WEDGE HOLOGRAPHY AS GENERALIZATION OF ADS/CFT

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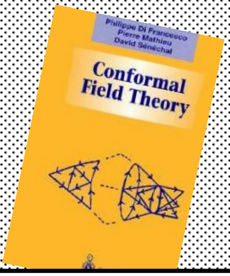
Based on [arXiv:2007.06800]
collaboration with
Ibrahim Akal, Tadashi Takayanagi, Zixia Wei

Introduction

Quantum blackhole?

Quantum gravity



Conformal field theory

AdS/CFT correspondence [Maldanena]

very useful tool to understand quantum gravity

- BH thermality means breakdown of unitality

  [Fitzpatrick, Kaplan, Walters, etc.]

- Contradiction between Hawking calculation and Page curve [Almheiri, Mahajan, Maldacena, Zhao, etc.]

Obviously, developments of AdS/CFT is important!

Summary①

[Akal, YK, Takayanagi, Wei]

Generalization

Original Holography	→	Our Wedge Holography
$AdS_{d+1} = CFT_d$		$AdS_{d+1} = CFT_{d-1}$

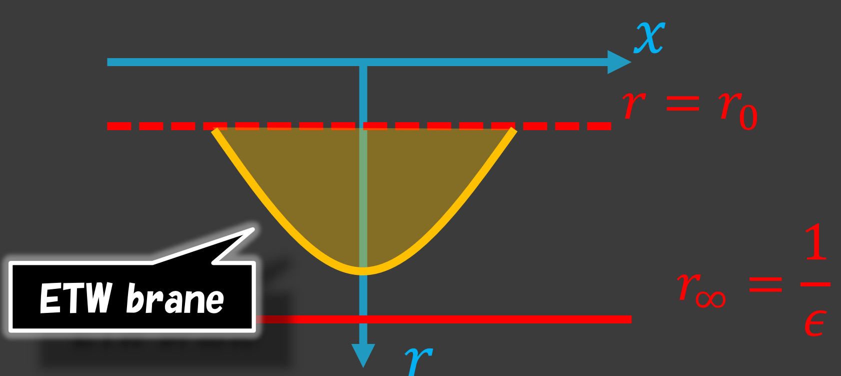
Motivation

- New laboratory
- New construction of CFT_1
- Construction is more definite than double holography
- Joining gravity from AdS_{d+1} and CFT_{d-1}

  ⇒radiation setup

Summary②

[Akal, YK, Takayanagi, Wei]



$T > 1/R$
dS brane

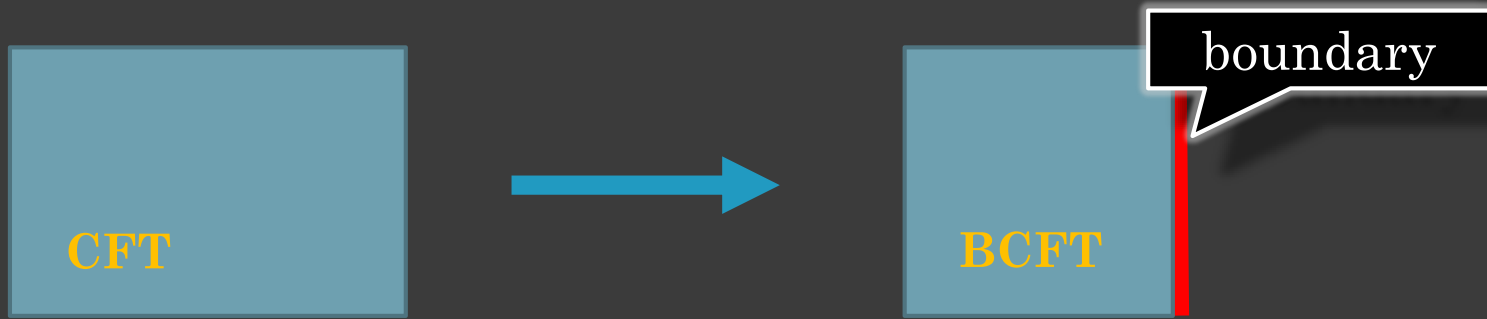
No asymptotic boundary
⇒ No CFT dual?

Motivation

- New laboratory
- dS braneworld
- Universe creation setup

Contents

Boundary CFT



Boundary breaks conformal symmetry…

Conformal Symmetry of BCFT is part of conformal symmetry preserving the bdy. position.

$$SO(2,d) \longrightarrow SO(2,d-1)$$

Bdy. Condition is

$$T_{xy} = 0$$

No energy/momentum flux across the bdy.

Boundary CFT

Definite definition of BCFT_2 is given by [Cardy].

Difference I: **Symmetry**

Symmetry of **CFT** is

$$\boldsymbol{Vir \times \overline{Vir}}$$

On the other hand, bdy. condition of **BCFT**,

$$\boldsymbol{T_{xy} = 0 \quad \Leftrightarrow \quad L_n - \bar{L}_{-n} = 0}$$

shows the dependency of Vir and $\overline{Vir}$.
Instead, symmetry of **BCFT** is given by

$$\boldsymbol{Vir}$$

Boundary CFT

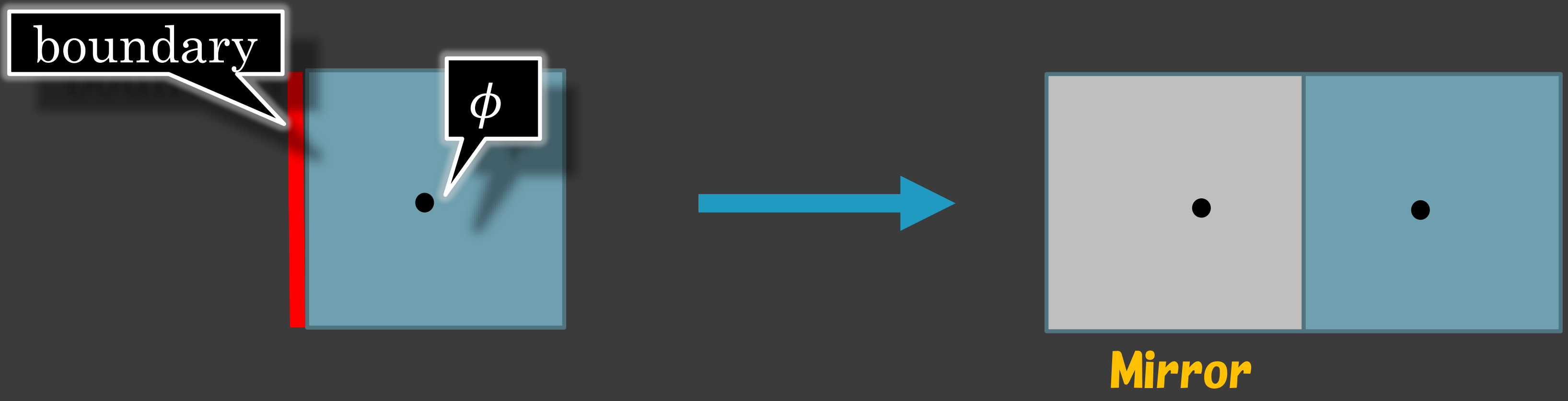
Difference II: ***Correlation function***

Kinematic part of **CFT** correlator is completely fixed by the conformal Ward id.

In fact, the ***same*** conformal Ward id. can be also applied to **BCFT** via ***mirror method***.

Therefore, we can explicitly calculate ***BCFT correlators*** like **CFT**, even though it does not have full conformal sym..

Boundary CFT

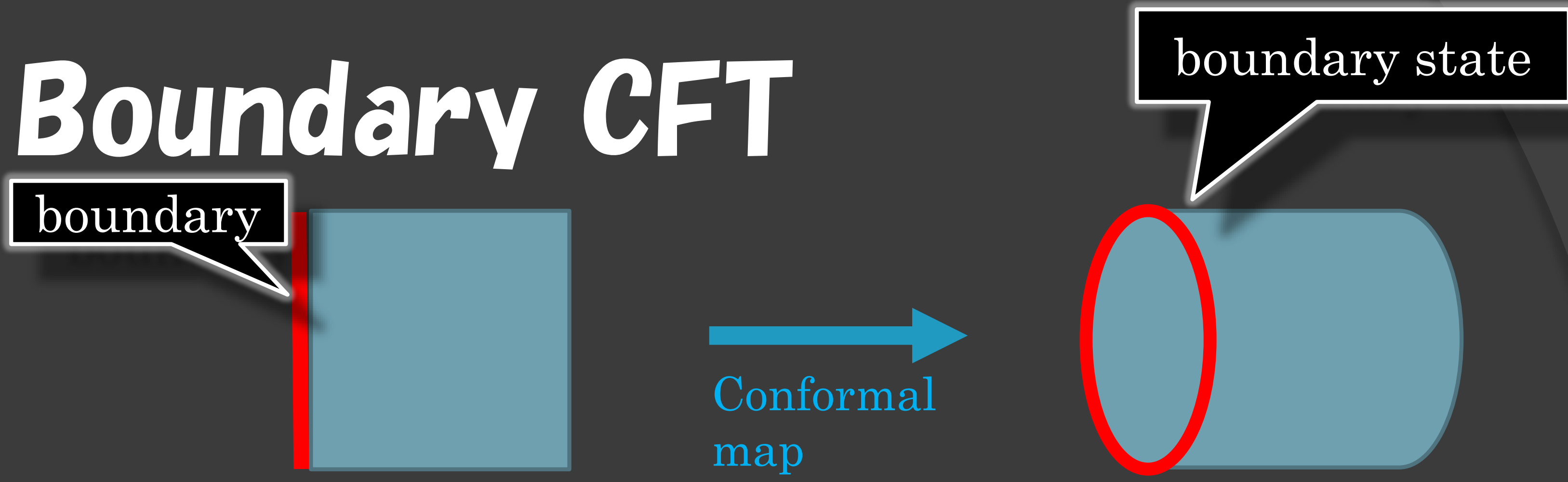


Mirror method:

$$\langle \phi(z) \rangle_{bdy} = \langle \phi(z) \phi(z^*) \rangle \propto \frac{1}{|z - z^*|^{2h_\phi}}$$

with $z = x + iy$.
The second eq. comes from standard conformal Ward id.

