\Sigma^0}$	$0.12 + 0.17i$
$X_{\bar{K}^0\Lambda}$	$-0.02 + 0.00i$
$X_{\pi^+\Xi^-}$	$0.00 + 0.00i$
$X_{\pi^0\Xi^0}$	$0.00 + 0.00i$
$X_{\eta\Xi^0}$	$0.01 + 0.02i$
Z	$0.06 + 0.11i$

□ From the result of compositeness, the $\bar{K}\Sigma$ compositeness really dominates the sum rule with small imaginary part.

--> **Strongly indicates that Ξ(1690) is indeed a $\bar{K}\Sigma$ molecular state.**

3. Results and discussions

++ Charged Ξ(1690) ++

- Finally we consider the charged Ξ(1690) in the same parameter set as the neutral one. As a result, we obtain the $\Xi(1690)^{-}$ pole as:

w_{pole}	$1693.4 - 10.5i$ MeV
$X_{\bar{K}^0\Sigma^-}$	$0.86 - 0.50i$
$X_{K^-\Sigma^0}$	$-0.27 + 0.31i$
$X_{K^-\Lambda}$	$-0.02 + 0.04i$
$X_{\pi^-\Xi^0}$	$0.00 + 0.00i$
$X_{\pi^0\Xi^-}$	$0.00 + 0.00i$
$X_{\eta\Xi^-}$	$0.07 + 0.03i$
Z	$0.36 + 0.12i$

- The $\Xi(1690)^{-}$ pole is located between the $K^-\Sigma^0$ and $\bar{K}^0\Sigma^-$ thresholds; The pole is in the first Riemann sheet of the $\bar{K}^0\Sigma^-$ and $\eta\Xi^-$ channels and in the second Riemann sheet of the $K^-\Lambda$, $K^-\Sigma^0$, $\pi^-\Xi^0$, and $\pi^0\Xi^-$ channels.

- The pole position has a larger imaginary part ~ 10 MeV compared to the neutral case, since it exists above the $\bar{K}^0\Sigma^-$ threshold in its second Riemann sheet and hence the decay to $\bar{K}^0\Sigma^-$ is allowed.

- Although both $X_{K0\Sigma-}$ and $X_{K-\Sigma0}$ have large imaginary part, sum of them is the dominant contribution with its small imaginary part, which implies that the $\Xi(1690)^{-}$ state is also a $\bar{K}\Sigma$ molecular state.

4. Summary

4. Summary

++ Summary ++

- We have investigated dynamics of $\bar{K}\Sigma$ and its coupled channels in the chiral unitary approach.
  - We employ the simplest interaction: Weinberg-Tomozawa term.
  - Subtraction constants as free parameters are fixed by fitting the $\bar{K}^0\Lambda$ and $K^-\Sigma^+$ mass spectra to the experimental data.

- As a result, we have found that:
  - The obtained scattering amplitude can qualitatively reproduce the experimental data of the $\bar{K}^0\Lambda$ and $K^-\Sigma^+$ mass spectra.
  - Dynamically generates a Ξ^* pole near the $\bar{K}\Sigma$ threshold as a $\bar{K}\Sigma$ molecule, which can be identified with the $\Xi(1690)^0$ resonance.
  - However, the $\bar{K}\Sigma$ interaction alone is slightly insufficient to bring a $\bar{K}\Sigma$ bound state, so multiple scattering is important for $\Xi(1690)$.
  - The small or vanishing couplings of the $\bar{K}\Sigma$ channel to others can naturally explain small decay width of $\Xi(1690)$.

4. Summary

++ Outlook ++

- **Theoretical study**:
  - **Propose reactions** which can **clarify properties** of the $\Xi(1690)$ resonance in experiments, both neutral and charged states.
  - **Predict the $\Xi(1690)$ production cross section**.
  - Improvement of model by, *e.g.*, introducing s- and u-channel Born terms.

- **Experimental study**:
  - **Determine J^P of the $\Xi(1690)$ resonance.**
  - Measure **the $\bar{K}\Lambda$ and $\bar{K}\Sigma$ mass spectra** and **ratio of their branching fractions**.
  - Furthermore, **precise determination of its pole position** should be important to discuss the internal structure of $\Xi(1690)$.

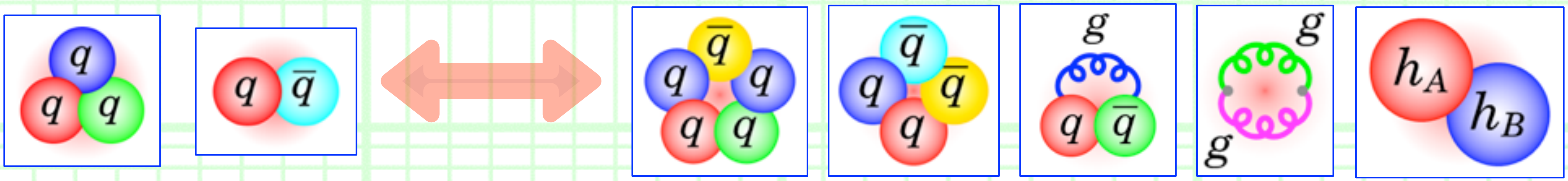
  --- **Flatte parameterization** may be necessary since it exists near the $\bar{K}\Sigma$ threshold.

5. Furthermore on exotic hadrons

5. Furthermore on exotic hadrons

++ Identify exotic hadrons ++

- **How can we identify exotic hadrons**, especially in Exps.?



- **Naïve**: compare with predictions from **constituent quark models**.
  - Mass, width, couplings, etc. of exotic hadrons **do not match the predictions from constituent quark models**.

  --> The constituent quark models can **support** the exotic nature of **exotic hadrons = not qqq nor $q\bar{q}$**.

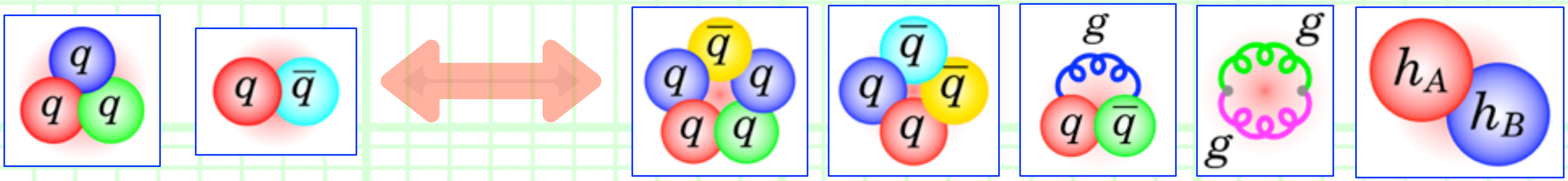
- However, constituent quark models (or, in general, **any effective models**) **cannot provide undoubted evidences of the exotic nature**, because constituent quarks are not "universal" for hadrons.
  - Constituent quarks are **not asymptotic states of QCD** !

--> We need some **approaches which do not rely on effective models of QCD** to identify the exotic hadrons.

5. Furthermore on exotic hadrons

++ Identify exotic hadrons ++

- **How can we identify exotic hadrons, especially in Exps.?**



--- What are crucial differences between ordinary and exotic ?

□ Spatial structure (= spatial size) of hadronic molecules.

--- Loosely bound hadronic molecules will have large spatial size.

T. S. , T. Hyodo and D. Jido (2008), (2011); T. S. and T. Hyodo (2013).

□ # of constituents is different.

--- However, # of constituents is usually not conserved due to the creation/annihilation of $q\bar{q}$ (*e.g.* $\bar{K}N$ <--> *uds* transition).

--> "Count" it by using the counting rule in high energy scattering.

H. Kawamura, S. Kumano and T. S. , *Phys. Rev.* D88 (2013) 034010.

□ Compositeness is introduced to identify hadronic molecules.

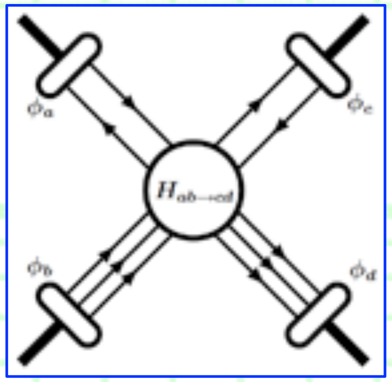
Hyodo, *Int. J. Mod. Phys.* A28 (2013) 1330045; T. S. , T. Hyodo and D. Jido, *PTEP* 2015 063D04; ...

