Holographic teleportation protocol in higher dimensions

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Overview

Introduction

- Gao, Jafferis and Wall realized that wormholes in AdS can be made traversable by coupling the two asymptotic boundaries;
- From the perspective of the boundary theory, the traversability of wormholes in AdS can be thought of as a teleportation protocol [Maldacena, Stanford and Yang 2017];
- Most works regarding traversable wormholes only consider lower dimensional systems
  e.g. BTZ eternal black hole, JT gravity, etc.

In this work, we study $d+1$ dimensional traversable wormholes that appear in the context of Rindler-AdS/CFT.

Motivation:

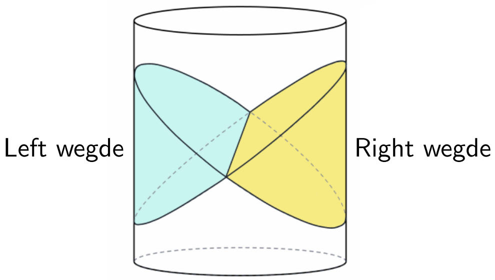
- The Rindler-AdS$_{d+1}$ geometry allows us to obtain analytic results;
- To understand how bounds on information transfer change in higher dimensional (possibly more realistic) setups;
- To study the interplay between **traversability** and **scrambling**:

  e.g. what is the role of the butterfly speed v_B in traversability?

  scrambling: $\langle V(0)W(t,x)V(0)W(t,x)\rangle = c_0 - c_1 e^{2\pi T\left(t-\frac{|x|}{v_B}\right)}$

Rindler wedges of AdS

AdS_{d+1} = universal cover of the hyperboloid: $-T_1^2 - T_2^2 + X_1^2 + \cdots + X_d^2 = -\ell^2$
with ambient metric: $ds^2 = -dT_1^2 - dT_2^2 + dX_1^2 + \cdots + dX_d^2$



Rindler coordinates:

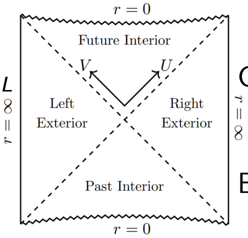
$T_1 = \sqrt{r^2-1}\sinh t\,, \quad T_2 = r\cosh\chi\,, \quad X_d = \sqrt{r^2-1}\cosh t\,, \quad X_1^2 + \cdots + X_{d-1}^2 = r^2\sinh^2\chi$

$$ds^2 = -(r^2-1)dt^2 + \frac{dr^2}{r^2-1} + r^2 dH_{d-1}^2\,, \quad dH_{d-1}^2 = d\chi^2 + \sinh^2\chi d\Omega_{d-2}^2$$

Maximally extended geometry in Kruskal coordinates

[Czech, Karczmarek, Nogueira, Raamsdonk 2012]

CFT_L



CFT_R

Boundary: $\mathbb{R} \times H_{d-1}$

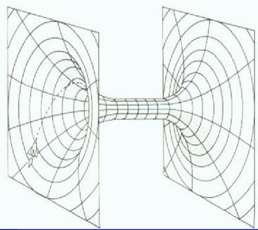
$$ds^2 = -\frac{4dUdV}{(1+UV)^2} + \frac{(1-UV)^2}{(1+UV)^2} dH_{d-1}^2$$

Thermofield double (TFD) state = Two-sided black hole Maldacena 2001

$$|TFD\rangle = \frac{1}{Z(\beta)^{1/2}} \sum_n e^{-\beta E_n/2} |n\rangle_L |n\rangle_R \;=$$

$$Z(\beta) = \mathrm{Tr} e^{-\beta H}$$

L



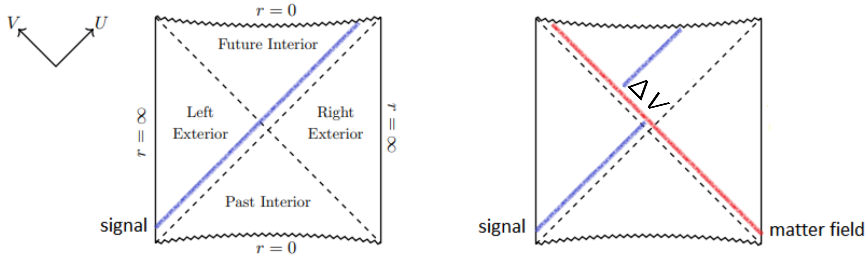
R

Average Null Energy Condition (ANEC):