Boundary CFT



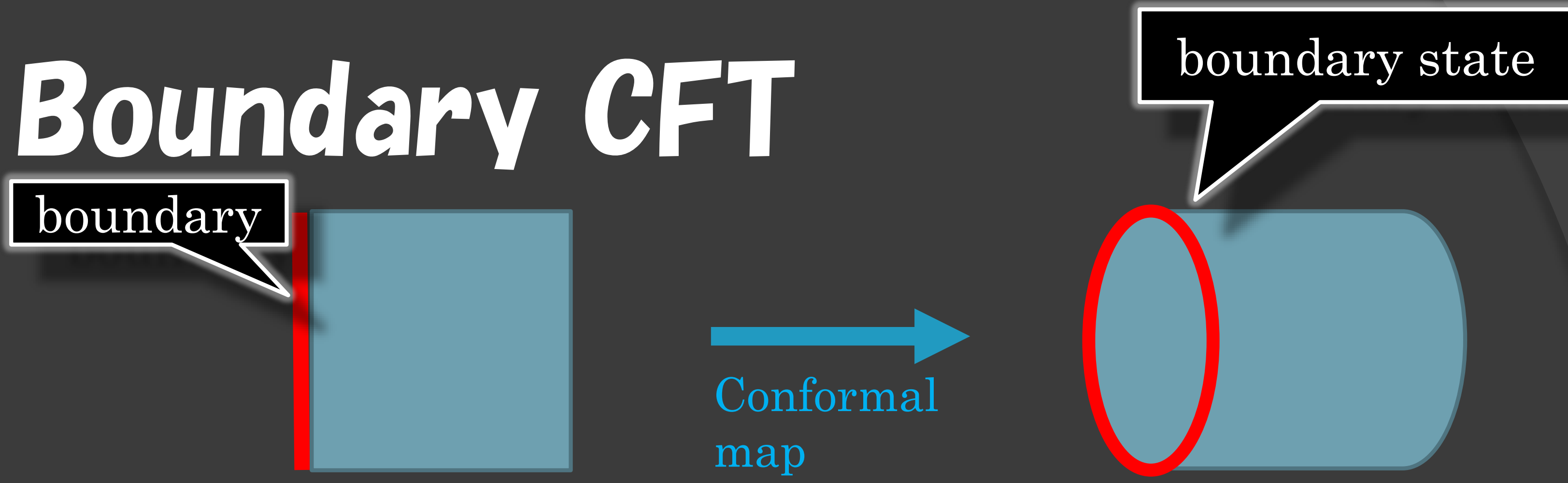
Difference III: ***Basic contents***

CFT is specified by central charge, primary state, OPE coefficient

BCFT has extra contents, boundary state, OPE coefficient among bulk states and boundary states

With all of them, we can explicitly calculate any correlators

Boundary CFT



Definition of **boundary state**

$$(L_n - L_{-n})|B\rangle = 0$$

Comment:

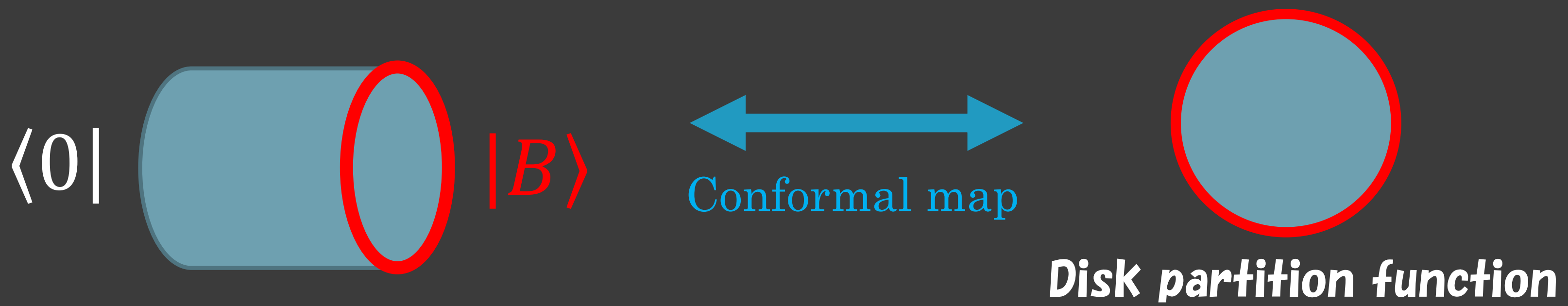
If dim. of irr. rep. is finite (i.e., RCFT), $|B\rangle$ can be completely classified,

$$|B\rangle = \frac{S_{i_B j}}{\sqrt{S_{0j}}}|j\rangle$$

Otherwise (including holographic CFT), no classification.

Boundary entropy [Affleck, Ludwig]

DoF of boundary is measured by ***boundary entropy***.

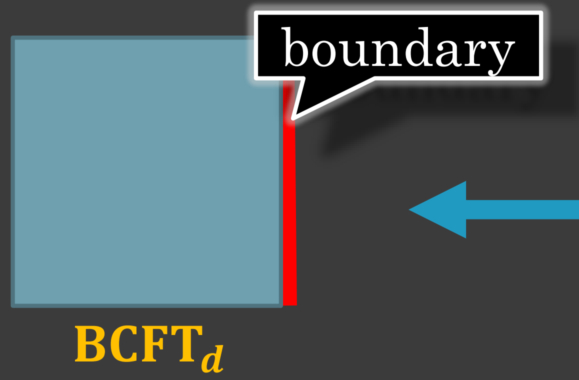


$$S_{bdy} = \log g_B, \quad g_B = \langle 0|B\rangle \simeq Z_{disk}$$

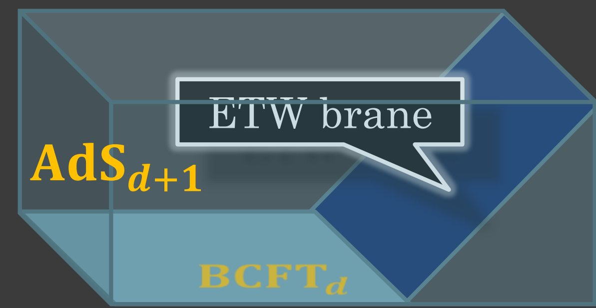
Interesting point:

- ***g-theorem*** [Friedan, Konechny]
  S_{bdy} decreases under the RG flow like central charge
  $\Rightarrow$consistent with DoF measure interpretation

Gravity dual of BCFT [Fujita, Tonni, Takayanagi]



BCFT



AdS with ETW brane

$\partial(\mathrm{ETW}) = \mathrm{bdy.\,of\,CFT}$

Induced metric: $h_{\mu\nu} = g_{\mu\nu} - n_\mu n_\nu$,

Extrinsic curvature: $K_{\mu\nu} = h_\mu{}^\rho h_\nu{}^\lambda \nabla_\rho n_\lambda$

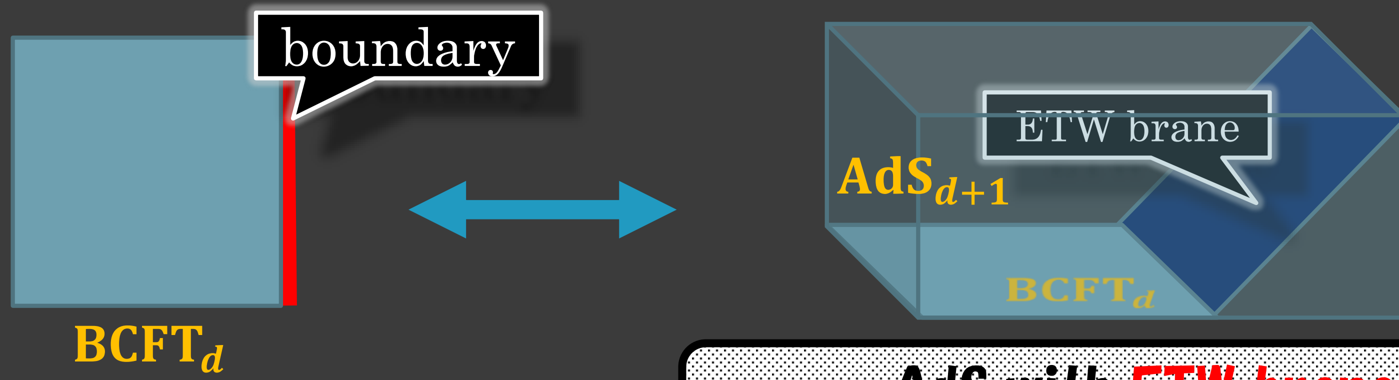
Gravity action:

$$I = -\frac{1}{16\pi G_N}\int_N \sqrt{g}(R - 2\Lambda) - \frac{1}{8\pi G_N}\int_Q \sqrt{h}(K - T)$$

Neumann b.c. is imposed on the brane (Einstein eq. of brane).

$$K_{ab} - K h_{ab} = -T h_{ab}$$

Gravity dual of BCFT [Fujita, Tonni, Takayanagi]



BCFT

AdS with ETW brane

$\partial(\mathrm{ETW}) = \mathrm{bdy.\,of\,CFT}$

Interesting point of AdS/BCFT:

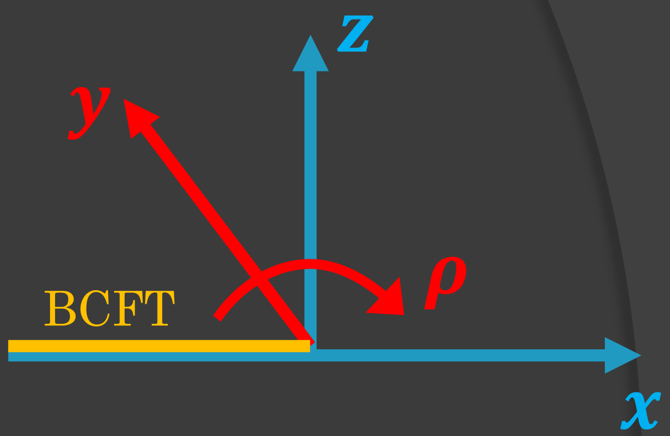
Boundary provides **dynamical** spacetime on ETW (see also [Randull-Sandrum])
This is the reason why resent progresses about Page curve utilize AdS/BCFT

Gravity dual of BCFT

Simple example:

Poincare patch:

$$ds^2 = R^2\left(\frac{dz^2 - dt^2 + dx^2 + d\vec{w}^2}{z^2}\right)$$



Let us consider a BCFT on a UHP (x < 0).

It is useful to use the coordinate $\left(x = \frac{y}{\cosh\frac{\rho}{R}}, z = \frac{y}{\cosh\frac{\rho}{R}}\right)$,

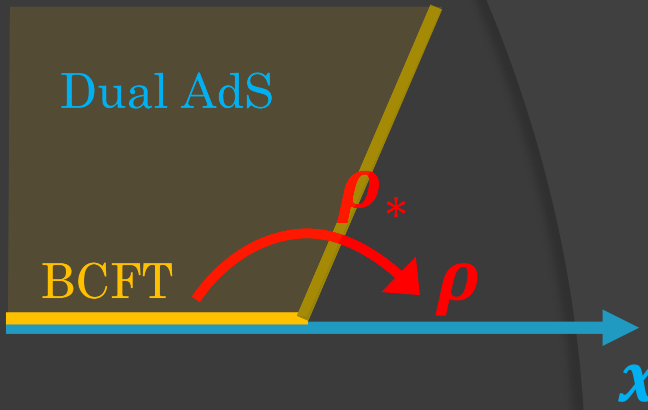
$$ds^2 = d\rho^2 + R^2\cosh^2\frac{\rho}{R}\left(\frac{dy^2 - dt^2 + d\vec{w}^2}{y^2}\right)$$

The bdy. position $\rho = \rho(y)$ is fixed by the Neumann b.c.

$$K_{ab} - Kh_{ab} = -Th_{ab}$$

Gravity dual of BCFT

$$K_{ab} - Kh_{ab} = -Th_{ab}$$



A solution to the Neumann b.c. is

$$\rho = \rho_* \qquad s.t. \quad T = \frac{d-1}{R}\tanh\frac{\rho_*}{R}$$

By an appropriate map from this UHP, the gravity dual of a disk partition function (= **boundary entropy**) is obtained.

$$S_{bdy} = -I_{disk} = \frac{\rho_*}{4G_N}$$

S_{bdy} becomes large as the size of the space is increased (i.e. ρ_* is increased)

⇒boundary becomes **classical** (explained later)

