Thermal radiation of de Sitter space in the semiclassical JT model

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based on 2010.09173 [gr-qc] (with Hwajin Eom)

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Outline

Black hole complementarity,

Firewall and quantum atmosphere

Quantum fields on the background of de Sitter space

De Sitter space embedded in Jackiw-Teitelboim model

Discussion

Black hole complementarity(BHC)

- Hawking (1975)

-black hole temperature and its entropy
-Hawking radiation may be a carrier of information of black holes.

- Black hole complementarity

-Susskind, Thorlacius and Uglum (1993)
-Stephens, `t Hooft and Whiting (1994)

BHC: two disconnected patches, two independent observers [solution to information cloning puzzle]

Firewall and quantum atmosphere

- Hawking(1975): highly blue-shifted infinite radiation at the horizon is the origin of Hawking radiation (via pair creations at the horizon) *Commun. Math. Phys.* **43** (1975) 199–220.
- Unruh(1976): no outgoing positive radiation at the horizon→ no firewall *Phys. Rev.* **D14** (1976) 870.

- AMPS firewalls (2013): monogamy principle,

JHEP **02** (2013) 062, [1207.3123].

incompatiblity between general covariance and semiclassical quantum field theory

- Giddings(2016): near horizon quantum atmosphere is the source of Hawking radiation → no firewall

Phys. Lett. **B754** (2016) 39–42

No firewalls in the Israel-Hartle-Hawking state

- The Tolman temperature should be written as the effective Tolman temperature,

$$T_{\mathrm{T}} = \frac{C}{\sqrt{-g_{00}(r)}} \quad \Rightarrow \quad T_{\mathrm{eff}} = \frac{1}{\sqrt{\gamma g}}\sqrt{C_0 - \frac{g}{2}T^\mu_\mu + \frac{1}{2}\int T^\mu_\mu dg}$$

$$g_{00} \equiv -g$$

- The Stefan-Boltzmann law should be written as the effective Stefan- Boltzmann law as

$$\varepsilon = \gamma T_{\mathrm{T}}^2 \quad \gamma = \pi/6 \quad \Rightarrow \quad \varepsilon = \gamma T_{\mathrm{eff}}^2 - \frac{1}{2}\langle T^\mu_\mu \rangle$$

- The black hole temperature is (i) non-vanishing even in the free-fall frame (ii) finite everywhere (iii) vanishing at the horizon.

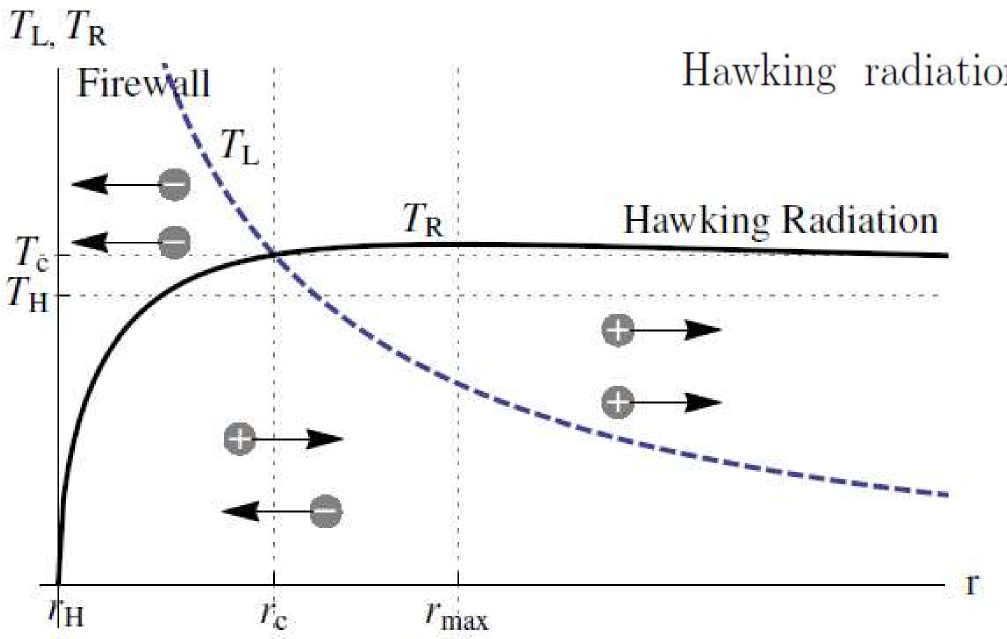
$$\gamma T_{\mathrm{eff}}^2 = \frac{1}{g}\left(\langle T_{++}\rangle_{\mathrm{HH}} + \langle T_{--}\rangle_{\mathrm{HH}}\right)$$