$$\int T_{\mu\nu} k^\mu k^\nu d\lambda \geq 0$$ Morris, Thorne, Yurtsever 88

k^μ tangent vector, λ affine parameter

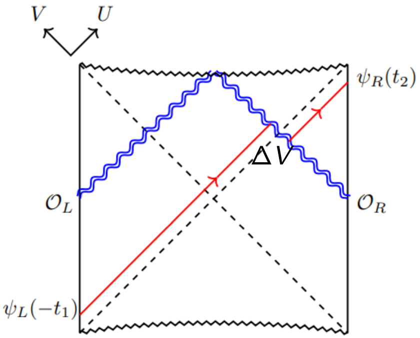
ANEC implies $\Delta V > 0$



Matter backreaction: $ds^2 \to ds^2 + h_{UU} dU^2$

Null shift: $\Delta V \sim \int h_{UU} dU \sim G_N \int T_{UU} dU \geq 0$

Gao-Jafferis-Wall traversable wormhole



Double trace deformation: $H \to H + \delta H$, with

$$\delta H = -\int dx\, h(t,x)\mathcal{O}_L(-t,x)\mathcal{O}_R(t,x)$$

Negative energy in the bulk violates ANEC: $\Delta V \sim \int T_{UU} dU < 0$

Computing $\langle T_{\mu\nu} \rangle$ by point-splitting

Scalar field action: $S_{\text{scalar}} = -\frac{1}{2}\int d^{d+1}x\,\sqrt{-g}\,\left(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + m^2\phi^2\right)$

Classical stress tensor: $T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi - \frac{1}{2}g_{\mu\nu}\,m^2\phi^2$

The 1-loop expectation value of $T_{\mu\nu}$ can be compute as follows:

$T_{\mu\nu}(x) = \partial_\mu\phi(x)\partial_\nu\phi(x) + \ldots \rightarrow T_{\mu\nu}(x,x') = \partial_\mu\phi(x)\partial'_\nu\phi(x') + \ldots$

$\langle T_{\mu\nu}\rangle = \lim_{x'\to x}\partial_\mu\partial'_\nu\langle\phi(x)\phi(x')\rangle + \ldots$

$$\langle T_{\mu\nu}\rangle = \lim_{x'\to x}\partial_\mu\partial'_\nu G(x,x') - -\frac{1}{2}g_{\mu\nu}g^{\alpha\beta}\partial_\alpha\partial'_\beta G(x,x') - \frac{1}{2}g_{\mu\nu}\,m^2 G(x,x')$$

where $G(x,x') = \langle\phi(x)\phi(x')\rangle$ (deformed bulk 2-pt function)

Evaluating $G(x,x')$ at first order in perturbation theory

Interaction picture:

$$
\begin{aligned}
G(x,x') &= \langle \phi^H(t,r,\mathbf{x})\phi^H(t',r',\mathbf{x}')\rangle \\
&= \langle U^{-1}(t,t_0)\phi^I(t,r,\mathbf{x})U(t,t_0)U^{-1}(t',t_0)\phi^I(t',r',\mathbf{x}')U(t',t_0)\rangle
\end{aligned}
$$

where $U(t,t_0) = \mathcal{T} e^{-i\int_{t_0}^{t}\delta H(t')dt'}$, with $\delta H = -h\int dx\, \theta(t-t_0)\mathcal{O}_L(-t,x)\mathcal{O}_R(t,x)$

Small h expansion:

$$
G(x,x') = G_0(x,x') + G_h(x,x')h + \ldots \qquad \langle T_{\mu\nu}\rangle = \langle T_{\mu\nu}\rangle_0 + \langle T_{\mu\nu}\rangle_h h + \ldots
$$

$$
\langle T_{\mu\nu}\rangle_0 = \lim_{x'\to x}\partial_\mu\partial'_\nu G_0(x,x') + \ldots \Longrightarrow \int dU \langle T_{UU}\rangle_0 = 0
$$

$$
\langle T_{\mu\nu}\rangle_h = \lim_{x'\to x}\partial_\mu\partial'_\nu G_h(x,x') + \ldots \Longrightarrow \int dU \langle T_{UU}\rangle_h \neq 0
$$