Ryu-Takayanagi formula

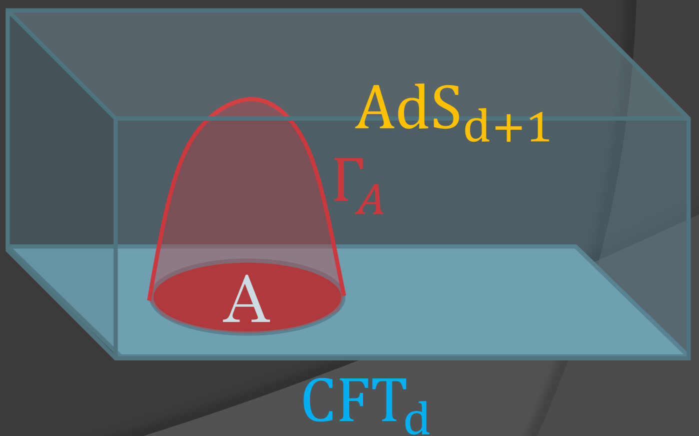
[Ryu, Takayanagi]

EE can probe how much information about $\bar{A}$ can be extracted from subregion A in CFT.

$$S_A = \mathrm{tr}\,\rho_A \log \rho_A, \qquad \rho_A = \mathrm{tr}_{\bar{A}}\,\rho$$

Gravity dual of EE is

$$S_A = \min_{\substack{\Gamma_A \\ \partial\Gamma_A = \partial A}} \left(\frac{\mathrm{Area}(\Gamma_A)}{4G_N} \right)$$



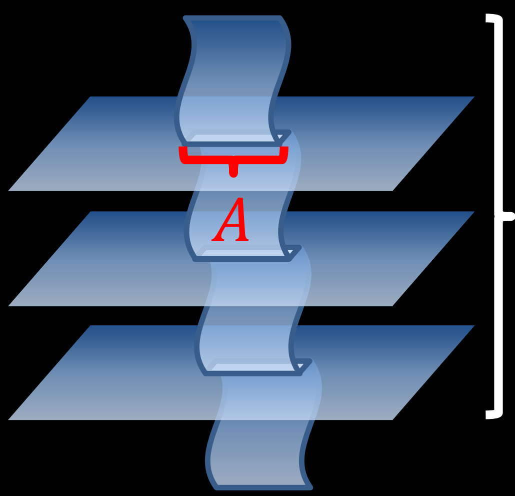
⇒ Gravity calculation is easy

Ryu-Takayanagi formula

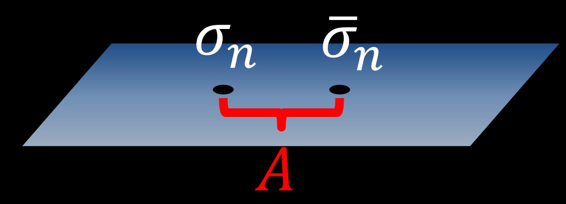
CFT calculation is also not so hard (replica trick).

$$S_A = \mathrm{tr}\,\rho_A \log \rho_A = \lim_{n\to 1} \frac{1}{1-n} \log \underline{\mathrm{tr}\rho_A^n}, \qquad \rho_A = \mathrm{tr}_{\bar{A}}\, \rho$$

$\mathrm{tr}\rho_A^n =$



n-sheets $=$



$\langle \sigma_n(l) \bar{\sigma}_n(0) \rangle$

Ryu-Takayanagi formula

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$$S_A = \mathrm{tr}\,\rho_A \log \rho_A = \lim_{n\to 1} \frac{1}{1-n} \log \underline{\mathrm{tr}\rho_A^n}, \qquad \rho_A = \mathrm{tr}_{\bar{A}}\,\rho$$

$$\mathrm{tr}\rho_A^n = \langle \sigma_n(l)\bar{\sigma}_n(0)\rangle = \frac{1}{l^{2h_{\sigma_n}}}$$

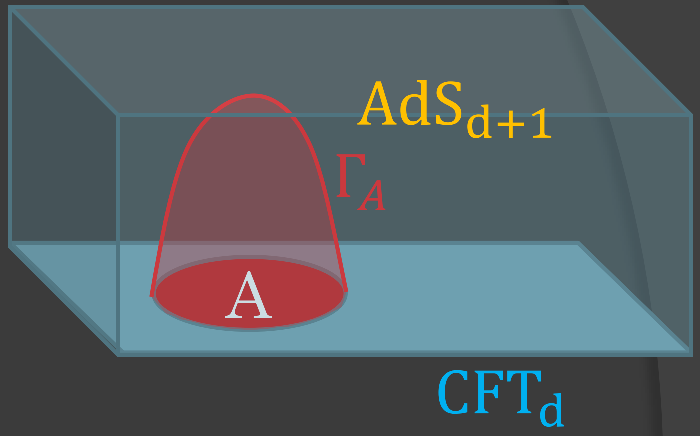
where $h_{\sigma_n} = \frac{c}{24}\left(n - \frac{1}{n}\right)$. The second eq. is just Ward id.

As a result,

$$S_A = \frac{c}{3}\log\frac{l}{\epsilon}$$

Ryu-Takayanagi formula

$$S_A = \min_{\substack{\Gamma_A \\ \partial\Gamma_A = \partial A}} \left(\frac{\mathrm{Area}(\Gamma_A)}{4G_N} \right)$$



Usefulness

- Both gravity and CFT calculation are ***easy***
  ⇒ For this reason, we will use this to justify our new holography.
- Useful to probe global feature
  - Which is part of gravity dual to subregion A in CFT?
    Sub-region/sub-region duality (see [Suzuki, YK, Takayanagi, Umemoto])
  - c-theorem [Myers-Sinha]

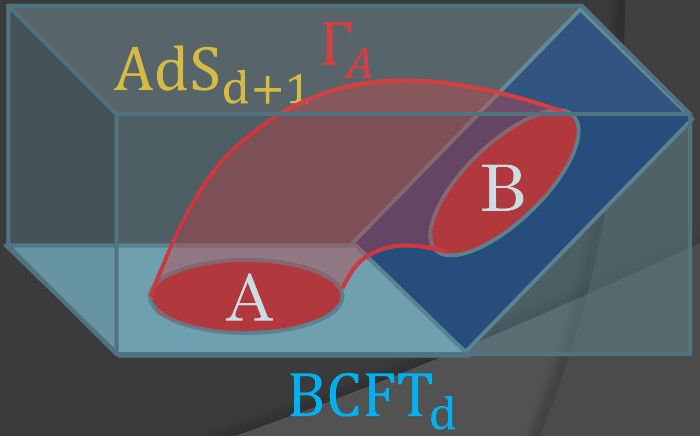
⇒ How is this generalized to AdS/BCFT?

Ryu-Takayanagi formula

Difference

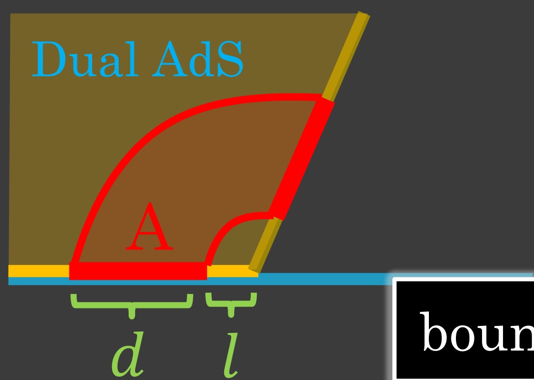
- RT surface can ***end on ETW brane*** (which satisfies homologous cond. from string perspective.)
- DoF of boundary (boundary entropy) also contributes to S_A.

$$S_A = \min_{\substack{\Gamma_A \\ \partial\Gamma_A = \partial A \cup \partial B}} \left(\min_B \frac{\mathrm{Area}(\Gamma_A)}{4G_N} \right)$$

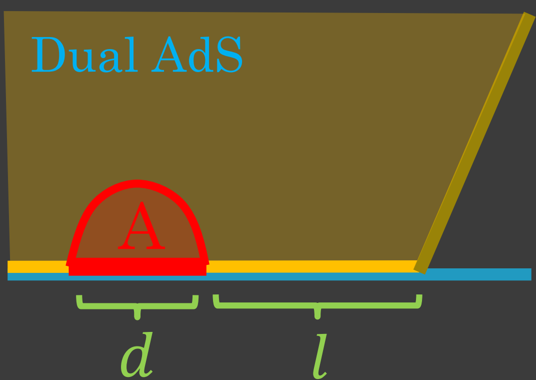


Ryu-Takayanagi formula

Simple example:



transition



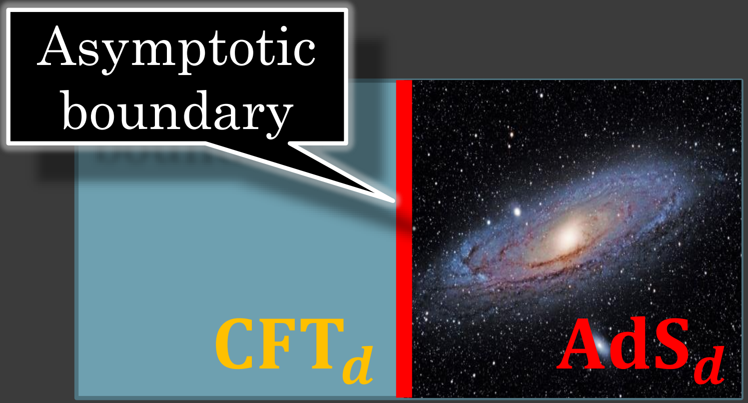
boundary entropy

$$S_A = \frac{c}{6}\log\frac{l}{\epsilon} + \frac{c}{6}\log\frac{l+d}{\epsilon} + 2\log g_B$$

$$S_A = \frac{c}{3}\log\frac{d}{\epsilon}$$

This entanglement entropy can be completely reproduced by **CFT** calculation (replica trick and ***mirror method***)

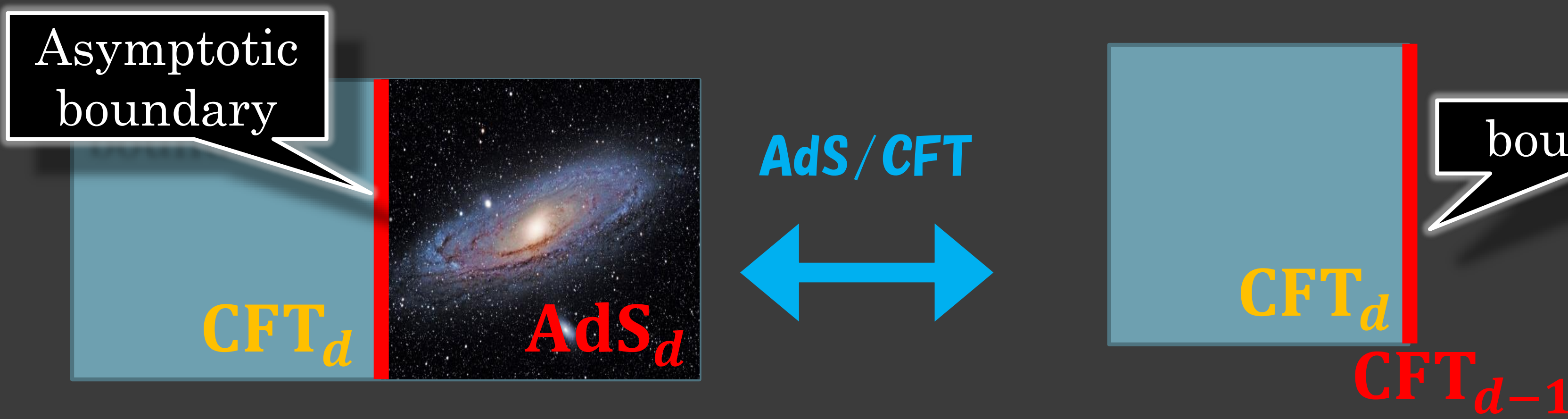
Developments from AdS/BCFT



Setup:
AdS$_d$ & CFT$_d$ are glued along the (asymptotic) boundary

This AdS$_d$ is dynamical.
Light can go through asymptotic boundary.
We can discuss the Page curve in this setup.