5. Furthermore on exotic hadrons

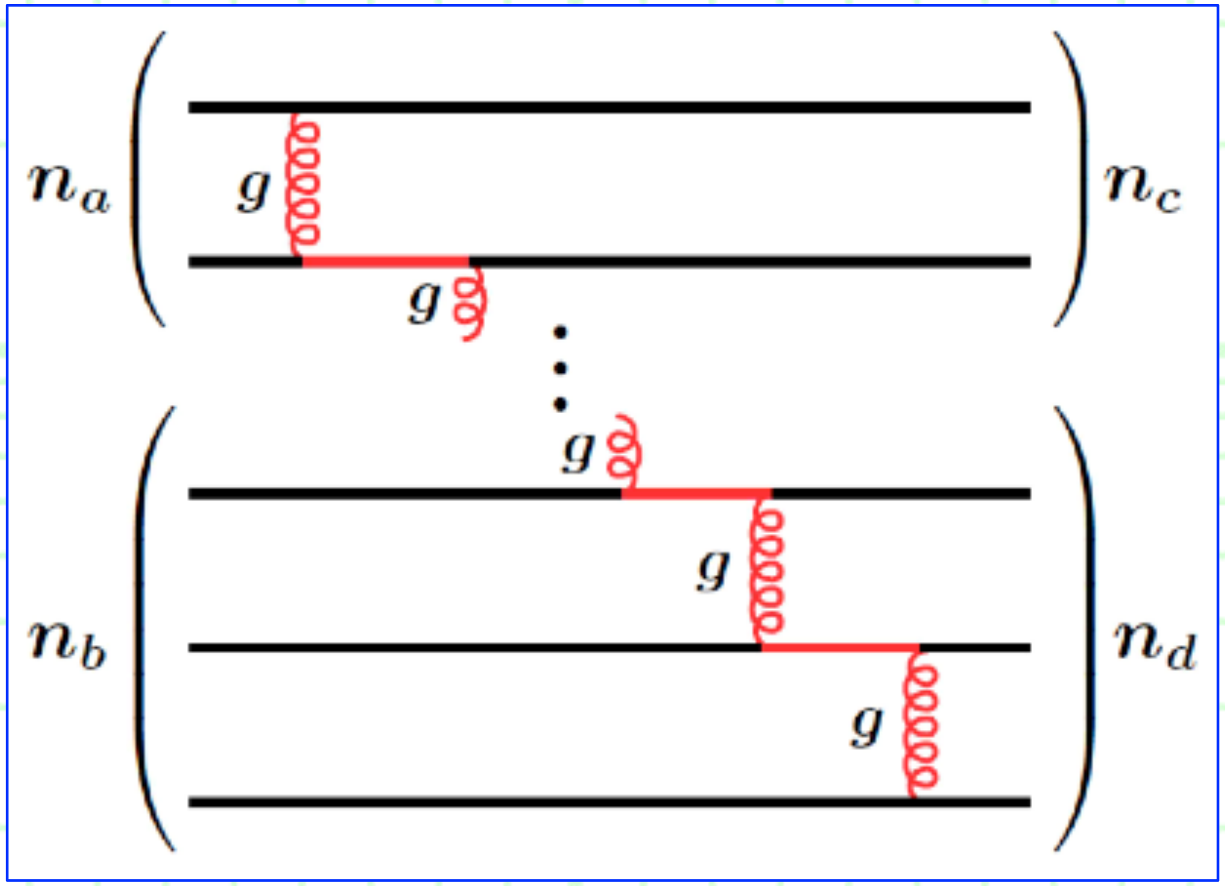
++ Counting rule for constituent quarks ++

- The constituent counting rule emerges in exclusive reactions at high energy and high momentum transfer region:

$$\left(\frac{d\sigma}{dt}\right)_{ab\to cd} \sim s^{2-n} \times f(\theta_{\rm cm}), \quad n \equiv n_a + n_b + n_c + n_d$$



Brodsky and Farar ('73, '75); Matveev *et al.* ('73).



- Consider $a\ b \to c\ d$ reaction in a large-angle exclusive process.

--- # of constituents: $n_a + n_b + n_c + n_d$.

□ Connect quarks by gluons.

□ Each gluon propagator ~ $1 / s$.

□ Each quark propagator ~ $1 / s^{1/2}$.

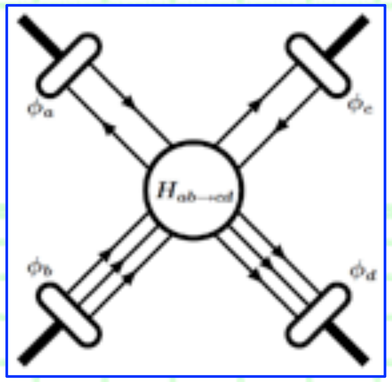
--> Count the power of $1 / s$ to obtain the scaling law.

5. Furthermore on exotic hadrons

++ Counting rule for constituent quarks ++

- The constituent counting rule emerges in exclusive reactions at high energy and high momentum transfer region:

$$\left(\frac{d\sigma}{dt}\right)_{ab\to cd} \sim s^{2-n} \times f(\theta_{\rm cm}), \quad n \equiv n_a + n_b + n_c + n_d$$



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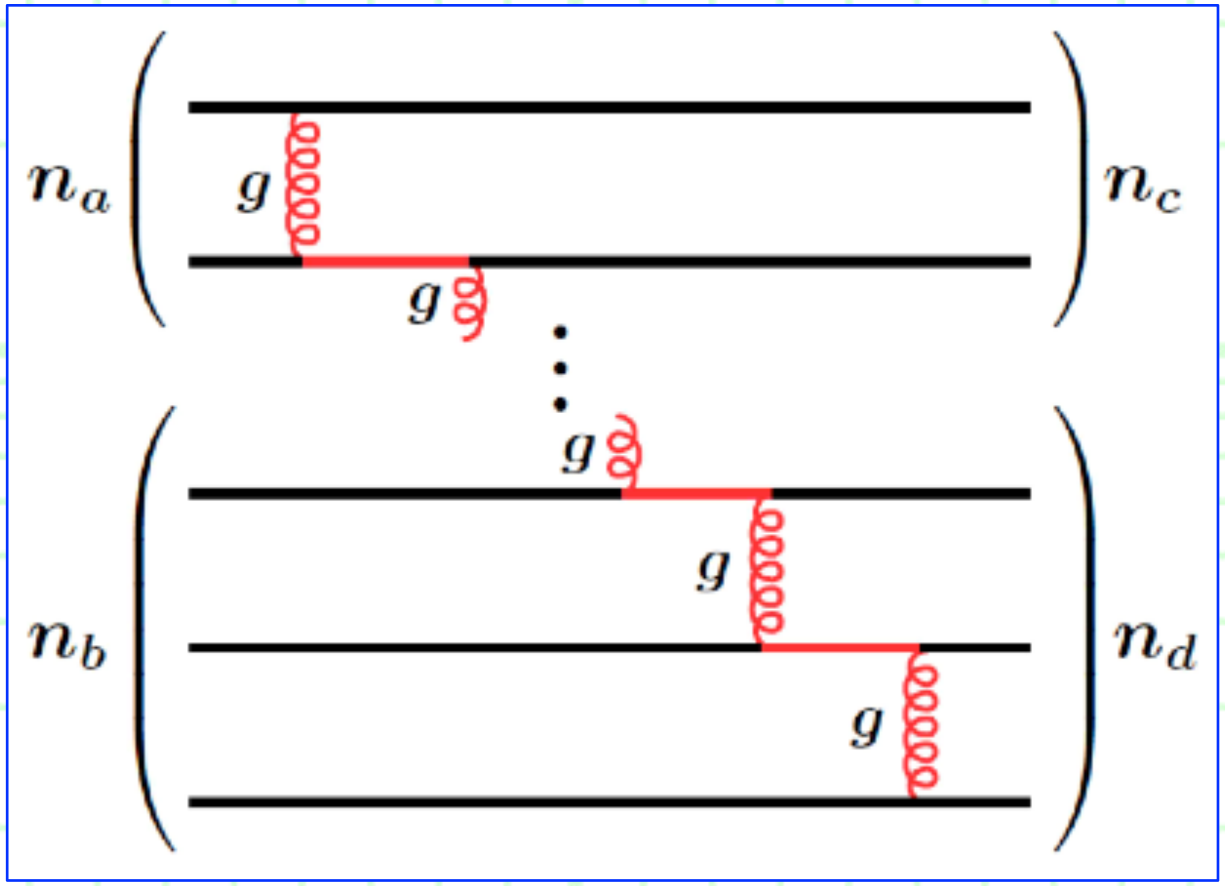

- Consider *a b --> c d* reaction in a large-angle exclusive process.
  1. High momentum reaction so as to apply pQCD.
  2. Large scattering angle so as to share the momenta.

--- Applicable to any hadrons as long as we can observe them.