Compatibility between firewall and Hawking radiation

arXiv:1604.00465

$$\text{Firewall}: T_{\mathrm{L}} = \frac{1}{\sqrt{2}\pi r}\sqrt{\frac{M}{r-2M}\left(1-\frac{3M}{2r}\right)},$$

$$\text{Hawking radiation}: T_{\mathrm{R}} = \frac{1}{8\pi M}\sqrt{1+\frac{2M}{r}+\left(\frac{2M}{r}\right)^2-3\left(\frac{2M}{r}\right)^3}.$$



- Hawking radiation for a asymptotically flat black hole

$$h(\sigma^-) = T^{\mathrm{QT}}_{--}(\sigma^+, \sigma^-)|_{\sigma^+\to\infty}$$

$$= T^{\mathrm{bulk}}_{--}|_{\sigma^+\to\infty} + T^{\mathrm{boundary}}_{--}|_{\sigma^+\to\infty}$$

$$= -\frac{1}{12\pi} t_-(\sigma^-).$$

- Asymptotically AdS geometry (HH vacuum)

$$\begin{aligned} T^{\rm bulk}_{--} &= -\frac{1}{12\pi}\left[(\partial_-\rho)^2 - \partial_-^2\rho\right] \\ &= -\frac{M}{48\pi\ell^2}, \\ T^{\rm boundary}_{--} &= -\frac{1}{12\pi}t_- \\ &= \frac{M}{48\pi\ell^2}, \end{aligned}$$

- The vanishing radiation in AdS(2)

$$\begin{aligned} h(\sigma^+,\sigma^-) &= T^{\rm QT}_{--}(\sigma^+,\sigma^-) \\ &= T^{\rm bulk}_{--} + T^{\rm boundary}_{--} \\ &= 0. \end{aligned}$$

Issues

- So far, we have discussed the divergent temperature at the horizon in the Unruh vacuum while the temperature vanishes at the horizon in the Hartle-Hawking vacuum. The question is

  " what happens in the de Sitter space"?

  cf: especially in the Bunch-Davies vacuum

One method

- Euclidean functional path integral

The Gibbons and Hawking analysis is based on the fact that a well-defined meaning for the Green's function can be given by analytically continuing back to the original Lorentzian spacetime from a region in which the metric is positive definite.

Ex) T in Euclidean = T in Lorentzian,

Other methods

- Bogoliubov transformations or scattering method

The condition for a thermal flux is calculated in a certain limit e.g., $\omega \ll 1$,

$$\langle 0|N|0\rangle = \frac{R}{1-R} = \frac{1}{e^{\omega/T_{GH}} - 1}$$

$$T_{GH} = \frac{H}{2\,\pi}$$

Here, I will take stress tensor approach in the Lorentzian spacetime.

Quantum fields on the background of the de Sitter space

- Hawking temperature (2D and 4D)

$$T_{\mathrm{GH}} = \frac{H}{2\pi}$$

- The temperature of the BD vacuum in 4D

$$\rho = -\langle \mathrm{BD}|T_0^0|\mathrm{BD}\rangle = \frac{H^4}{960\pi^2} = \gamma_{(4)} T_{\mathrm{GH}}^4, \qquad \gamma_{(4)} = \frac{\pi^2}{60}$$

The temperature of the BD vacuum in 2D

$$\rho = -\langle \mathrm{BD}|T_0^0|\mathrm{BD}\rangle = -\frac{H^2}{24\pi} = \gamma_{(2)} T_{\mathrm{GH}}^2, \qquad \gamma_{(2)} = \frac{\pi}{6}$$

Explicitly, in 2D

- The line element of dS space (SO(1,2))

$$ds^2 = -g(r)dt^2 + \frac{1}{g(r)}dr^2,$$
$$g(r) = 1 - H^2r^2 \ (r_H = 1/H)$$

- Conformal coordinates (or Kruskal coordinates)

$$ds^2 = -\mathrm{sech}^2\left[\frac{H}{2}(\sigma^+ - \sigma^-)\right]\mathrm{d}\sigma^+\mathrm{d}\sigma^-,$$
$$\sigma^\pm = t \pm r^*, \ r^* = r_H \tanh^{-1}(Hr)$$