Developments from AdS/BCFT



Setup:

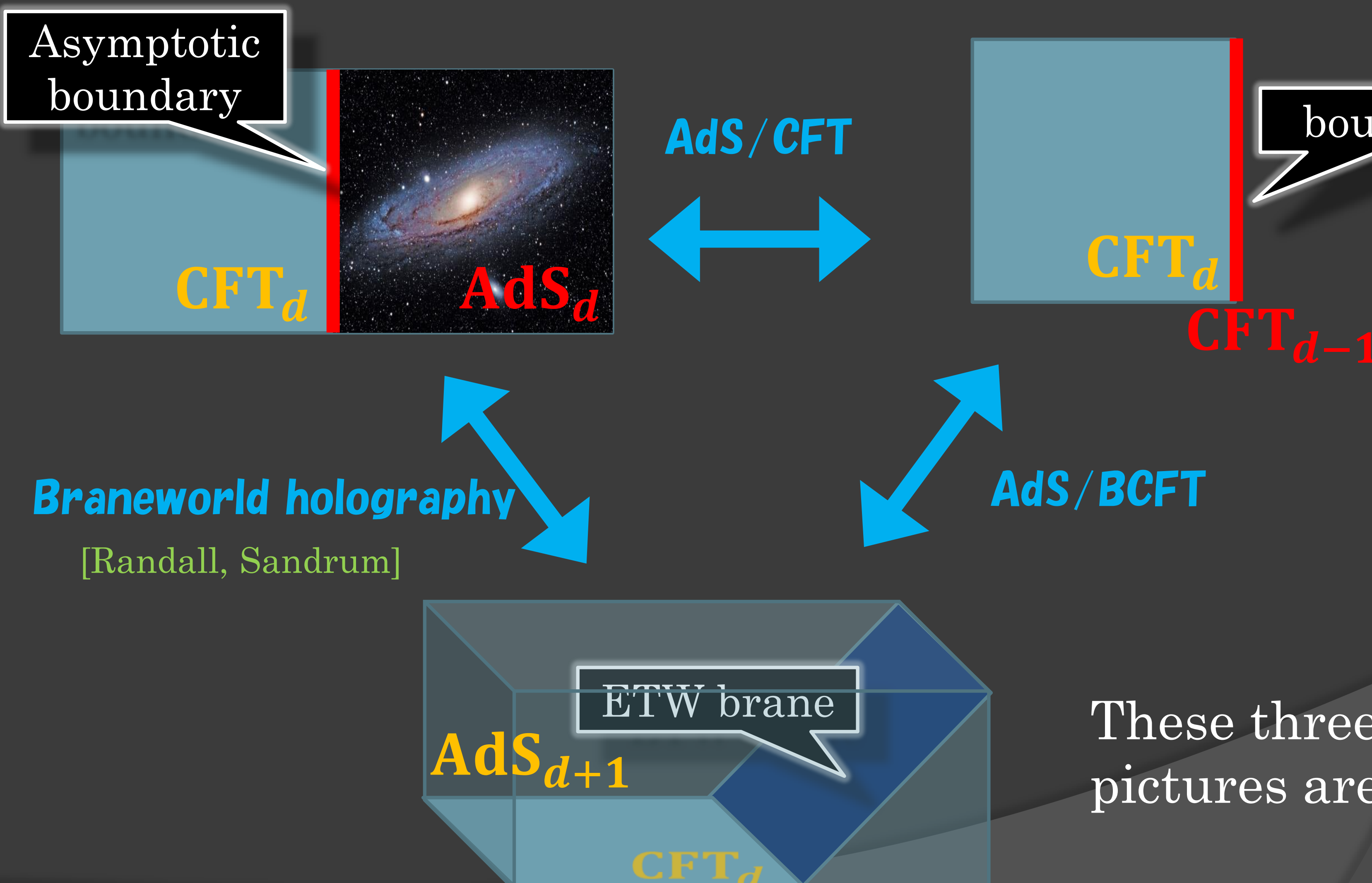
AdS$_d$ & CFT$_d$ are glued along the (asymptotic) boundary

AdS/CFT correspondence:

AdS$_d$ = CFT$_{d-1}$

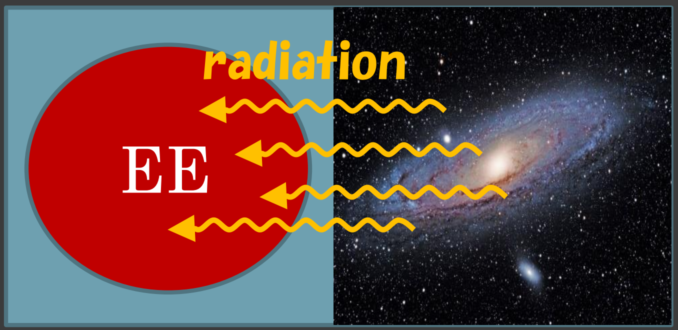
This CFT$_{d-1}$ can be thought of as boundary object of CFT$_d$

Developments from AdS/BCFT



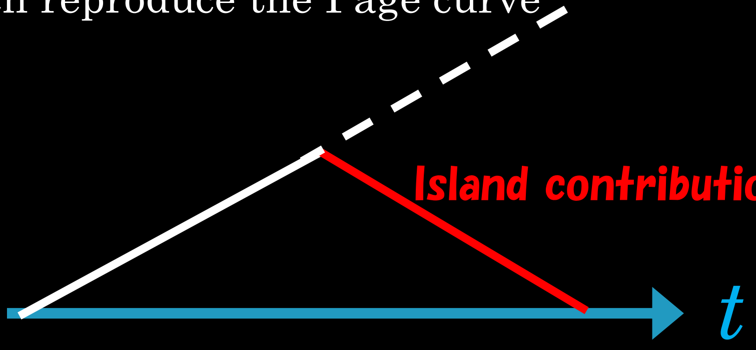
These three pictures are same

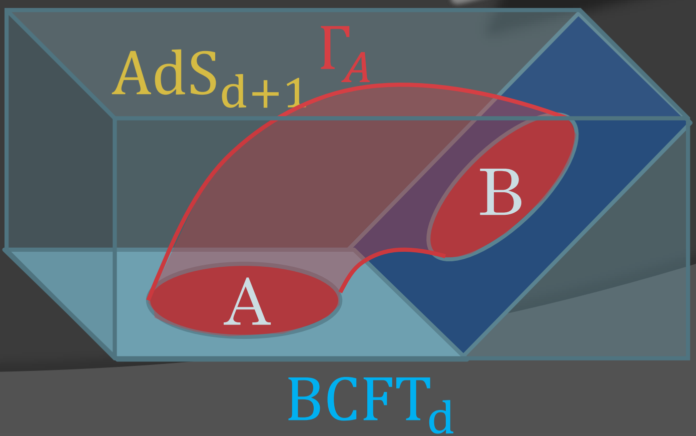
Developments from AdS/BCFT



Braneworld holography

By utilizing ***AdS/BCFT***,
We can find the contribution from ***Island***
(effective quantum contribution),
which reproduce the Page curve





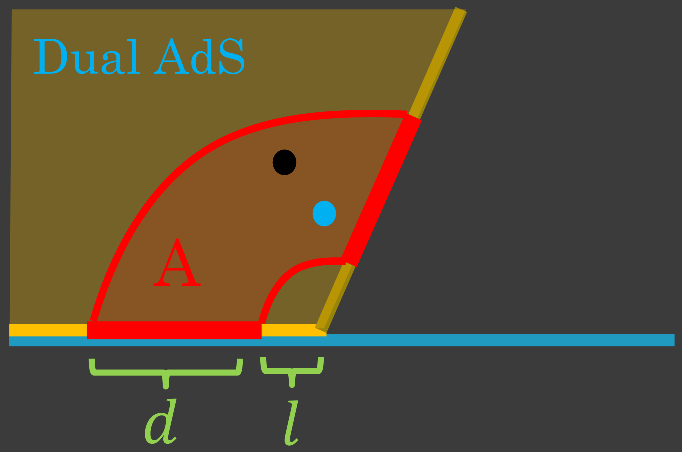
double holography
[Almheri, Mahajan, Maldacena, Zhao]

Developments from AdS/BCFT

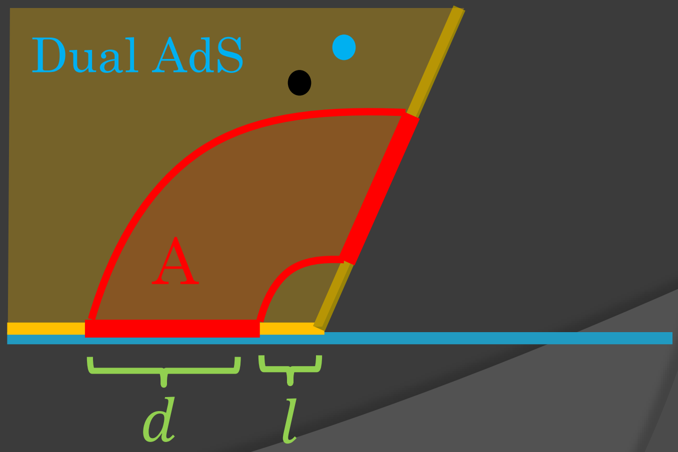
Subregion/Subregion duality implies

EE for bath $\supset$ information of gravity (island)

It means

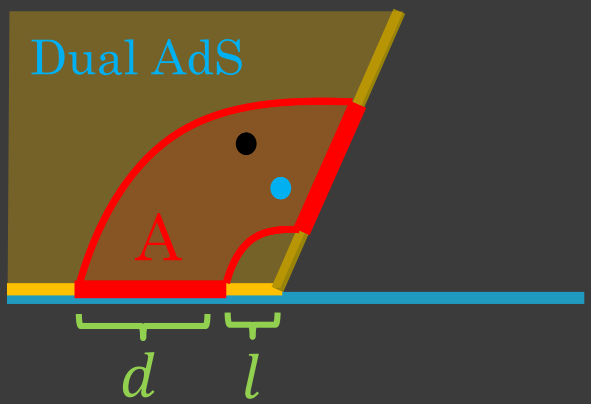


Distinguishable from subregion A

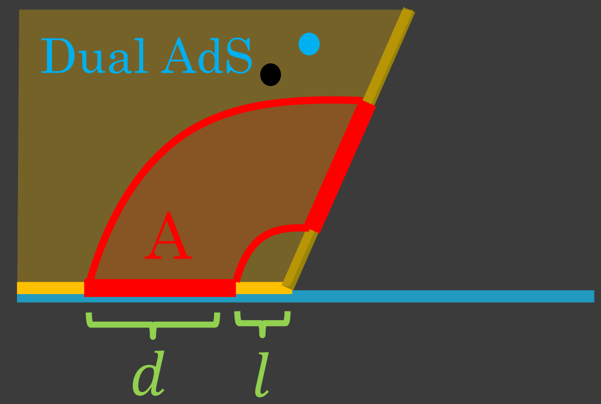


Not distinguishable from subregion A

Developments from AdS/BCFT



Distinguishable from subregion A



Not distinguishable from subregion A

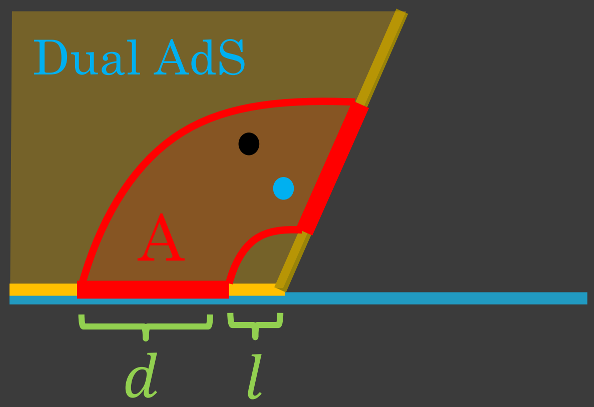
Fidelity (one of distinguishabilities)

$$F(\rho,\rho') = \mathrm{tr}\left(\sqrt{\sqrt{\rho}\rho'\sqrt{\rho}}\right)$$

$F = 1$ if and only if $\rho = \rho'$

can be used to show this subregion/subregion duality.