5. Furthermore on exotic hadrons

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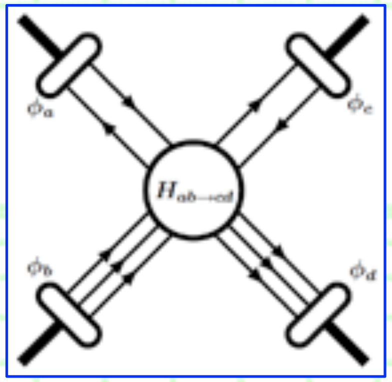
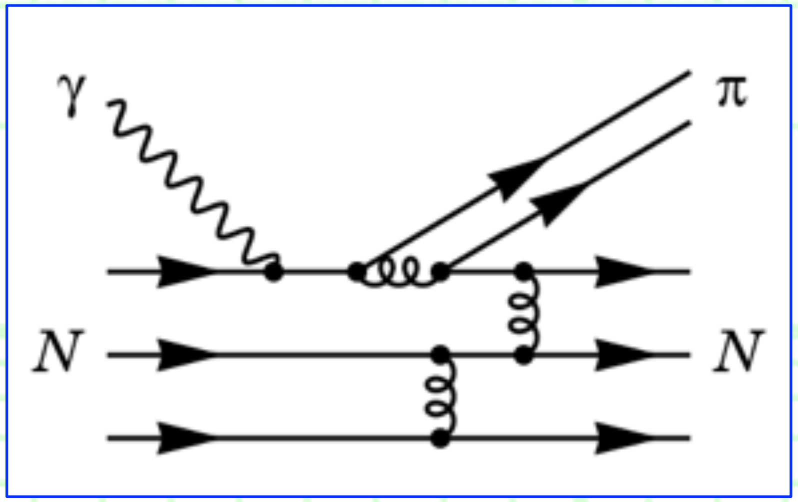
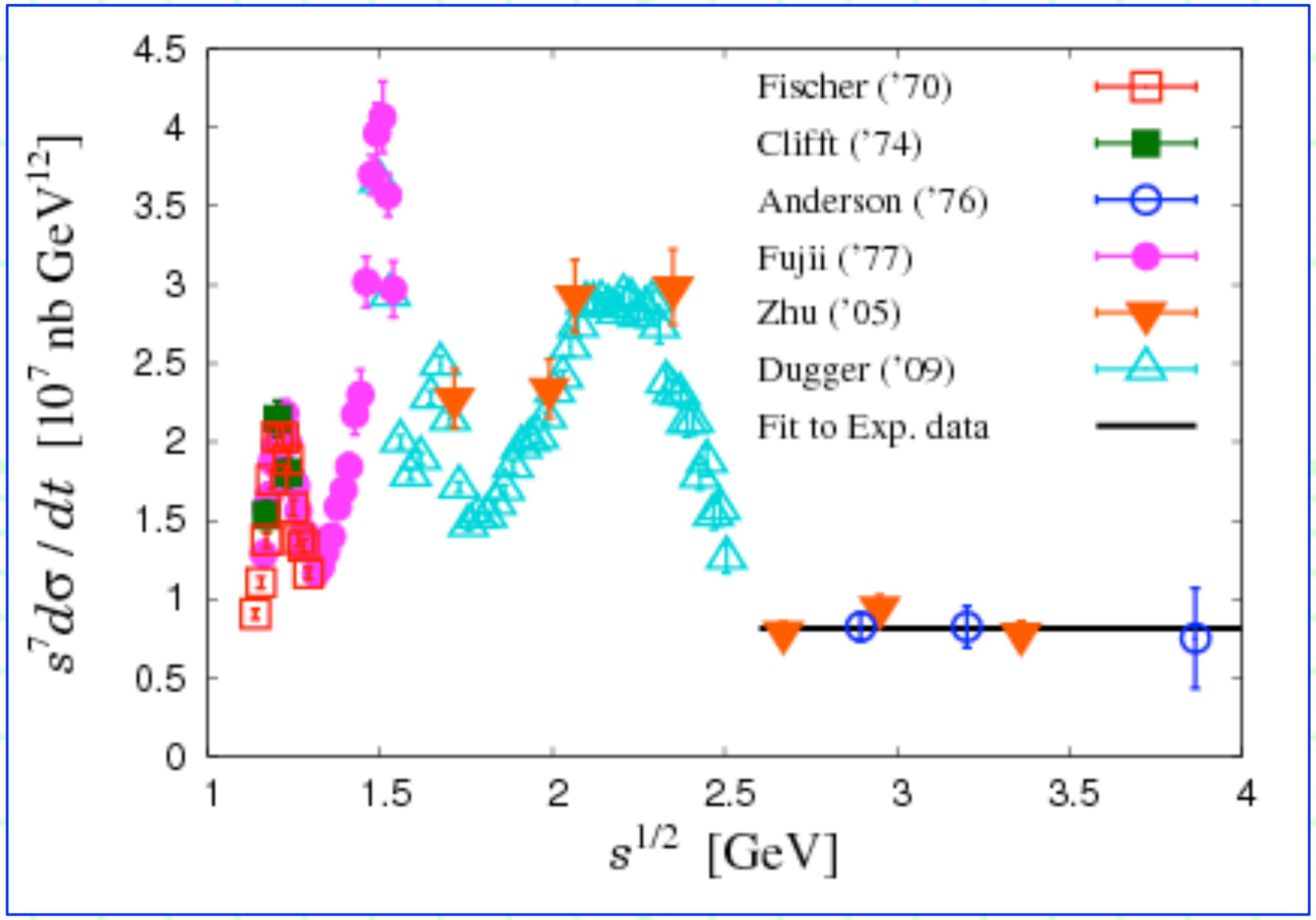


Brodsky and Farar ('73, '75); Matveev *et al.* ('73).

- **Ex. 1: $\gamma p \to \pi^+ n$ at $\theta_{\rm cm} = 90^{\rm o}$.**



***n* = 1+3+2+3 = 9.**



--- At High energy and high momentum transfer region, propagators scales as $\sim 1/t \sim 1/u \sim 1/s$.

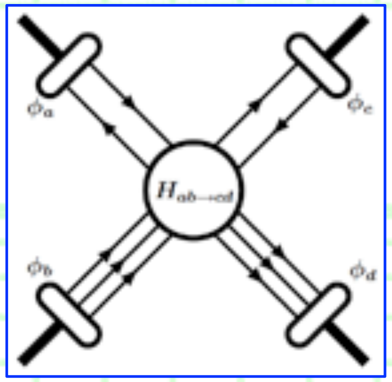
L.Y. Zhu *et al.*, *Phys. Rev. Lett.* 91 (2003) 022003; H. Kawamura, S. Kumano, and T. S. (2013).

5. Furthermore on exotic hadrons

++ Counting rule for constituent quarks ++

- **The constituent counting rule** emerges **in exclusive reactions at high energy and high momentum transfer region**:

$$\left(\frac{d\sigma}{dt}\right)_{ab\to cd} \sim s^{2-n} \times f(\theta_{\rm cm}), \quad n \equiv n_a + n_b + n_c + n_d$$



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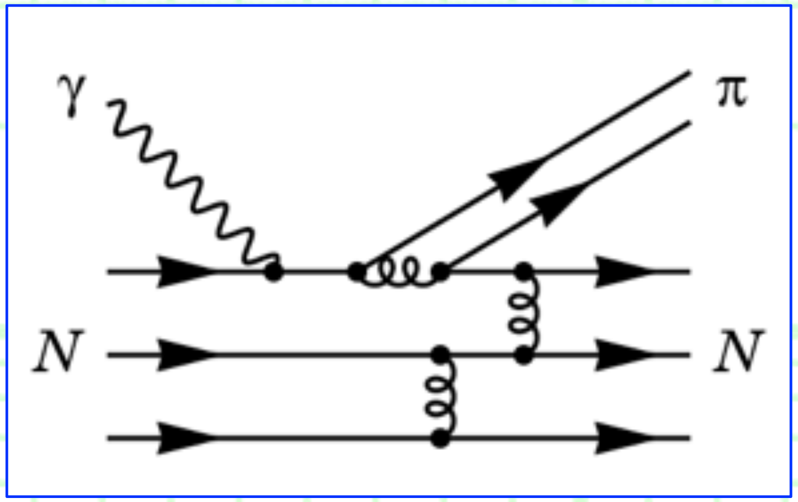
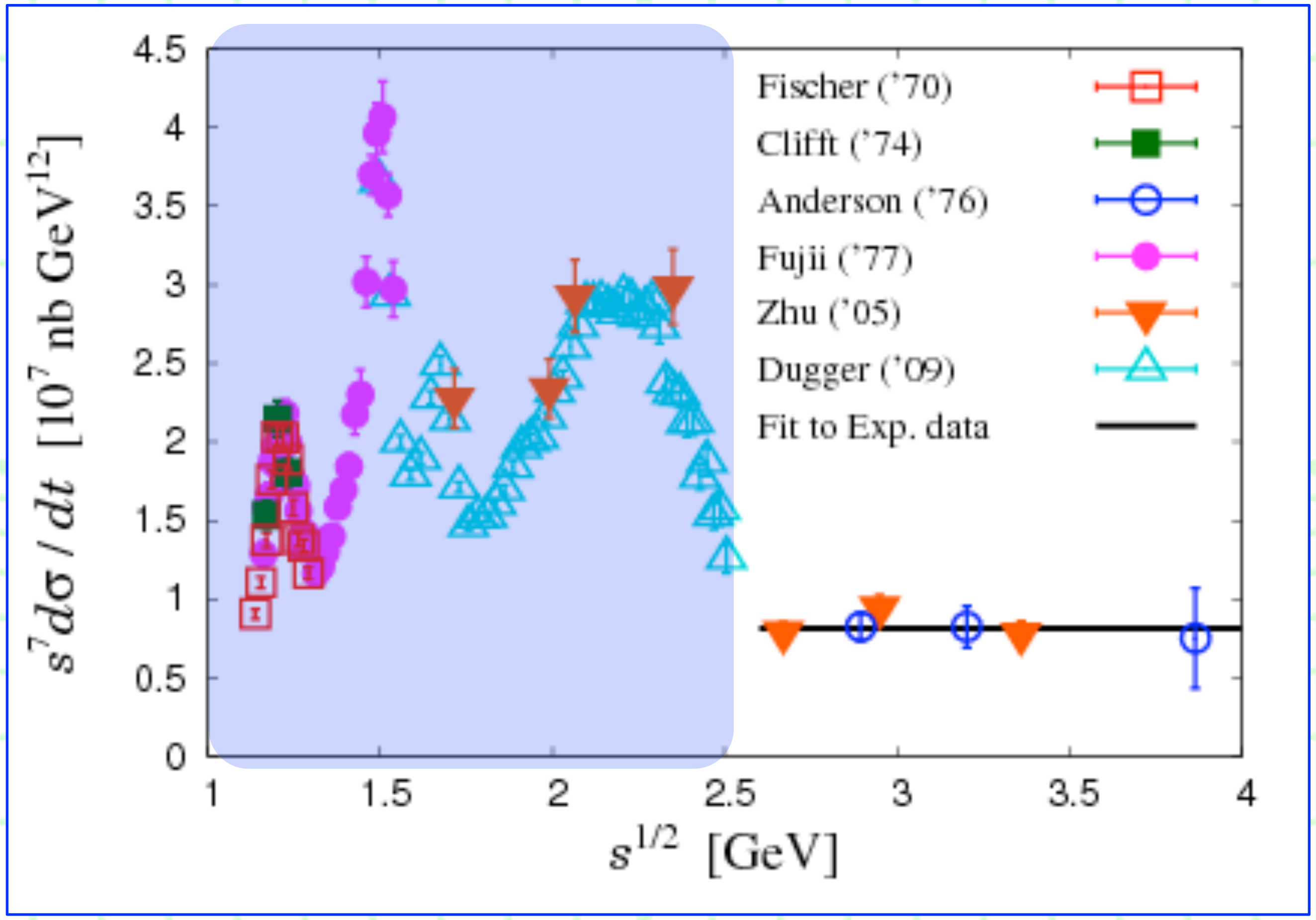