- Stress tensor

$$\langle T_{\pm\pm}(\sigma)\rangle = \kappa\left[\frac{1}{8}\left(gg'' - \frac{1}{2}(g')^2\right) - t_\pm(\sigma^\pm)\right],$$

$$\langle T_{+-}(\sigma)\rangle = \frac{\kappa}{8}gg''$$

$g' = dg/dr,\ t_\pm(\sigma^\pm)$: integration functions,

$\kappa = N/(12\pi),\ N$: the number of classical matter fields

- Bunch-Davies (BD) vacuum $|\mathrm{BD}\rangle$

$$t_{\pm}(\sigma^{\pm}) = -\frac{H^2}{4}$$

➡ [2d] $\langle \mathrm{BD}|T_{\mu\nu}|\mathrm{BD}\rangle = \frac{\kappa H^2}{2} g_{\mu\nu}$ [Aalsma *et el.*, JHEP11(2019)136(arXiv:1905.02714)]

(cf) [4d] $\langle \mathrm{BD}|T_{\mu\nu}|\mathrm{BD}\rangle = -\frac{H^4}{960\pi^2} g_{\mu\nu}$ [Page, Phys.Rev.D.25,1499(1982)]

The sign difference between 2d and 4d was pointed out.
[Markkanen arXiv:1703.06898

- The proper energy density

$$\rho = \langle T_{\mu\nu} \rangle u^\mu u^\nu, \quad u^\mu = \Big(u^+(\sigma), u^-(\sigma) \Big) = \left(\frac{1}{\sqrt{g}}, \frac{1}{\sqrt{g}} \right)$$

(free-fall at rest)

- Usual Stefan-Boltzmann law $\rho = \gamma T^2$

➡ $$\gamma T^2 = \frac{1}{g} \Big[\langle \mathrm{BD} | T_{++} | \mathrm{BD} \rangle + \langle \mathrm{BD} | T_{--} | \mathrm{BD} \rangle + 2 \langle \mathrm{BD} | T_{+-} | \mathrm{BD} \rangle \Big] = -\frac{\kappa H^2}{2}$$

- The temperature becomes imaginary.

- Modified Stefan-Boltzmann law (Gim and Kim, arXiv:1508.00312)

$$\rho = \gamma T^2 - \frac{1}{2}\langle T^{\mu}_{\mu} \rangle$$

$$\Rightarrow \quad \gamma T^2 = \frac{1}{g}\Big[\langle \mathrm{BD}|T_{++}|\mathrm{BD}\rangle + \langle \mathrm{BD}|T_{--}|\mathrm{BD}\rangle\Big] = 0$$

- The temperature becomes zero (arXiv:2010.09173)

$$T = 0$$

- Another quantum state $|\Psi\rangle$

$$t_{\pm}(\sigma^{\pm}) = -\frac{H^2}{2}$$

$$\Rightarrow \quad \langle\Psi|T_{\pm\pm}(\sigma)|\Psi\rangle = \frac{\kappa H^2}{4}, \quad \langle\Psi|T_{+-}(\sigma)|\Psi\rangle = -\frac{\kappa H^2}{4} g(r)$$

- Note that the proper temperature in thermal equilibrium is

$$T = \frac{T_{\mathrm{GH}}}{\sqrt{g(r)}}$$

De Sitter space embedded in the JT model

- Actions

$$S_{\mathrm{JT}} = \frac{1}{2\pi}\int d^2x\sqrt{-g}\phi\left[R - 2H^2\right],$$

$$S_{\mathrm{cl}} = \frac{1}{2\pi}\int d^2x\sqrt{-g}\sum_{i=1}^{N}\left[-\frac{1}{2}(\nabla f_i)^2\right],$$

$$S_{\mathrm{qt}} = \frac{\kappa}{2}\int d^2x\sqrt{-g}\left[-\frac{1}{4}R\frac{1}{\Box}R\right]$$

- By using an auxiliary field ψ,

$$S_{\mathrm{qt}} = \frac{\kappa}{4}\int d^2x\sqrt{-g}\left[R\psi - \frac{1}{2}\left(\nabla\psi\right)^2\right].$$

- Equations of motion

$$\partial_+\partial_-\phi + \frac{H^2}{2}e^{2\rho}\phi + \langle T_{+-}(\sigma)\rangle = 0,$$

$$\partial_+\partial_-\rho - \frac{H^2}{4}e^{2\rho} = 0,$$

$$\partial_+\partial_- f_i = 0,$$

$$2\partial_+\partial_-\rho - \partial_+\partial_-\psi = 0,$$

$$\partial_\pm^2\phi - 2\partial_\pm\rho\partial_\pm\phi + \langle T_{\pm\pm}(\sigma)\rangle = 0$$ **(**constraint equations**)**