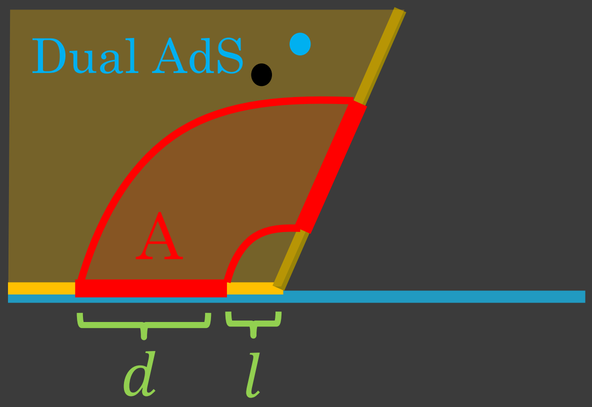
Developments from AdS/BCFT



Distinguishable from subregion A

$\rho =$ black excitation,
$\rho' =$ blue excitation,

$$F(\rho, \rho') \neq 1$$



Not distinguishable from subregion A

$\rho =$ black excitation,
$\rho' =$ blue excitation,

$$F(\rho, \rho') = 1$$

See more details in [Suzuki, YK, Takayanagi, Umemoto]

Contents

Developments from AdS/BCFT

[Akal, YK, Takayanagi, Wei]

Original Holography

$d+1$ dimensional AdS = d dimensional CFT

↓ Generalization

Our Wedge Holography

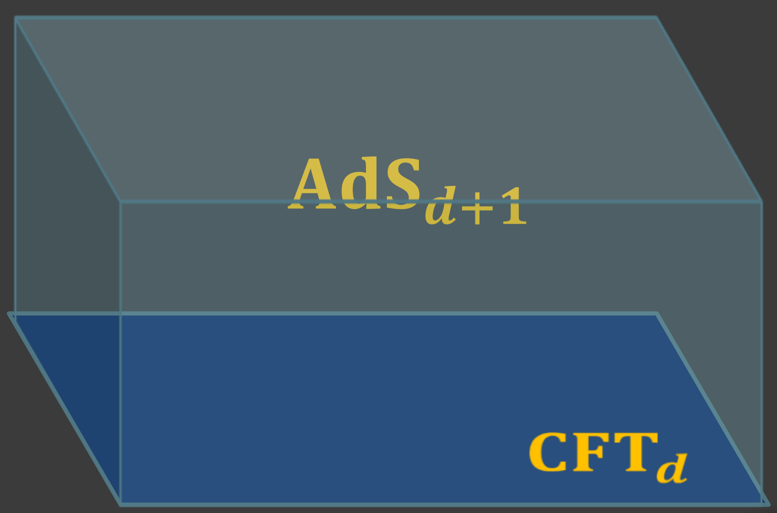
$d+1$ dimensional AdS = $\boldsymbol{d-1}$ dimensional CFT

Developments from AdS/BCFT

[Akal, YK, Takayanagi, Wei]

Original Holography

$d+1$ dimensional AdS = d dimensional CFT

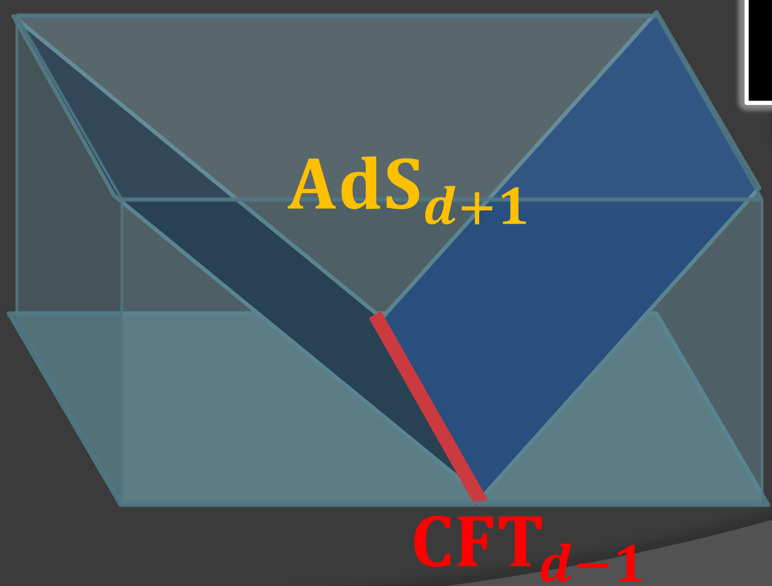


Developments from AdS/BCFT

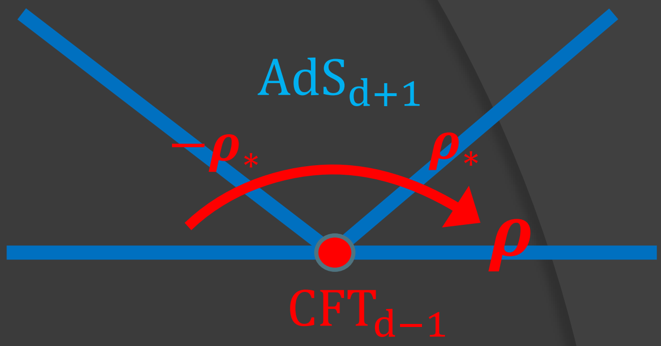
[Akal, YK, Takayanagi, Wei]

Our Wedge Holography

$d+1$ dimensional AdS = $d-1$ dimensional CFT



Wedge Holography



Definition of wedge:

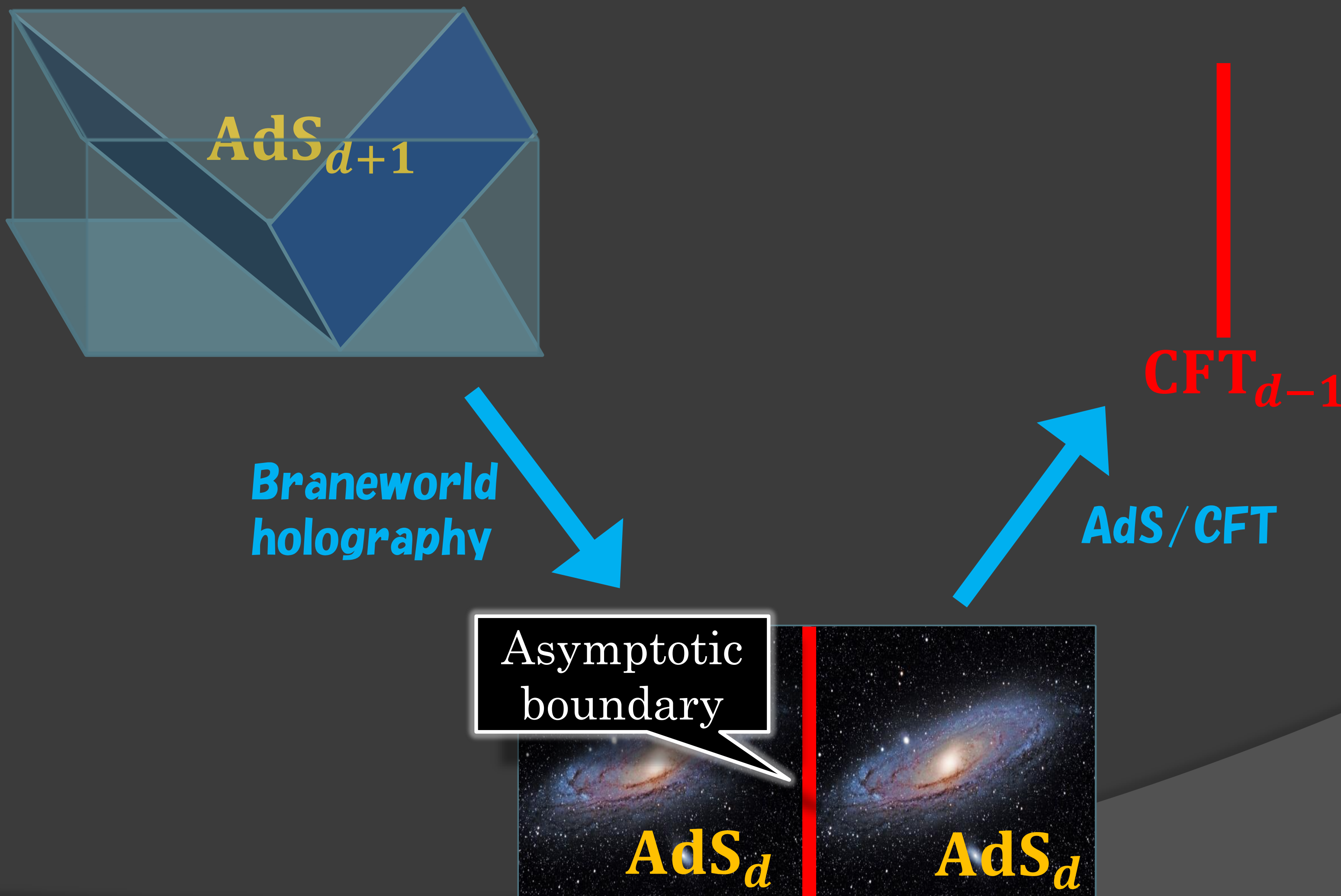
$$ds^2_{AdS_{d+1}} = d\rho^2 + R^2 \cosh^2\frac{\rho}{R}\left(\frac{dy^2 - dt^2 + d\vec{w}^2}{y^2}\right)$$
$$= d\rho^2 + \cosh^2\frac{\rho}{R}\, ds^2_{AdS_d}$$