***n* = 1+3+2+3 = 9.**

--- At High energy and high momentum transfer region, propagators scales as ~ 1 / *t* ~ 1 / *u* ~ 1 / *s*.



L.Y. Zhu *et al.*, *Phys. Rev. Lett.* 91 (2003) 022003; H. Kawamura, S. Kumano, and T. S. (2013).

5. Furthermore on exotic hadrons

++ Counting rule for constituent quarks ++

- **The constituent counting rule** emerges **in exclusive reactions at high energy and high momentum transfer region**:

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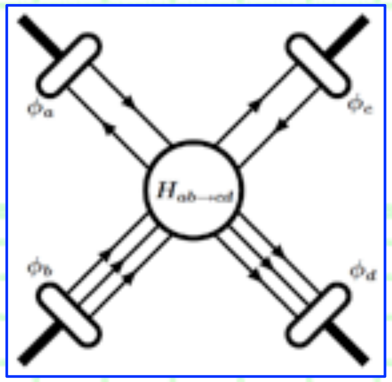

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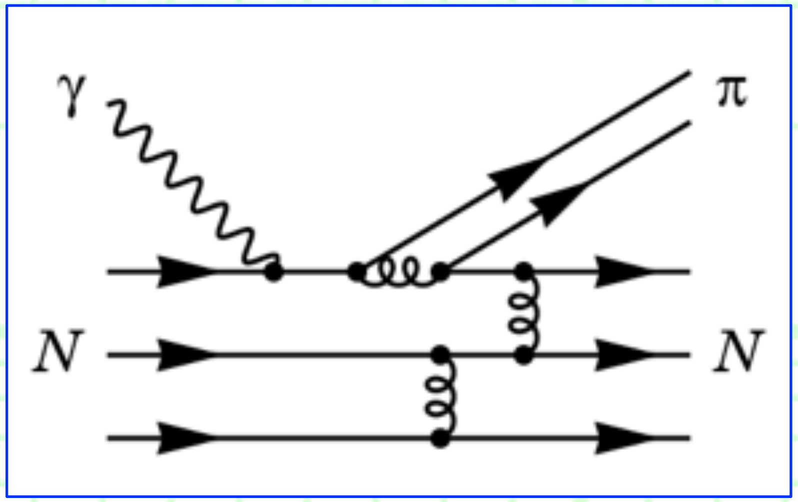
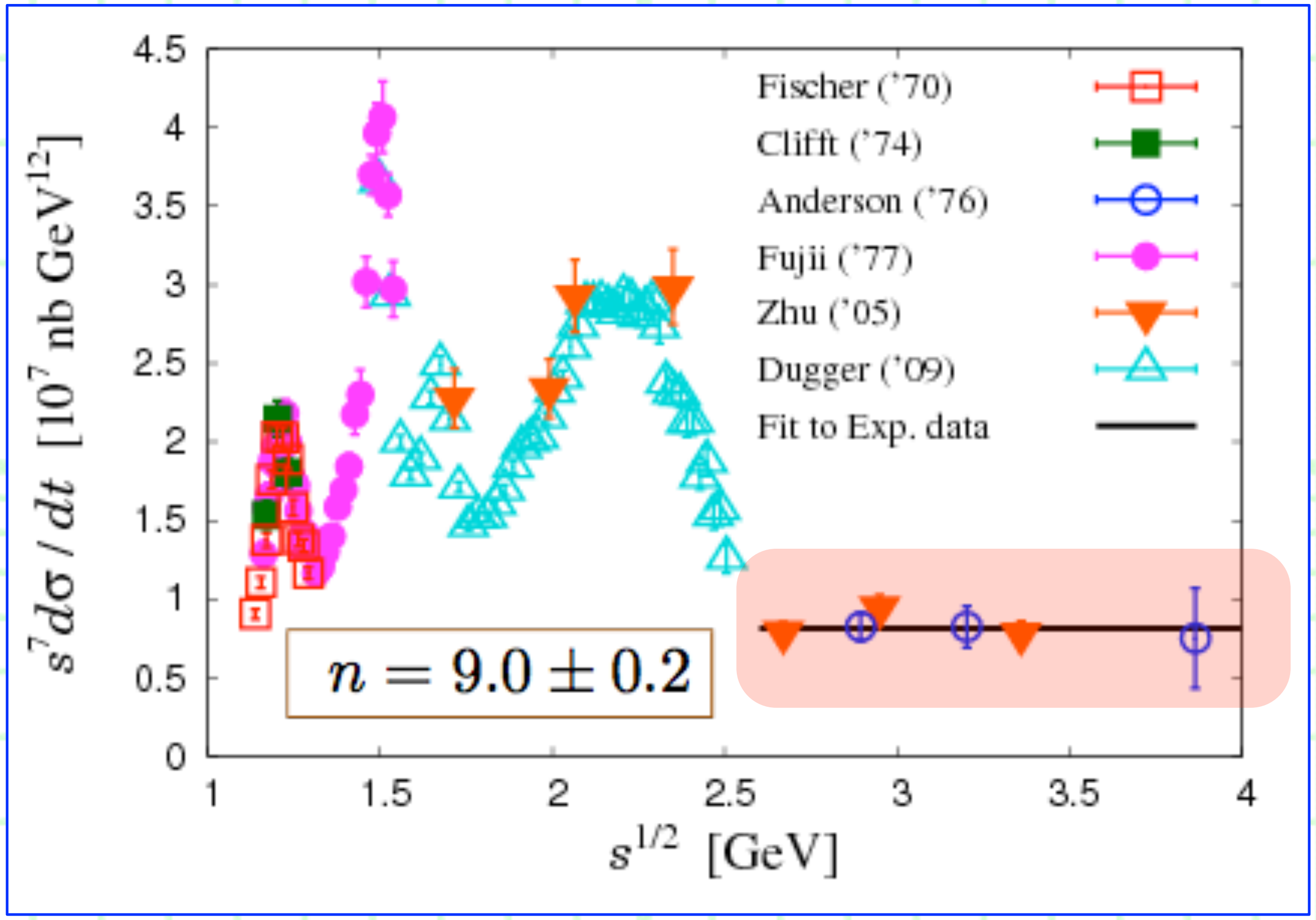
- **Ex. 1:** $\gamma\, p \to \pi^+ n$ at $\theta_{\rm cm} = 90^{\rm o}$.



$n = 1+3+2+3$
$= 9.$



--- At High energy and high momentum transfer region, **propagators scales as** $\sim 1/t \sim 1/u \sim 1/s$.

L.Y. Zhu *et al.*, *Phys. Rev. Lett.* 91 (2003) 022003;
H. Kawamura, S. Kumano, and T. S. (2013).

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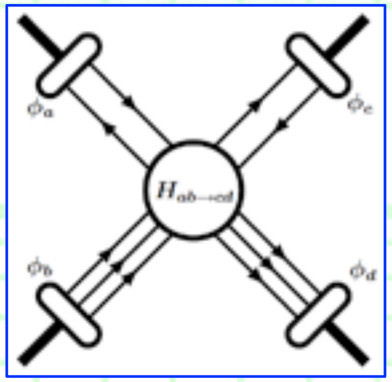