- Quantum stress tensor

$$\langle T_{\pm\pm}(\sigma)\rangle = -\kappa\left[(\partial_\pm\rho)^2 - \partial_\pm^2\rho\right] - \kappa t_\pm(\sigma^\pm), \quad \langle T_{+-}(\sigma)\rangle = -\frac{\kappa}{2}\partial_+\partial_-\psi$$

($t_\pm(\sigma^\pm)$ reflects the nonlocality of the Polyakov action)

- $f_i = 0$ (for simplicity)
- Equations of motion reduce to

$$\partial_+\partial_-\rho - \frac{H^2}{4}e^{2\rho} = 0,$$
$$\partial_+\partial_-\phi + \frac{H^2}{4}e^{2\rho}\left(\kappa - 2\phi\right) = 0 \ .$$

- Solutions

$$e^{2\rho(\sigma^+,\sigma^-)} = \mathrm{sech}^2\left[\frac{H}{2}\left(\sigma^+ - \sigma^-\right)\right],$$
$$\phi(\sigma^+,\sigma^-) = \left(\tilde{\alpha} + \frac{\kappa H}{4}(1-\alpha)\left(\sigma^+ - \sigma^-\right)\right)\tanh\left[\frac{H(\sigma^+ - \sigma^-)}{2}\right] + \frac{\kappa\alpha}{2}$$

($\alpha,\ \tilde{\alpha}$: integration constants)

- From the solutions, the quantum flux is given as

$$\langle T_{\pm\pm}(\sigma)\rangle = -\frac{\kappa H^2}{4} - \kappa t_\pm(\sigma^\pm).$$

- This should be compatible with the constraint equation so that

$$t_\pm(\sigma^\pm) = -\frac{H^2}{4}\alpha \, .$$

- Quantum stress tensor

$$\langle T_{\mu\nu}(\sigma)\rangle = \frac{\kappa H^2}{2} g_{\mu\nu} - \frac{\kappa H^2}{4}(1-\alpha) I_{\mu\nu}$$

$(\, I_{\pm\pm} = 1, \; I_{\pm\mp} = 0 \,)$

For $\alpha = 1$, $\langle \mathrm{BD}|T_{\mu\nu}(\sigma)|\mathrm{BD}\rangle = \frac{\kappa H^2}{2} g_{\mu\nu}$ (symmetric vacuum)

For $\alpha = 2$, $\langle \Psi|T_{\mu\nu}(\sigma)|\Psi\rangle = \frac{\kappa H^2}{2} g_{\mu\nu} + \frac{\kappa H^2}{4} I_{\mu\nu}$ (symmetry breaking vacuum)

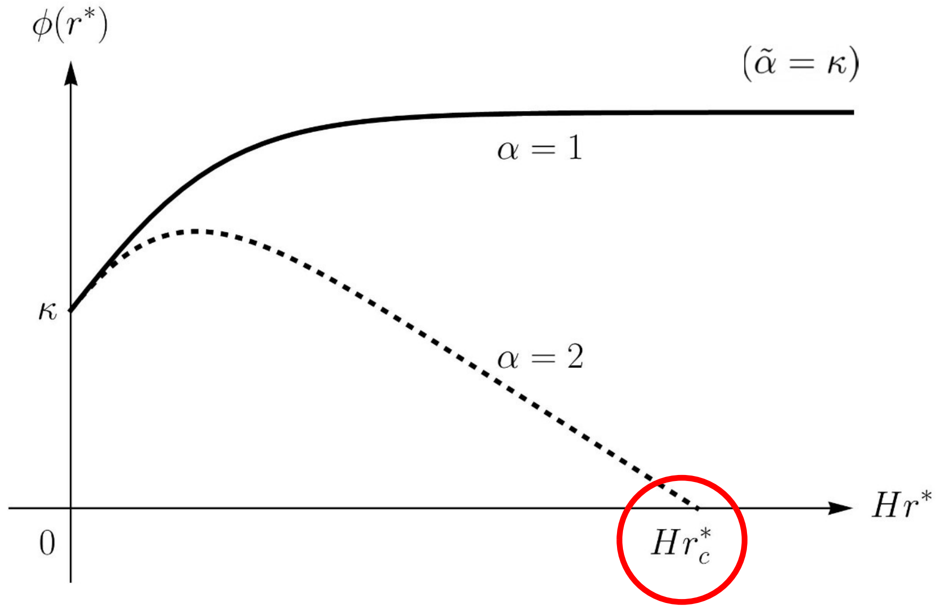
- From the modified Stefan-Boltzmann law, the local temperature for an arbitrary α is obtained as

$$T = \sqrt{\alpha - 1}\left(\frac{T_{\rm GH}}{\sqrt{g}}\right).$$

- For a real temperature, $\alpha \geq 1$.
- $\alpha = 1$: BD vacuum ➡ $T = 0$
- $\alpha > 1$ ➡ Tolman's form (Especially, $T = T_{\rm GH}/\sqrt{g}$ for $\alpha = 2$.)