Restricted to **wedge** subspace $-\rho_* \leq \rho \leq \rho_*$

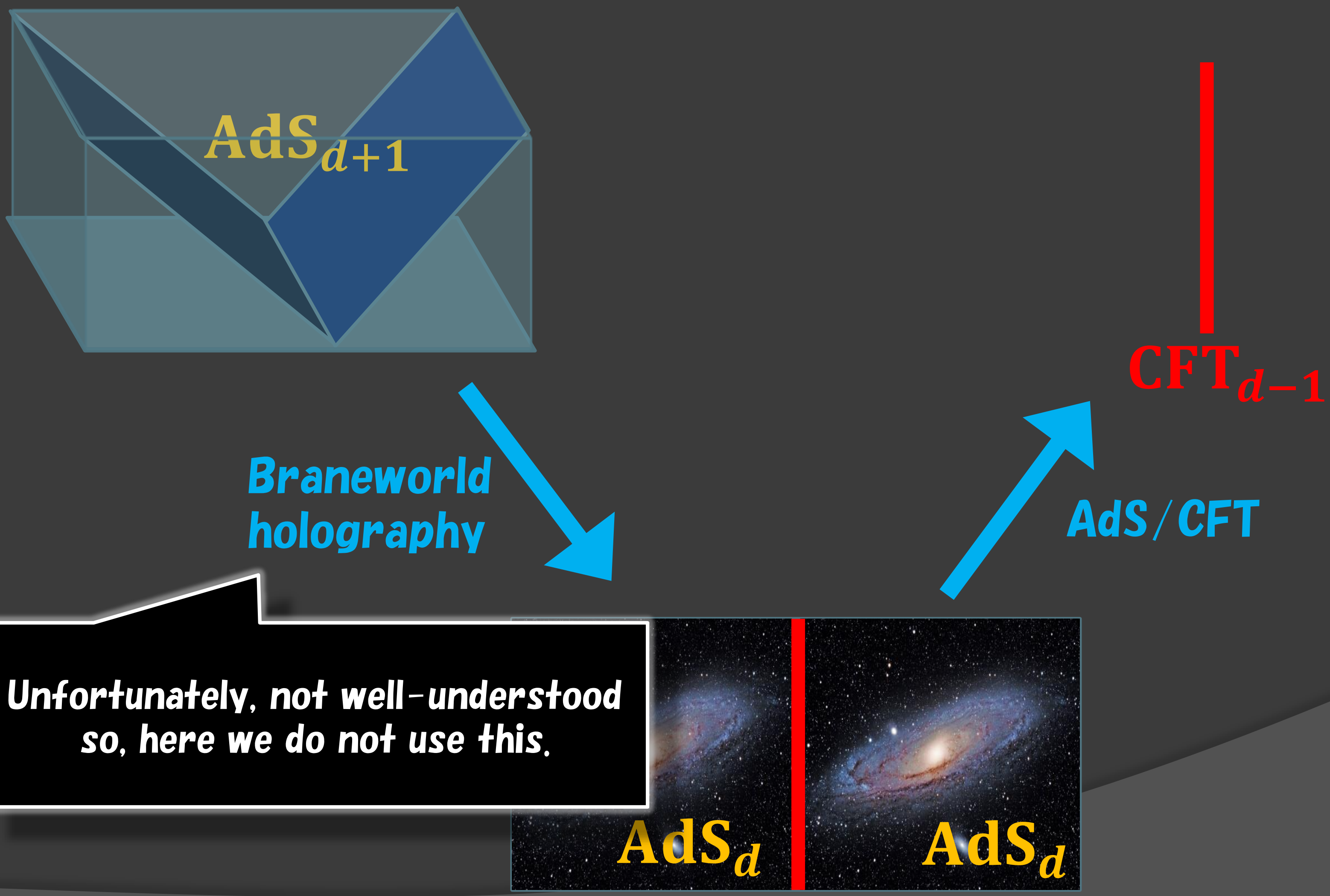
⇒asymptotic boundary is $d-1$ dimensional theory

Our proposal: This is CFT_{d-1}

Naive derivation?

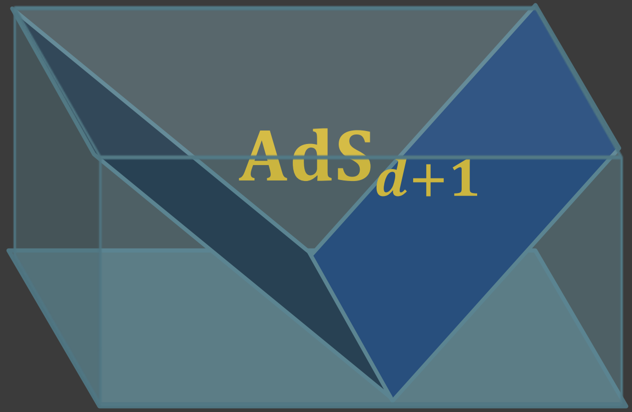


Naive derivation?



Unfortunately, not well-understood so, here we do not use this.

Naive derivation?

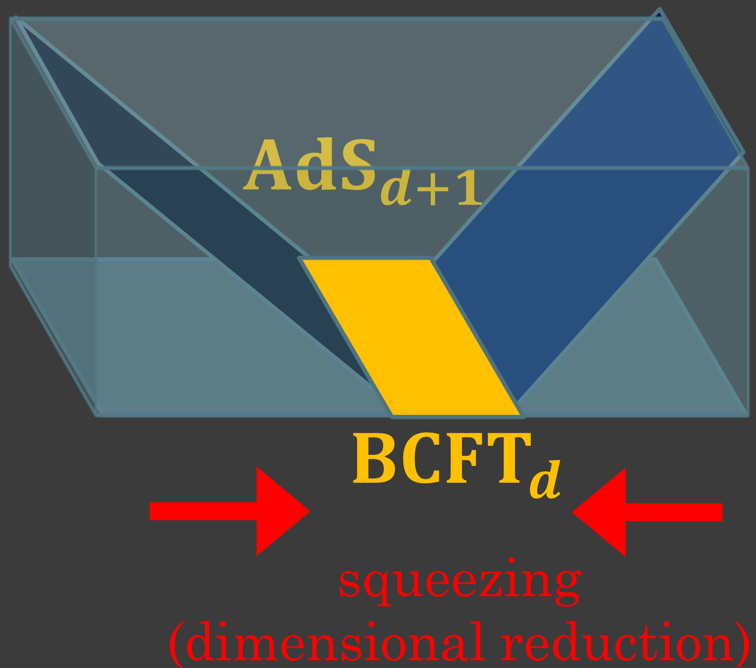


AdS/BCFT

CFT_{d-1}

Instead, we employ AdS/BCFT,
clear understanding,
some justifications

Derivation from AdS/BCFT



Definition: **Limit of BCFT** (very clear!)

Derivation from AdS/BCFT

Definition: Limit of BCFT (very clear!)

⇒But is this really CFT?

Three Justifications:

- Brane world holography + AdS/CFT (at least large ρ_*)
- Boundary of BCFT_d can be thought of as CFT_{d-1} , because OPE of boundary state also satisfies axiom of CFT. In this sense, our CFT_{d-1} can be interpreted as two boundaries interacting with each other through bulk.
- ***From now on, we see matching of gravity calculation and CFT calculation for some physical quantities.***

Free energy on Sphere

One check of holography principle can be done by seeing ***free energy***.

The on-shell action of $\mathrm{AdS}_{\mathrm{d+1}}$ (without boundaries) has the following form (see [Henningson, Skendteries]),

$$I = \#\frac{1}{\epsilon^d} + \#\frac{1}{\epsilon^{d-2}} + \cdots + \#\frac{1}{\epsilon^2} - \#\log\epsilon + O(1) \qquad (\text{if } d = odd)$$

$$I = \#\frac{1}{\epsilon^d} + \#\frac{1}{\epsilon^{d-2}} + \cdots + \#\frac{1}{\epsilon} + O(1) \qquad (\text{if } d = even)$$

In fact, $\mathrm{CFT}_{\mathrm{d}}$ has the same form.

$$\boxed{I_{AdS_{d+1}} = I_{CFT_d}}$$

This is one consistency check of the $\mathrm{AdS}_{\mathrm{d+1}}/\mathrm{CFT}_{\mathrm{d}}$ correspondence.

Free energy on Sphere

Let us consider $\mathrm{AdS}_{\mathrm{d+1}}/\mathrm{CFT}_{\mathrm{d-1}}$.

Of course, without the wedge, we can conclude

$$\boxed{I_{AdS_{d+1}} \neq I_{CFT_{d-1}}}$$

Therefore, the contribution from the **wedge** should have some non-trivial effects. We will see this.

For simplicity, let us focus on $\mathrm{AdS}_4/\mathrm{CFT}_2$.

We know

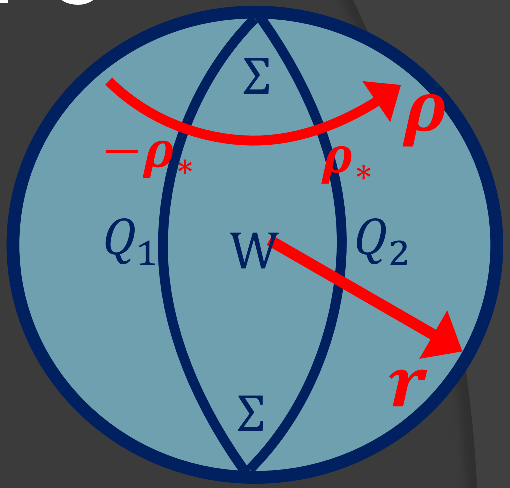
$$I_{CFT_2} = \#\frac{1}{\epsilon^2} + \frac{\mathrm{c}}{6}\chi(\Sigma)\log\epsilon + O(1)$$

Let's calculate the gravity action of wedged AdS_4 directly!

Free energy on Sphere

Metric of Euclidian AdS_{d+1}:

$$ds^2 = d\rho^2 + R^2 \cosh^2\frac{\rho}{R}\left(d\eta^2 + \sinh^2\eta\, d\Omega_{d-1}^2\right)$$
$$= dr^2 + R^2 \sinh^2\frac{r}{R}\left(d\theta^2 + \cos^2\theta\, d\Omega_{d-1}^2\right)$$



On-shell action gravity action on this wedge geometry,

$$I = -\frac{1}{16\pi G_N}\int_W \sqrt{g}(R - 2\Lambda) - \frac{1}{8\pi G_N}\int_{Q_1\cup Q_2}\sqrt{h}(K - T) - \frac{1}{8\pi G_N}\int_\Sigma \sqrt{h}\,K$$

For the wedged AdS_4, we have

$$I_{WAdS_4} = -\frac{R^2}{2G_N\epsilon^2}\sinh\frac{\rho_*}{R} + \frac{R^2}{G_N}\sinh\frac{\rho_*}{R}\log\epsilon + O(1)$$

Free energy on Sphere

$$I_{WAdS_4} = -\frac{R^2}{2G_N\epsilon^2}\sinh\frac{\rho_*}{R} + \frac{R^2}{G_N}\sinh\frac{\rho_*}{R}\log\epsilon + O(1)$$

The form for CFT_2 is

$$I_{CFT_2} = \#\frac{1}{\epsilon^2} + \frac{\mathrm{c}}{6}\chi(\Sigma)\log\epsilon + O(1)$$

The ϵ dependence perfectly matches !!

The central charge (i.e. degrees of freedom) of the wedged AdS_4 is ($\chi(\mathrm{S}^2) = 2$)

$$c = \frac{3R^2}{G_N}\sinh\frac{\rho_*}{R}$$

Next, we will check this result in an independent way.

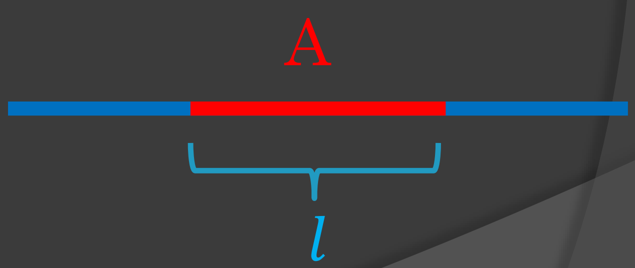
Entanglement entropy

Entanglement entropy can be evaluated from both **CFT** and **gravity** side. Therefore, it is very useful to check our new holographic principle.

For simplicity, let us focus on AdS_4/CFT_2 and consider a single interval A with size l.