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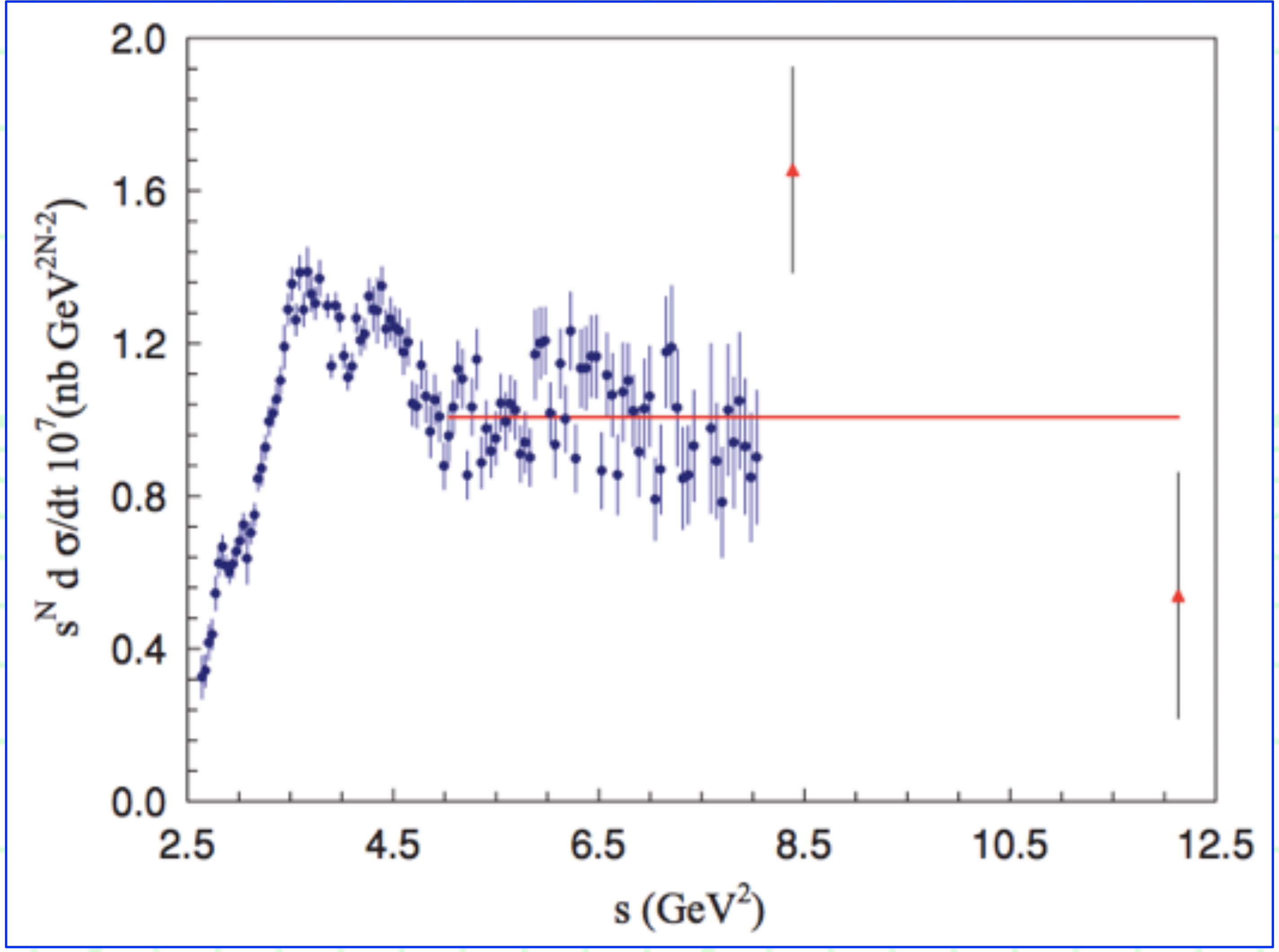
  - CLAS and SLAC data.



Schumacher and Sargsian, *Phys. Rev.* C83 (2011) 025207.

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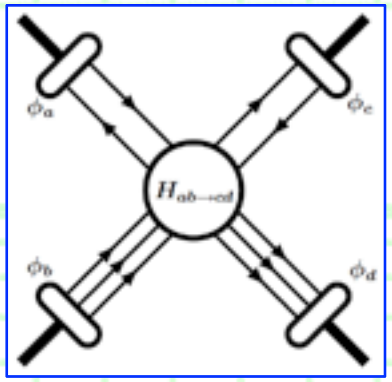

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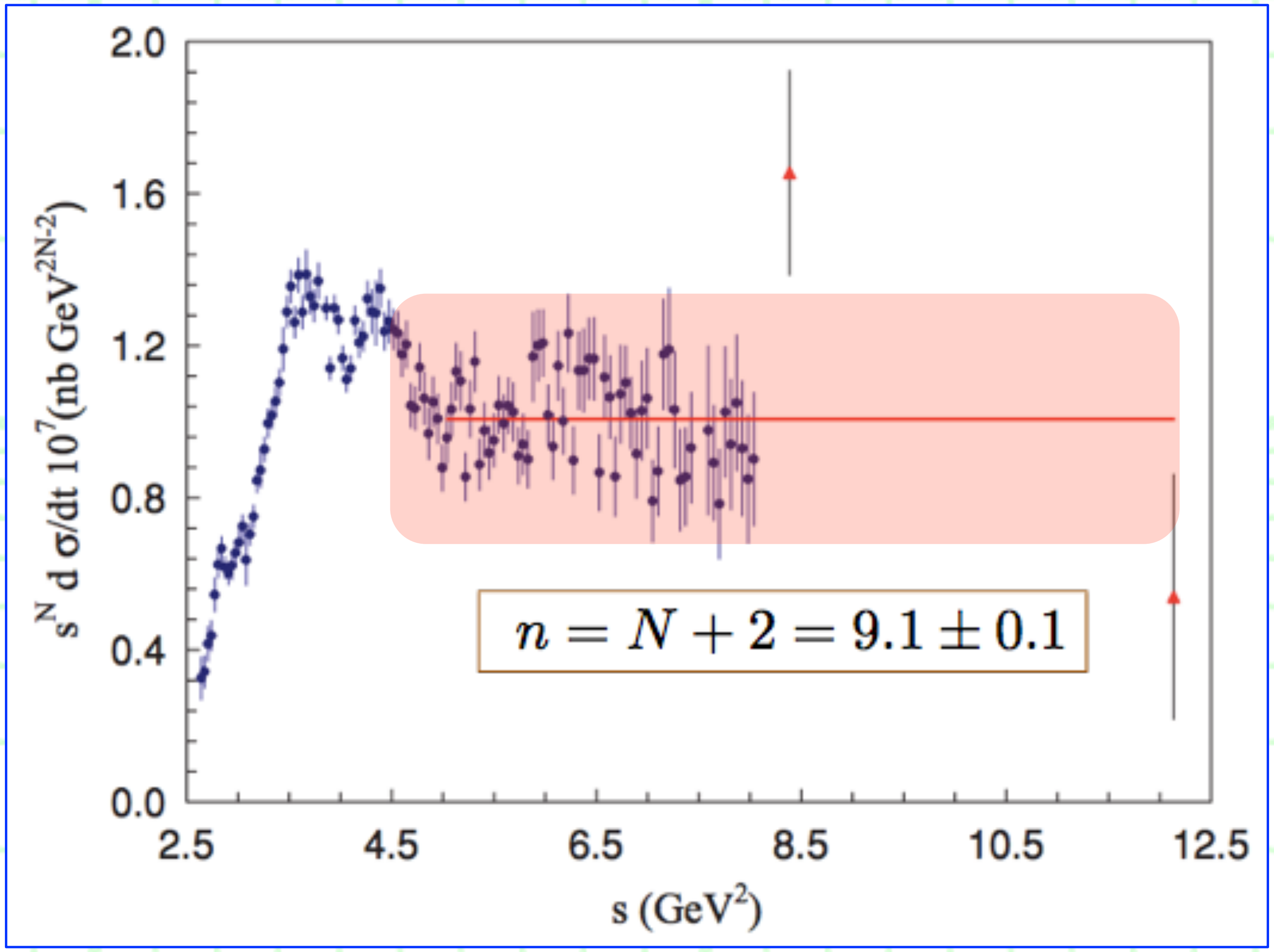

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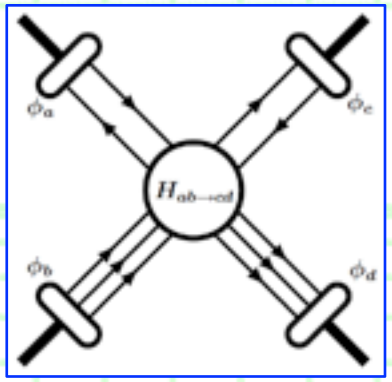

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Bertanza ('62); Yoder ('63); Goussu ('66); Dahl ('69); Binford ('69); Knasel ('75); Baker ('78); Saxon ('80).