The redshift factor indicates the inhomogeneity of the temperature due to the breaking of de Sitter symmetry except the time-like Killing symmetry ($\xi^{\mu}_{(t)}\partial_{\mu} = \partial_t$) .

- The singularity free condition for dilaton
  $0 < \alpha \leq 1, \ \tilde{\alpha} > -(\kappa\alpha)/2$
- For $\alpha = 2$, a time-like naked singularity occurs.



- The reality condition for the temperature ($\alpha \geq 1$)
  \+ The singularity free condition ($0 < \alpha \leq 1$) ➡ $\alpha = 1$ (BD vacuum)

- That is, in the JT model, α is determined uniquely as $\alpha = 1$.

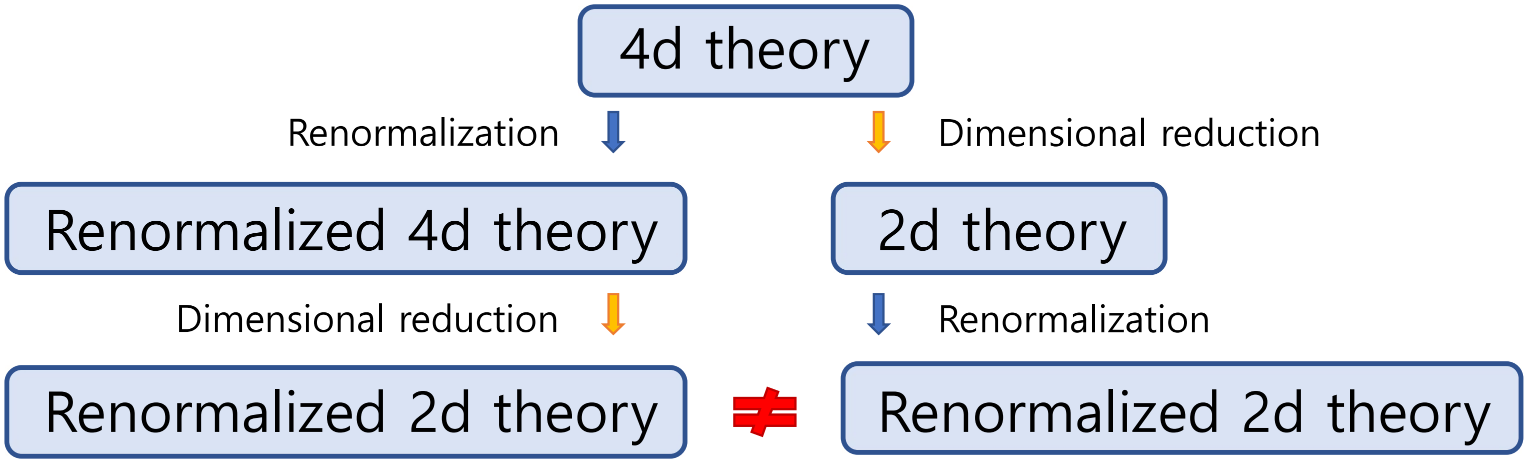
Discussion

- Different definitions between the Euclidean path-integral approach and the Lorentzian stress tensor approach
- Opposite sign of the stress tensors between 2D and 4D is

  due to the dimensional-reduction anomaly.

  [Frolov *et el.* Phys.Rev.D61, 024021(2000) (arXiv:hep-th/9909086)]

- Dimensional-reduction anomaly
  - Before renormalization the original higher-dimensional action can be presented as a sum over modes of dimensionally reduced actions, but this property is violated after renormalization.



- 4d Extension (background method)

In 4d dS space, $\langle T^{\mu}_{\nu} \rangle = \mathrm{diag}(-\rho, p, p, p), \quad -\rho + 3p = \langle T^{\mu}_{\mu} \rangle$

➡ Modified Stefan-Boltzmann law

$$\rho = 3\gamma T^4 - \frac{1}{4}\langle T^{\mu}_{\mu} \rangle$$

4d stress tensor in BD vacuum

$$\langle \mathrm{BD} | T_{\mu\nu} | \mathrm{BD} \rangle = -\frac{H^4}{960\pi^2} g_{\mu\nu}$$

➡ The temperature becomes zero.

$$T = 0$$