ANEC is violated for $h > 0$

ANEC violation in Rindler-AdS$_{d+1}$

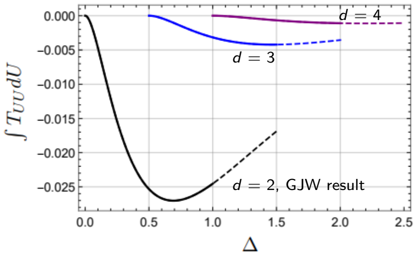
$$\frac{1}{\mathrm{vol}(S_{d-2})}\int_{U_0}^{\infty} dU T_{UU} = -\frac{h\pi^{\frac{1}{2}-d}\Gamma(\frac{d-1}{2})}{(2\Delta+1)}\frac{\Gamma(\Delta+\frac{1}{2})\Gamma(\Delta+\frac{3-d}{2})}{\Gamma(\Delta+1-\frac{d}{2})^2}\frac{{}_2F_1\left(\Delta+\frac{1}{2},\frac{1}{2}-\Delta,\Delta+\frac{3}{2};\frac{1}{1+U_0^2}\right)}{(1+U_0^2)^{\Delta+\frac{1}{2}}}$$

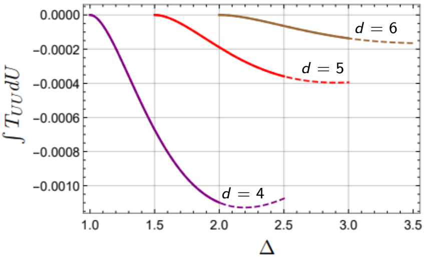
$U = e^{t_0}$, t_0 = time at which the deformation is turned on

deformation: $\delta H \sim -h\int \mathcal{O}_L\mathcal{O}_R$

Δ = scaling dimension of $\mathcal{O}_L$ and $\mathcal{O}_R$

$\frac{d-2}{2} \leq \Delta \leq \frac{d}{2}$, our formulas hold at least up to $\Delta = \frac{d+1}{2}$





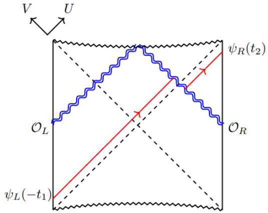
For fixed h, $|\int T_{UU}dU|$ quickly decreases as we increase d

Maldacena-Stanford-Yang two-sided correlator

Two-sided correlator: $\langle [\psi_R, e^{-igV}\psi_L e^{igV}] \rangle$

$$V = \frac{1}{K}\sum_{j=1}^{K} \int dt' d\mathbf{x}'\, h(t', \mathbf{x}') \mathcal{O}_L^j(-t', \mathbf{x}') \mathcal{O}_R^j(t', \mathbf{x}')$$

It is convenient to work with:



$$C = \mathrm{Im}\left(\langle [\psi_R, e^{-igV}\psi_L e^{igV}] \rangle \right)$$

$$C = \langle e^{-igV}\psi_L e^{igV}\psi_R \rangle \approx e^{-ig\langle V \rangle} \langle \psi_L e^{igV} \psi_R \rangle$$ large-K, small-G_N approx.

$\tilde{C} = \langle \psi_L e^{igV} \psi_R \rangle$ is an OTOC-like quantity that can be computed using the eikonal approximation OTOCs via eikonal approx.: [Shenker, Stanford 2014]

$\tilde{C} = \langle \psi_L e^{igV} \psi_R \rangle$ via eikonal approximation

At first order in g: $\tilde{C}_1 = ig\langle \psi_L V \psi_R \rangle = \frac{ig}{K} \sum_{j=1}^{K} \int h \underbrace{\langle \psi_L \mathcal{O}_L^j \mathcal{O}_R^j \psi_R \rangle}_{\text{OTOC}}$

Using the eikonal approximation, we find:

$$\tilde{C}_1 = ig\alpha \underbrace{\int dp\, d\mathbf{x}\, p\, \Psi_{\psi_R} \Psi_{\psi_L}(\mathbf{x}, p)}_{signal} \underbrace{\int dt'\, dx'\, h\, dq\, d\mathbf{y}\, q\, \Psi_{\mathcal{O}_R} \Psi_{\mathcal{O}_L}(\mathbf{y}, q)}_{\mathcal{O}-\text{quanta}} \underbrace{e^{i\delta(p\, q)}}_{interaction}$$