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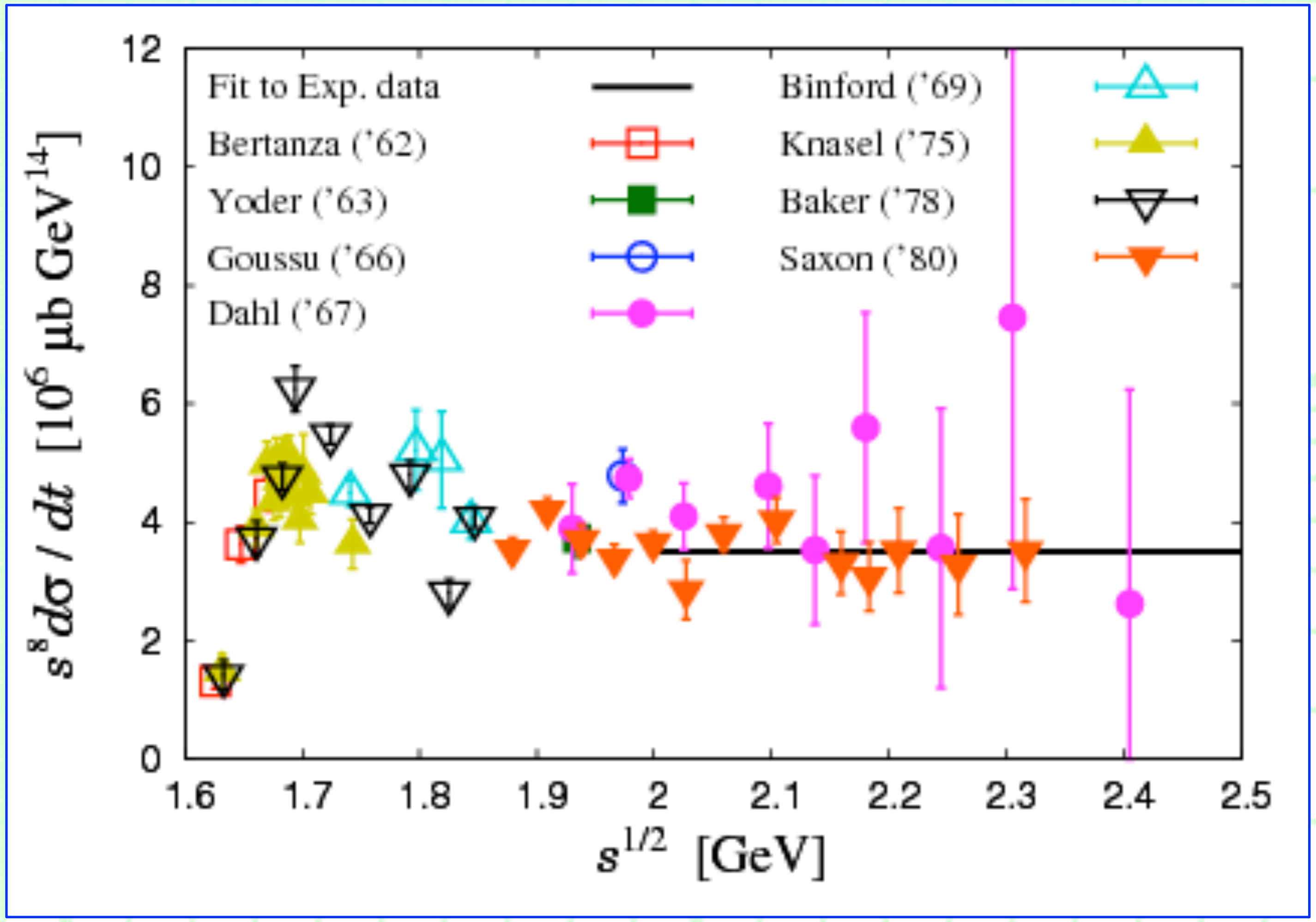

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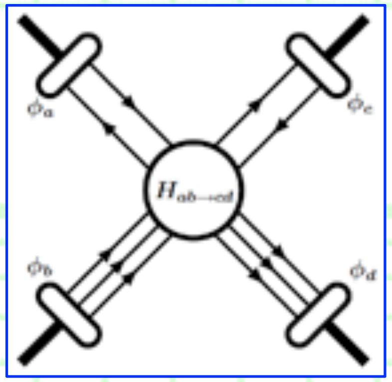

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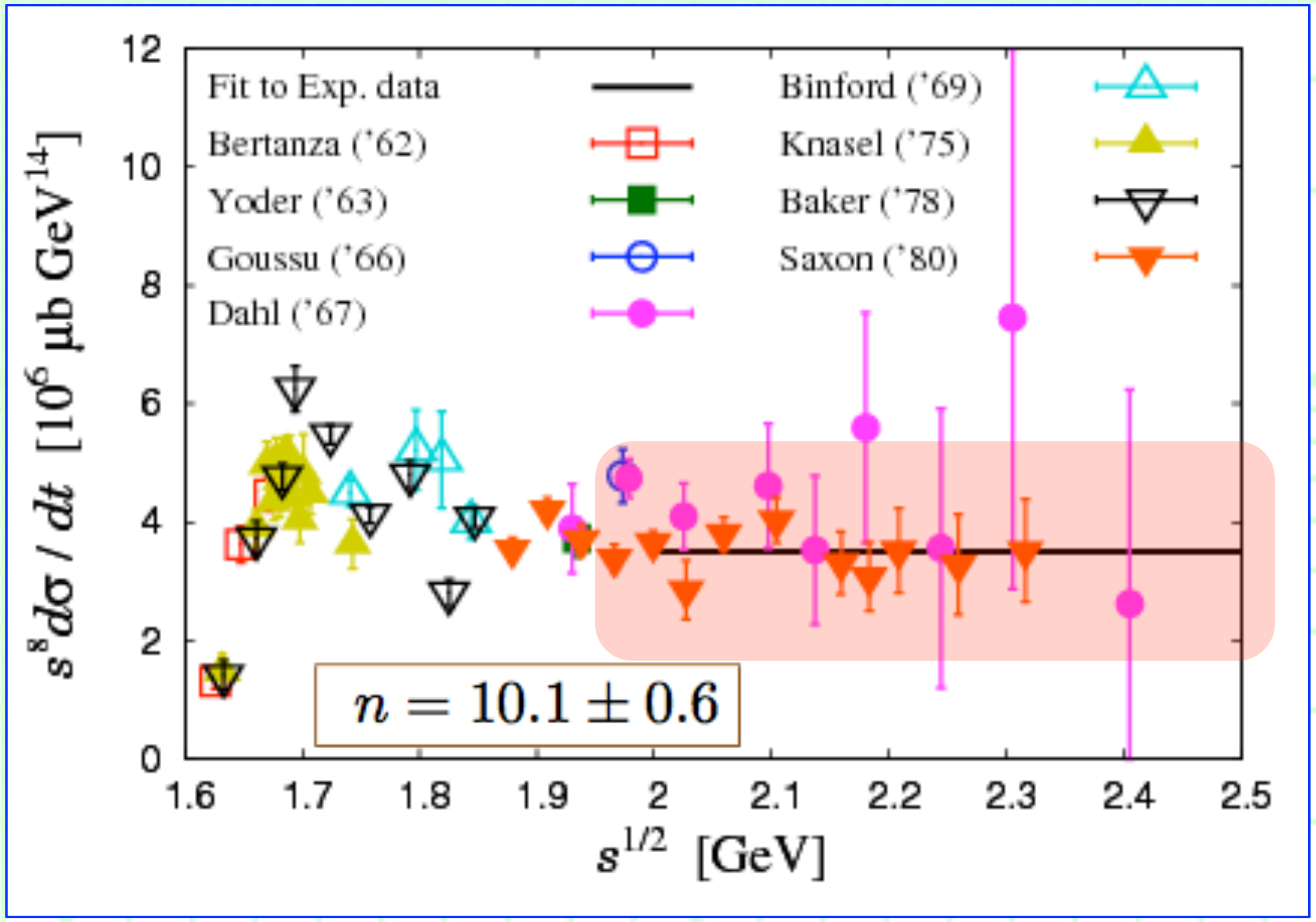

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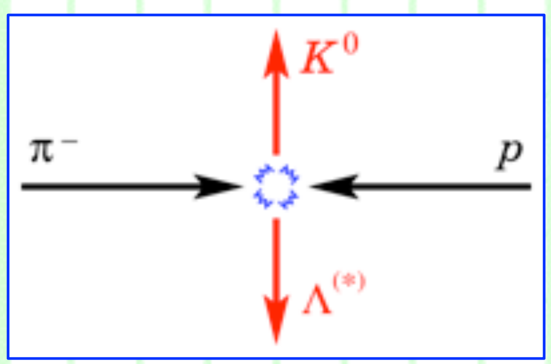
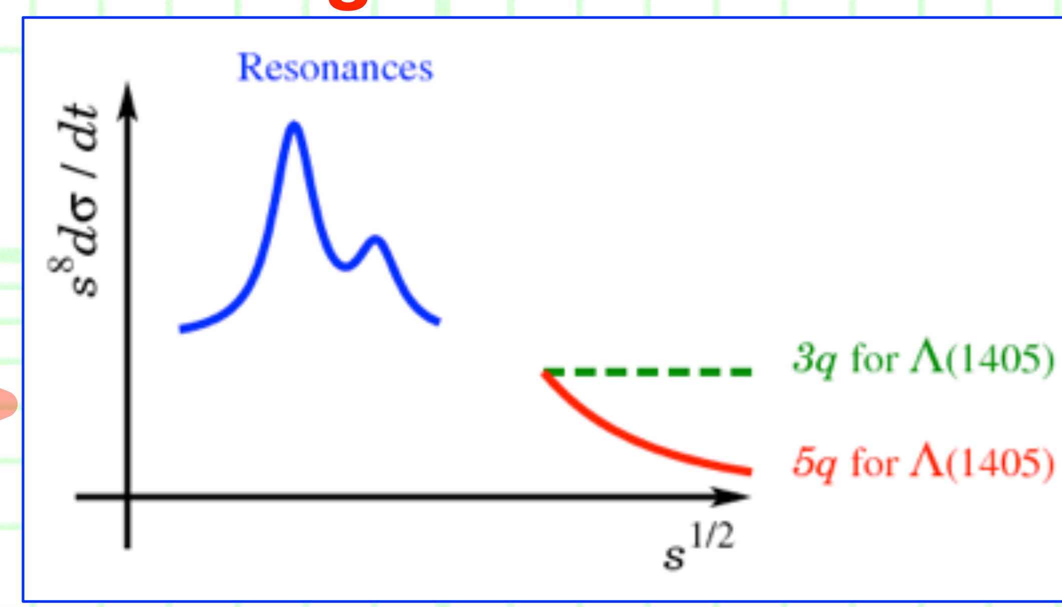
- **Then how cross section of $\pi^{-} p \to K^0 \Lambda(1405)$ at $\theta_{\rm cm} = 90^{\rm o}$ behaves at high energy and high momentum transfer region?**