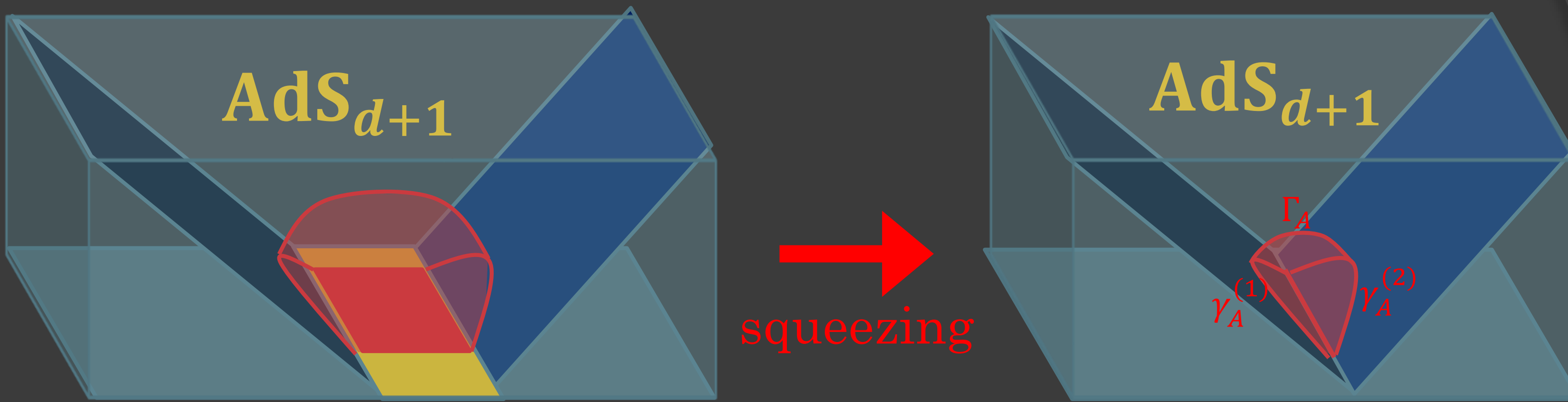
In this case, the entanglement entropy in CFT_2 is known as

$$\boxed{S_A = \frac{c}{3} \log \frac{l}{\epsilon}}$$



Let us compare this with gravity calculation.

Holographic EE



well-known HEE formula **our HEE formula**

The holographic entanglement entropy for our holography is naturally defined by recalling its definition,

$$S_A = \min_{\substack{\gamma_A^{(1)},\gamma_A^{(2)} \\ \partial\gamma_A^{(1,2)}=\partial A}} \left(\min_{\substack{\Gamma_A \\ \partial\Gamma_A=\gamma_A^{(1)}\cup\gamma_A^{(2)}}} \left(\frac{\mathrm{Area}(\Gamma_A)}{4G_N} \right) \right)$$

Holographic EE

This formula in 4d case leads to

$$S_A = \frac{R^2}{G_N}\sinh\frac{\rho_*}{R}\log\frac{l}{\epsilon}$$

This form matches with the CFT_2 result.

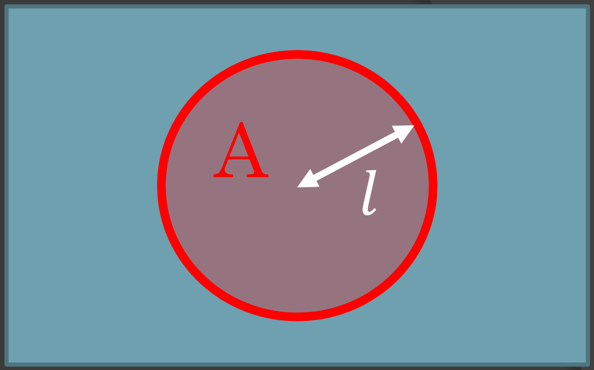
We can extract the **central charge** from this result as

$$c = \frac{3R^2}{G_N}\sinh\frac{\rho_*}{R}$$

This is completely same as the previous result!

Note: we can see this matching for more general cases (higher dimension and intervals).

Holographic EE



More consistency check:

HEE for a round disk with radius l

- AdS_5/CFT_3

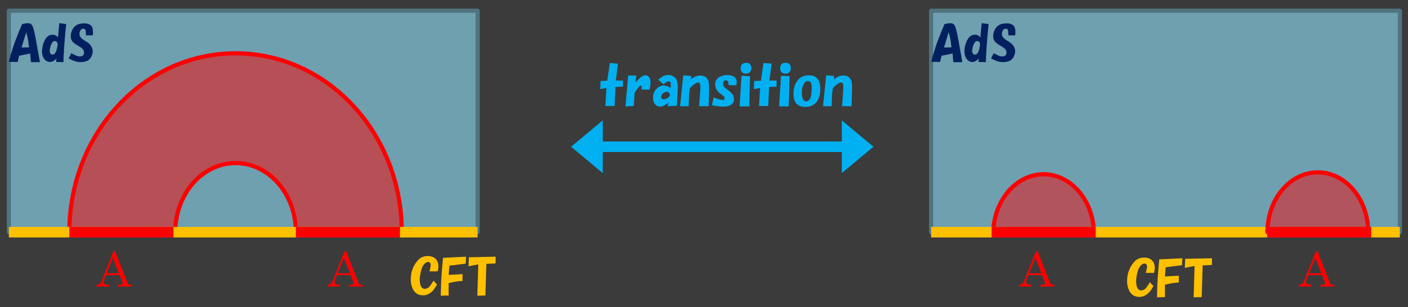
$$S_A = \left(\frac{\pi R^3}{G_N^{(5)}}\sinh\frac{\rho_*}{R}\right)\frac{l}{\epsilon} - \frac{\pi R^3}{G_N^{(5)}}\left(\frac{\rho_*}{2R} + \frac{1}{4}\sinh\frac{2\rho_*}{R}\right) + O\left(\frac{\epsilon}{l}\right)$$

- AdS_6/CFT_4

$$S_A = \left(\frac{\pi R^4}{G_N^{(6)}}\sinh\frac{\rho_*}{R}\right)\left(\frac{l}{\epsilon}\right)^2 - \frac{\pi R^4}{G_N^{(6)}}\left(\frac{\sinh\frac{3\rho_*}{R}}{12} + \frac{3}{4}\sinh\frac{\rho_*}{R}\right)\log\frac{l}{\epsilon} + O(1)$$

This scaling profile of HEE agrees with CFT_{d-1} results

Holographic EE



Double interval case: We found ***transition*** like the usual HEE.

Implication:
CFT dual to Wedge geometry is similar to holographic CFT (i.e. gapped CFT)

But in this work, we do not identify any specific features of the dual CFT in a definite way, except for central charge.
This is very interesting future direction!

Comments

- CFT_{d-1} ***dual to*** AdS_{d+1}
  - $c = \frac{3R^2}{G_N} \sinh \frac{\rho_*}{R}$
  - Large DoF of AdS_{d+1} leads to infinite tower of primary operators, which are interpreted as KK modes in brane world.

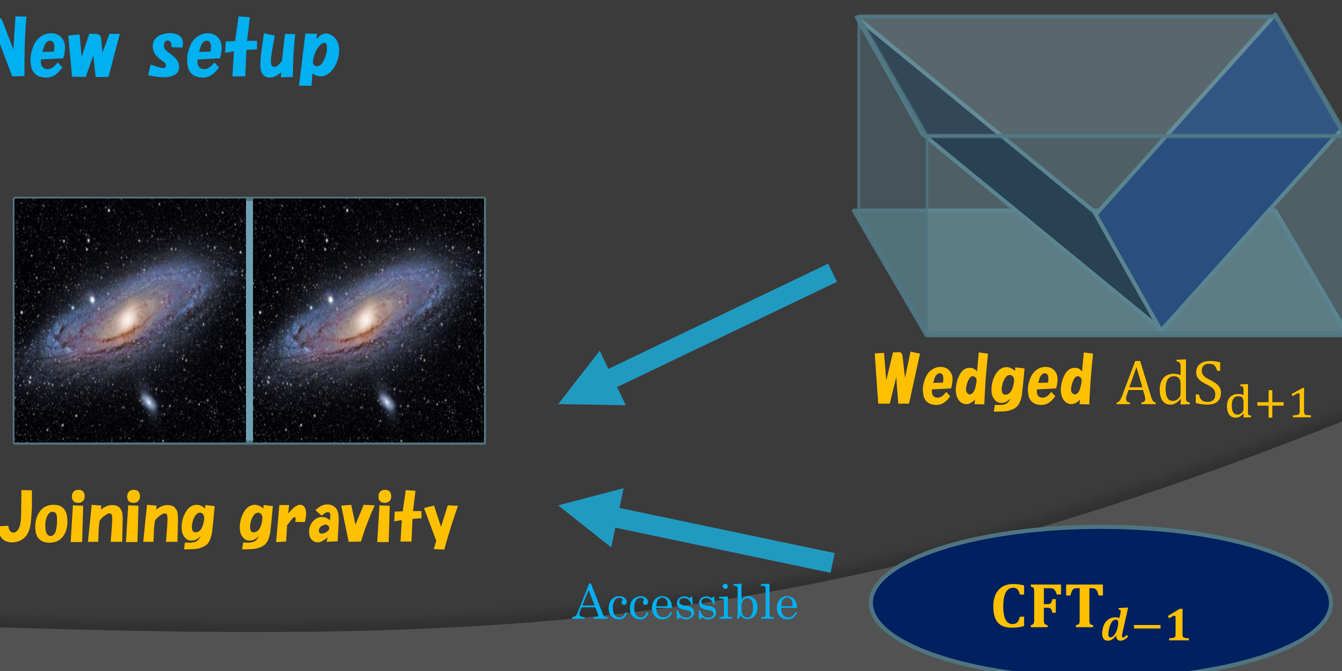
    $\Rightarrow$NOT holographic CFT
  - Zero size limit seems to be singular, but from gravity perspective, quantity in CFT_{d-1} are well-defined as wee saw.

Comments

- **New construction of CFT_1?**

  One construction of CFT_1 is known as the dual of JT gravity.
  But more precisely this is not conformal but near conformal.
  From our $\mathrm{AdS}_3/\mathrm{CFT}_1$, the dual theory is just a part of AdS_3, therefore, the CFT_1 is really conformal field theory.

- **New setup**



Comments

- **New setup**

Joining gravity

From compactification,

$$\frac{1}{G_N^{(d)}} = \frac{R}{G_N^{(d+1)}} \sinh\frac{\rho_*}{R}$$

- $$c_{AdS_3} \times 2 = \frac{3R}{2G_N^{(3)}} \times 2 = \frac{3R^2}{G_N^{(4)}} \sinh\frac{\rho_*}{R}$$

  is consistent with the central charge derived from EE.
- In general, if ρ_* is large, effective d-dimensional gravity is well-described (like R-S model).

W-holography at finite temp.

Let us consider more general case.

BTZ is simplest one other than Poincare AdS.

Metric of BTZ:

$$ds^2 = -(r^2 - r_0^2)dt^2 + R^2 \frac{dr^2}{r^2 - r_0^2} + r^2 dx^2$$

Coordinate transformation

$$t' \pm x' = \pm e^{\frac{r_0}{R}(x \pm t)} \sqrt{1 - \frac{r_0^2}{r^2}}, \quad z = \frac{r_0}{r} e^{\frac{r_0}{R} x}$$

Poincare:

$$ds^2 = R^2 \left(\frac{dz^2 - dt'^2 + dx'^2}{z^2} \right)$$

W-holography at finite temp.