At all orders in g the result exponentiates

$$\tilde{C} = \alpha \int dp\, d\mathbf{x} \Psi_{\psi_R}(\mathbf{x}, p)\, p\, \Psi_{\psi_L}(\mathbf{x}, p) e^{\left[ig \int dt'\, dx'\, h\, dq\, d\mathbf{y}\, q\, \Psi_{\mathcal{O}_R}(\mathbf{y},q) \Psi_{\mathcal{O}_L}(\mathbf{y},q) e^{i\delta(p\, q)}\right]}$$

JT gravity: [Maldacen, Stanford, Yang 2017] BTZ: [Almheiri, Mousatov, Shyani1 2018],

In this work, we study $C = e^{-ig\langle V \rangle} \tilde{C}$ in a Rindler-AdS$_{d+1}$ geometry

Probe approximation

Treating the signal in the probe approximation, we can show that

$$C_{\text{probe}} = \langle \psi_L e^{i\Delta V \hat{P}_V} \psi_R \rangle$$

For homogeneous operators, we find

$$\Delta V = -\alpha\, b_{\mathcal{O}}^2 \frac{\Delta_{\mathcal{O}} \Gamma(2\Delta_{\mathcal{O}})}{2} \mathrm{vol}(S_{d-1}) \int dt' d\chi' \frac{16\pi G_N}{\cosh \chi'^{2\Delta_{\mathcal{O}}+1}} \frac{h(t')}{\cosh t'^{2\Delta_{\mathcal{O}}+1}} ,$$

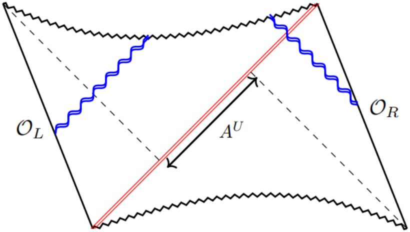
which is perfectly consistent with the result obtained via point splitting.

- This shows that Gao-Jafferis-Wall and Maldacena-Stanford-Yang methods agree at the quantitative level.
- roughly speaking, the reduction of $|\Delta V|$ as we increase d comes from the window for the conformal dimension of $\mathcal{O}$: $\frac{d-2}{2} \leq \Delta_{\mathcal{O}} \leq \frac{d}{2}$.

Backreaction effect

The backreaction of the signal closes the wormhole: ΔV becomes less negative

$$\Delta V \to \Delta V_{\text{back}} \text{ with } |\Delta V_{\text{back}}| < |\Delta V|$$



$$\Psi_{\mathcal{O}_R} \to e^{iA^U p_U} \Psi_{\mathcal{O}_R}$$

$A^U = 16\pi G_N q^{\text{tot}}$, $\quad$ q^{tot} is the total momentum of the signal

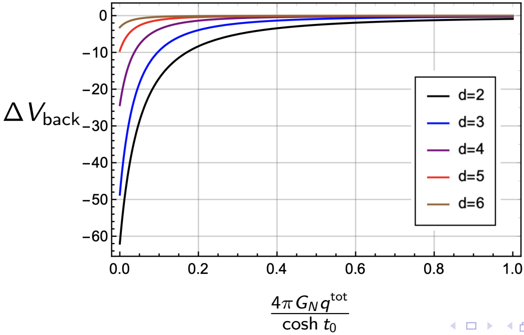
Backreaction effect

Considering homogeneous perturbations and taking $h(t,x) = \delta(t-t_0)$, we find:

$$\Delta V_{\text{back}} = d_{\mathcal{O}} \frac{\Gamma(2\Delta_{\mathcal{O}} - d + 3)\Gamma(\frac{d-1}{2})}{\Gamma(2\Delta_{\mathcal{O}} - \frac{d-5}{2})} \frac{F_1\left(2\Delta_{\mathcal{O}} - d + 3, 2\Delta_{\mathcal{O}} + 1, \frac{3-d}{2}, 2\Delta_{\mathcal{O}} - \frac{d-5}{2}, -\frac{4\pi G_N q^{\text{tot}}}{\cosh t_0}, -1\right)}{(\cosh t_0)^{2\Delta_{\mathcal{O}}+1}}$$

where F_1 is the Appel hypergeometric function and $d_{\mathcal{O}} = -8\pi G_N \Delta_{\mathcal{O}} \alpha\, g\, b_{\mathcal{O}}^2 \text{vol}(S_{d-2})$.