$n \overset{?}{=} 2+3+2+3 = 10.$





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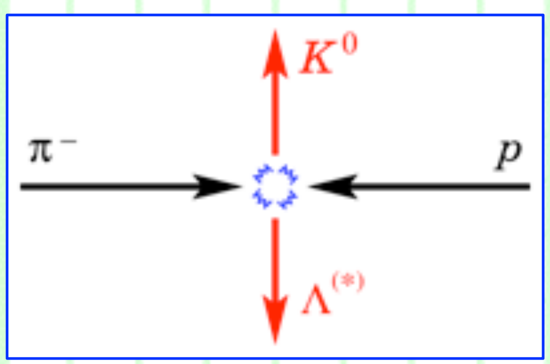
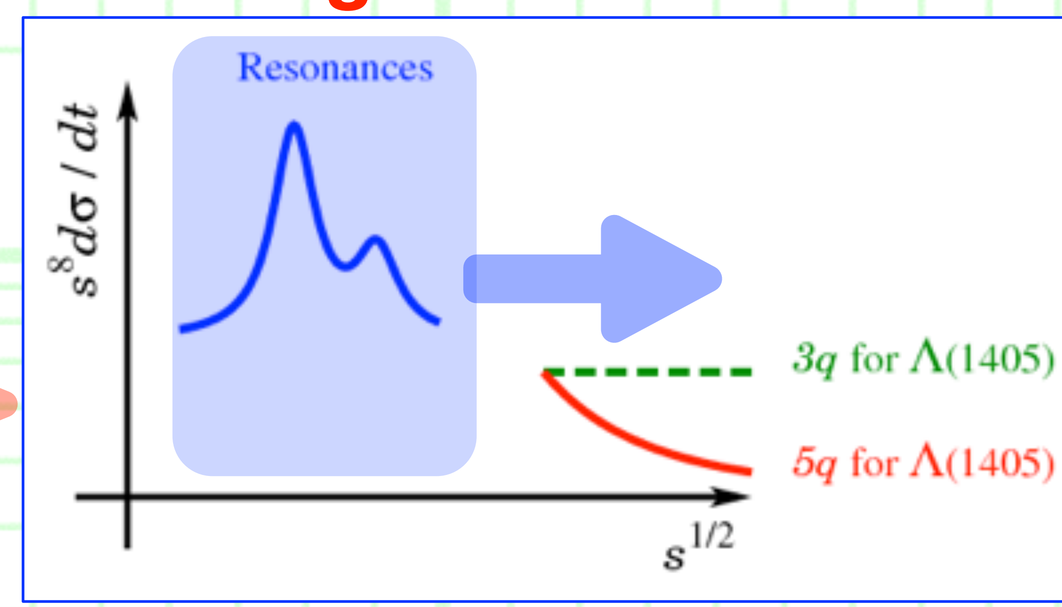




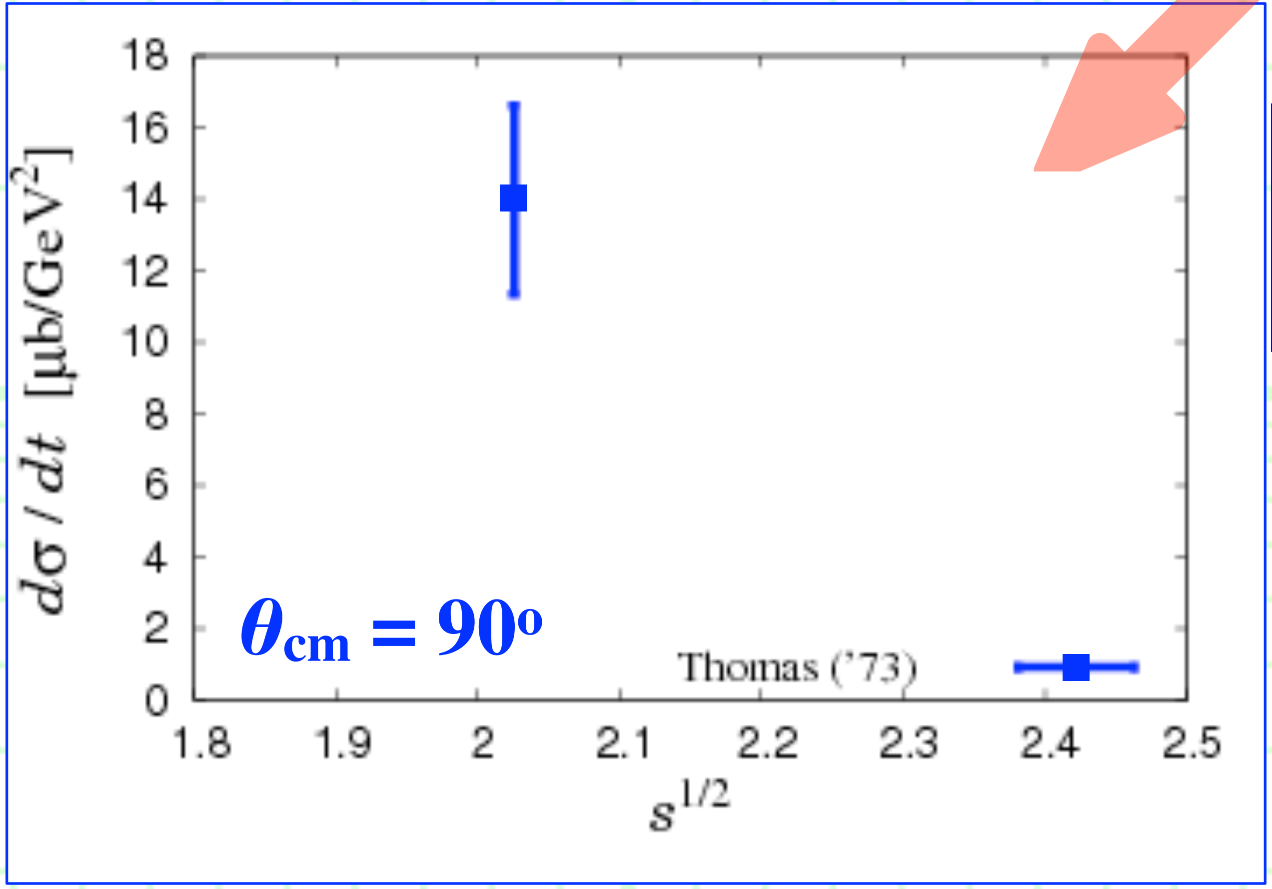
--> We "estimate" cross section of $\pi^{-} p \to K^0 \Lambda(1405)$ at $\theta_{\rm cm} = 90^{\rm o}$ as a function of s from the resonance region to the pQCD one.