Results for Rindler-AdS$_{d+1}$



Here: $\Delta_{\mathcal{O}} = \frac{d-1}{2}$

Bounds on information transfer

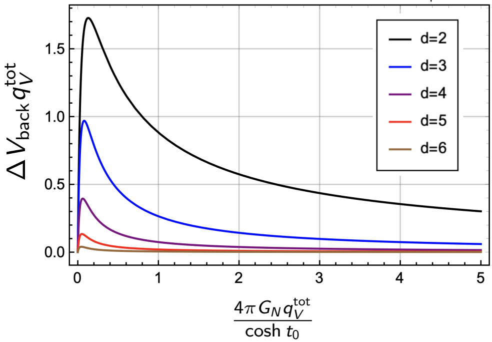
For a signal with N_{bits} particles and total momentum q_V^{tot}, one can show that:

$$N_{\text{bits}} \lesssim \max\left[q_V^{\text{tot}}|\Delta V_{\text{back}}|\right]$$

$N_{\text{bits}} = \frac{q_V^{\text{tot}}}{q_V^{\text{each}}}$, uncertainty principle: $\Delta V_{\text{each}} q_V^{\text{each}} \gtrsim 1$, condition for each particle to fit in the wormhole: $\Delta V_{\text{each}} \leq |\Delta V_{\text{back}}|$

$\frac{1}{q_V^{\text{each}}} \lesssim \Delta V_{\text{each}} \leq |\Delta V_{\text{back}}| \implies N_{\text{bits}} = \frac{q_V^{\text{tot}}}{q_V^{\text{each}}} \lesssim q_V^{\text{tot}}|\Delta V_{\text{back}}|$

Results for Rindler-AdS$_{d+1}$



Here: $\Delta_{\mathcal{O}} = \frac{d-1}{2}$

Parametric bounds on information transfer

$$N_{\text{bits}} \lesssim q_V^{\text{tot}} |\Delta V|$$

The probe approximation implies: $q_V^{\text{tot}} \lesssim \frac{r_0^{d-1}}{G_N}$

Our calculation gives: $|\Delta V| \sim h\, G_N\, K$

Combining the above results we find: $N_{\text{bits}} \lesssim h\, K\, r_0^{d-1}$

[Freivogel et al 2020] argue that $K \lesssim \frac{1}{G_N}$. This implies:

$$N_{\text{bits}} \lesssim h\, \frac{r_0^{d-1}}{G_N} \sim h\, S_{\text{BH}} \quad \text{(homogeneous shocks)}$$

We are currently investigating whether is possible to derive sharper bounds for localized shocks

The two-sided correlator:

$$C(T,X) = \langle e^{-igV}\psi_L(-T,X)e^{igV}\psi_R(T,X)\rangle$$

In the probe approximation, we find:

$$C_{\text{probe}}(T,X) \sim \int d\chi_1 \left[\cosh(\chi_1 - X) + \frac{D_1(\chi_1)}{4}e^T\right]^{-2\Delta_\psi}$$

where

$$D(\chi_1) \sim G_N \int_0^\infty d\chi_4 \frac{e^{-\mu|\chi_1-\chi_4|}}{(\cosh\chi_4)^{2\Delta_{\mathcal{O}}+1}}\,, \quad \mu = d-1 = \frac{1}{v_B}\,.$$

where $\mu = d - 1 = \frac{1}{v_B}$.

Sweet spot for traversability: the butterfly cone

[Couch et al 2020] studied the correlator $C(T, X)$ for a BTZ black hole and showed that the sweet spot for traversability is controlled by a light-cone:

Here $v_B = 1$

Local perturbation:
$\delta H = h\,\mathcal{O}_L(t_0, x_0)\mathcal{O}_R(t_0, x_0)$

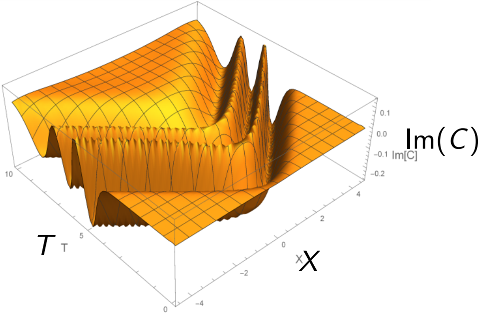


Figure from [Couch et al 2020, arXiv:1908.06993]

For a Rindler-AdS$_{d+1}$, we find a butterfly cone, with $v_B = \frac{1}{d-1}$.

Conclusions

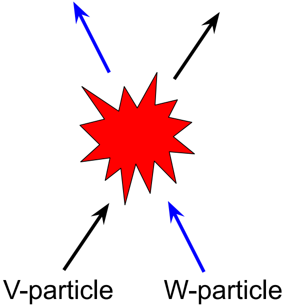
- We show the equivalence between the Gao-Jafferis-Wall point splitting method and Maldacena-Stanford-Yang eikonal method and extend their calculations to a higher dimensional setup;
- The ANEC violation (the opening of the wormhole) reduces as we increase the dimensionality of the spacetime;
- The amount of information that can be transferred through the wormhole also reduces as we increase d;
- The Rindler-AdS$_{d+1}$ geometry allows us to obtain several analytic results which can be used for further investigations;

  e.g. we have an analytic result for δS_{BH} that can in principle be computed in terms of quantum extremal surfaces
- Our preliminary results suggest that the sweet spot for traversability is defined by the butterfly cone.

THANK YOU

Gravity Dual of OTOCs Shenker Stanford 2013-2014

$$\langle\, V(0)\, W(t)\, V(0) W(t)\,\rangle \quad =$$

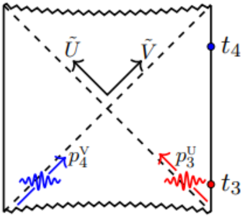


- OTOCs are related to a high energy collision that takes place close to the black hole horizon;
- the special properties of the black hole horizon give rise to the universal behavior of OTOCs.

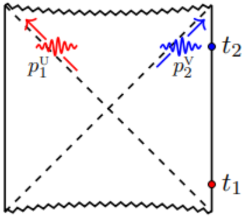
Bulk representation of 'In' and 'Out' states

$$\langle \mathrm{TFD}|V_{\mathbf{x}_1}(t_1)W_{\mathbf{x}_2}(t_2)V_{\mathbf{x}_3}(t_3)W_{\mathbf{x}_4}(t_4)|\mathrm{TFD}\rangle = \langle \mathrm{out}|\mathrm{in}\rangle$$

$$|\mathrm{in}\rangle = V_{\mathbf{x}_3}(t_3)W_{\mathbf{x}_4}(t_4)|\mathrm{TFD}\rangle\,, \quad |\mathrm{out}\rangle = W_{\mathbf{x}_2}(t_2)^\dagger V_{\mathbf{x}_1}(t_1)^\dagger|\mathrm{TFD}\rangle\,.$$



$V_{\mathbf{x_3}}(t_3)W_{\mathbf{x_4}}(t_4)|\mathrm{TFD}\rangle$



$W_{\mathbf{x_2}}(t_2)^\dagger V_{\mathbf{x_1}}(t_1)^\dagger|\mathrm{TFD}\rangle$

These two-particle states are described by a shock wave geometry:

$$ds^2_{\mathrm{shock}} = ds^2_0 + h_{UU}dU^2 + h_{VV}dV^2$$

The eikonal approximation 't Hooft 87 Verlinde[2] 92 Kabat, Ortiz 92

$$\langle V_0(0)\, W_{\mathbf{x}}(t)\, V_0(0)\, W_{\mathbf{x}}(t)\rangle = \int \overbrace{K_V K_W K_V K_W}^{\text{bulk-bdry propagators}} \underbrace{\langle \phi_V \phi_W \phi_V \phi_W \rangle}_{\text{bulk 4pt-function}}$$

Using the eikonal approximation, we can write

$$\langle \phi_V \phi_W \phi_V \phi_W \rangle = e^{i\delta(s,b)}$$

where $s = -(p_1 + p_2)^2$ and $b = |\mathbf{x}|$ is the collision impact parameter.

Assumptions:

- Linearized gravity: $G_N << 1$
- Regge limit: $s = E_{\mathrm{CM}}^2 >> 1$ and fixed b

$e^{i\delta} =$  $+ \cdots$

The gravitational interaction dominates $\rightarrow$ Universality of OTOCs