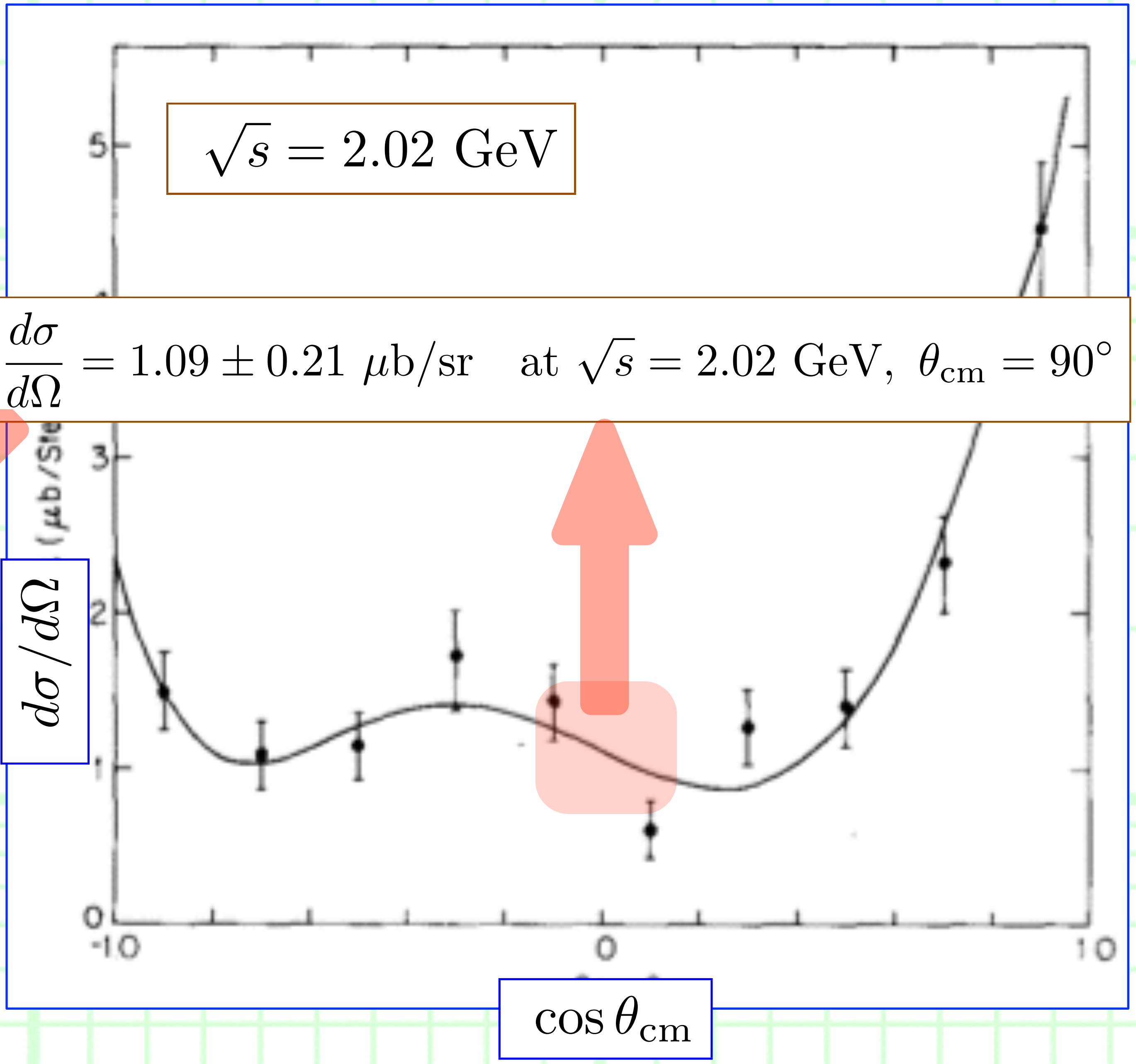
5. Furthermore on exotic hadrons

++ Λ(1405) production: Experimental data ++

- Now we consider
  $\pi^{-} p \to K^0\,\Lambda(1405)$ reaction.

--- Very few Exp. data have been taken, and (as far as I know) only one data is available for $d\sigma / dt$ at $\theta_{cm} = 90^o$:



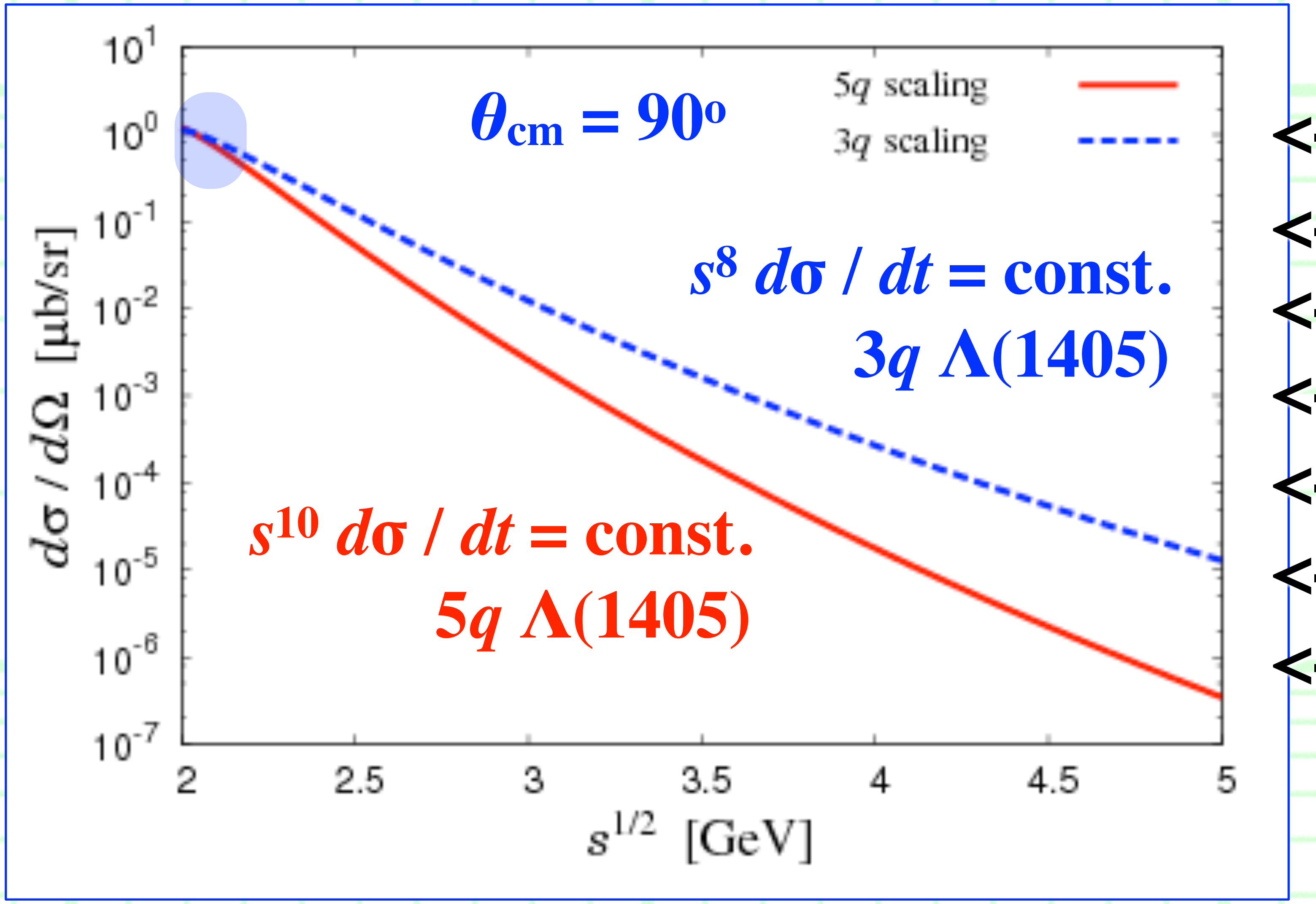


Thomas *et al*., *Nucl. Phys.* B56 (1973) 15.

5. Furthermore on exotic hadrons

++ Λ(1405) production: Estimation ++

- Estimate cross section at higher energies by using Exp. data at $\sqrt{s}$ = 2.02 GeV with $s^{10}\, d\sigma / dt$ = const. or $s^{8}\, d\sigma / dt$ = const.



- Ratio of the cross section for 3q and 5q Λ(1405) is about 10:1 (~ 10 nb : 1 nb) at $\sqrt{s}$ = 3 GeV and more at higher energies.

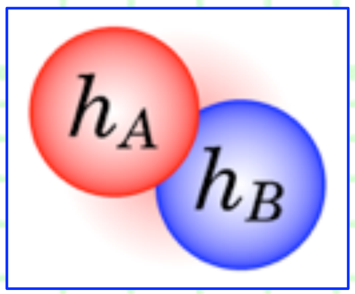
Thank you very much for your kind attention !

Appendix

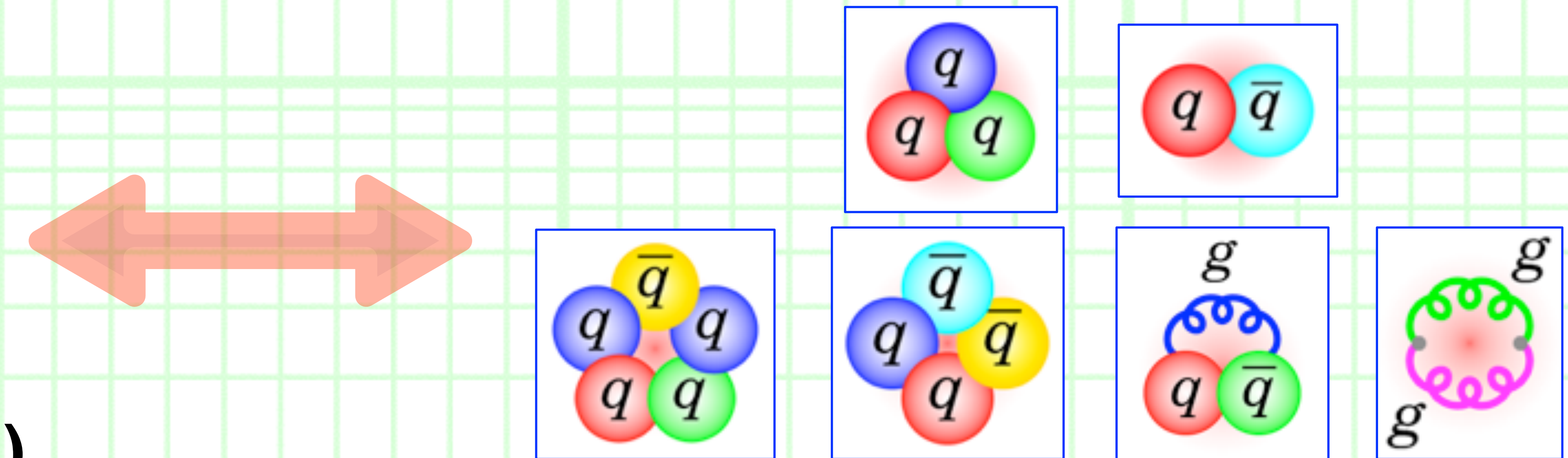
Appendix

++ Uniqueness of hadronic molecules ++

- **Hadronic molecules** should be **unique**, because they are **composed of hadrons themselves**, which are **color singlet**.



Hadronic molecules
(cf. deuteron)



...

--> Various quantitative/qualitative diff. from other compact hadrons.

□ **Large spatial size** due to **the absence of strong confining force**.

□ **Hadron masses are "observable"**, in contrast to quark masses.
--> Expectation of the existence around two-body threshold.

□ **Treat them without complicated calculations of QCD.**
--- With appropriate interactions, we can use quantum mechanics.
--> Wave function and its norm = compositeness.