General solution to Neumann bdy. in Poincare metric

$$(z-\alpha)^2+(x'-p)^2-(t'-q)^2=\beta^2$$

Its tension is $T=\frac{\alpha}{\beta R}$

Coordinate transformation

$RT<1$: AdS_2 brane

$$r(x)=\frac{r_0 TR}{\sqrt{1-T^2R^2}\sinh\frac{r_0 x}{R}}$$

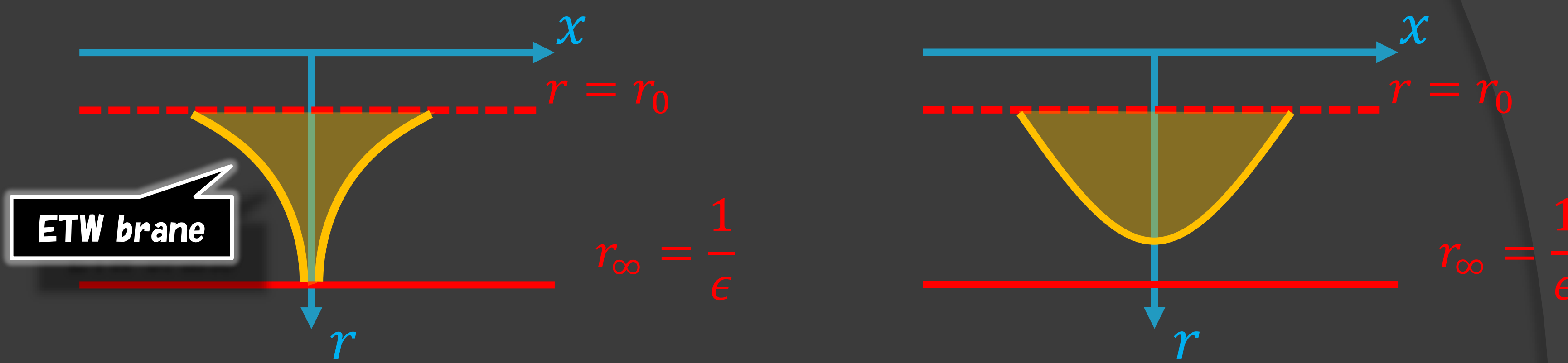
$RT=1$: R_2 brane

$$r(x)=2r_0 e^{\frac{r_0 x}{R}}$$

$RT>1$: dS_2 brane

$$r(x)=\frac{r_0 TR}{\sqrt{T^2R^2-1}\cosh\frac{r_0 x}{R}}$$

W-holography at finite temp.



$T < 1/R$
AdS_2 brane

$T > 1/R$
dS_2 brane

AdS brane case:

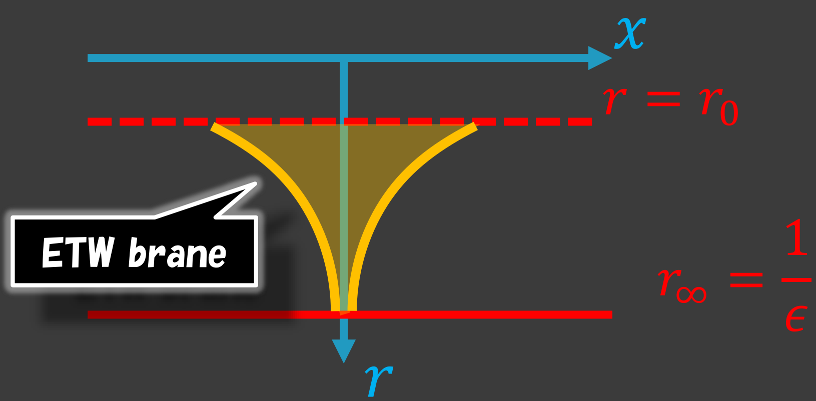
$$I_W = -\frac{\beta \sinh \rho_*}{4\pi G_N \epsilon} - \frac{\rho_*}{2G_N}$$

$= 2 \times S_{bdy}$

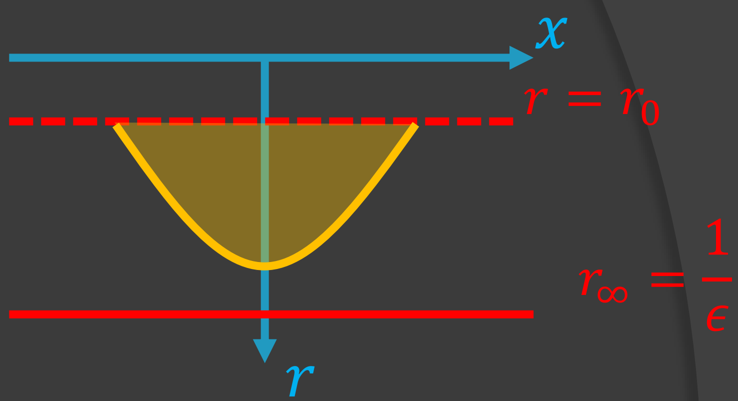
Gravity action is consistent with anomaly of CFT_1.

DoF of CFT_1 is equal to DoF of two AdS branes.

W-holography at finite temp.



$T < 1/R$
AdS_2 brane



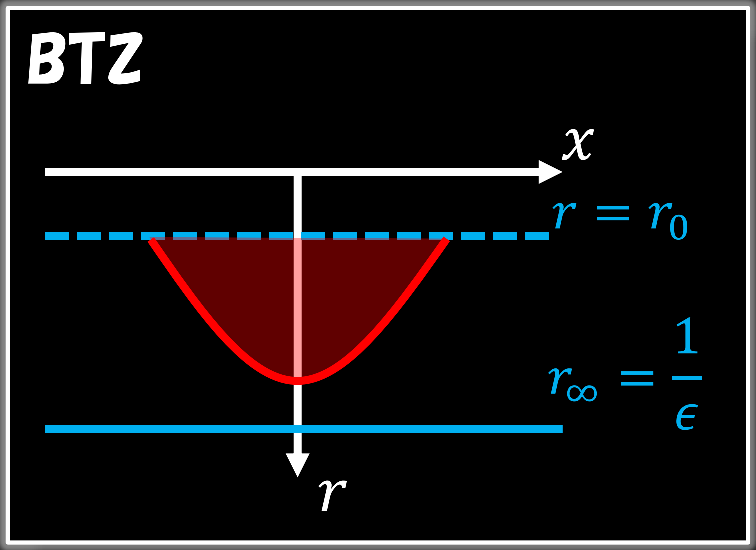
$T > 1/R$
dS_2 brane

dS brane case?

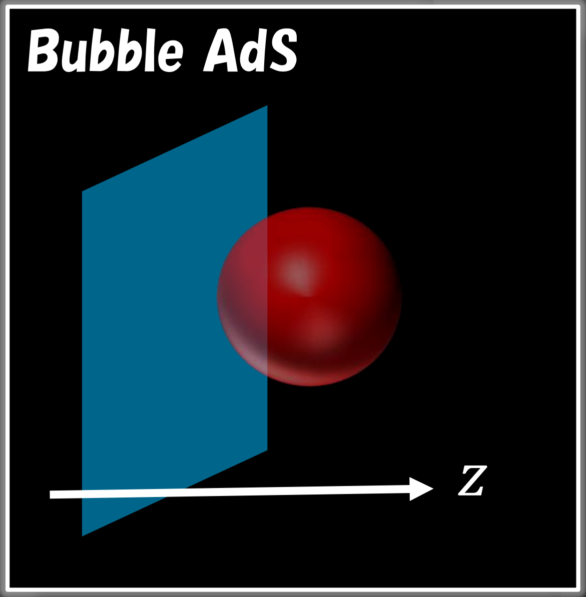
No asymptotic boundary ⇒ No CFT dual ?

Contents

Imaginary BCFT



back to
(Euclidean)
Poincare

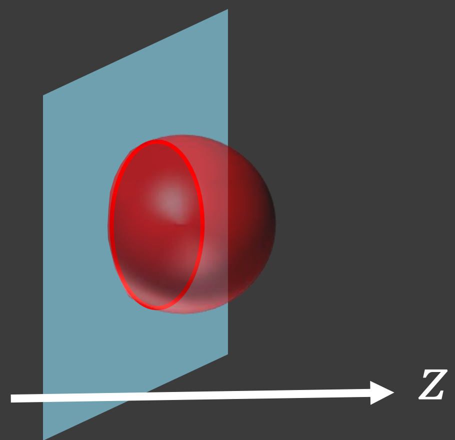


Profile of **bubble** brane is

$$(z-\alpha)^2 + x^2 + t^2 = \beta^2 \qquad \text{with} \qquad |\alpha| > |\beta|$$

Bubble AdS had not been considered in AdS/BCFT context.
Here, we give the CFT dual of this bubble AdS.

Imaginary BCFT

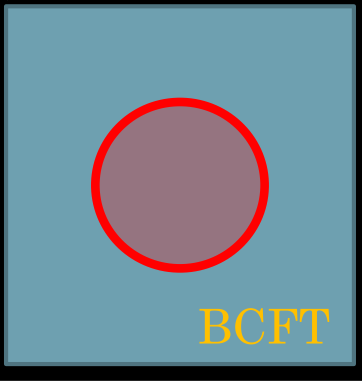


$T < 1/R$: AdS_2 brane

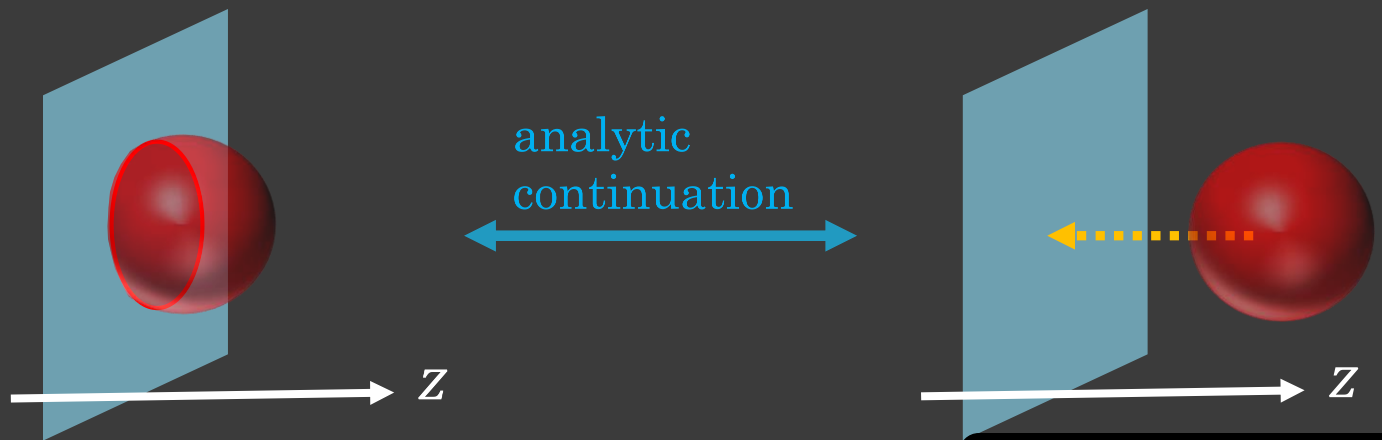
This geometry has BCFT dual (**Known!**).
Its bdy. is circle with radius

$$r^2 = \beta^2 - \alpha^2$$

Note that $T < 1/R \iff \beta^2 > \alpha^2$



Imaginary BCFT



$T < 1/R$: AdS_2 brane

$T > 1/R$: dS_2 brane

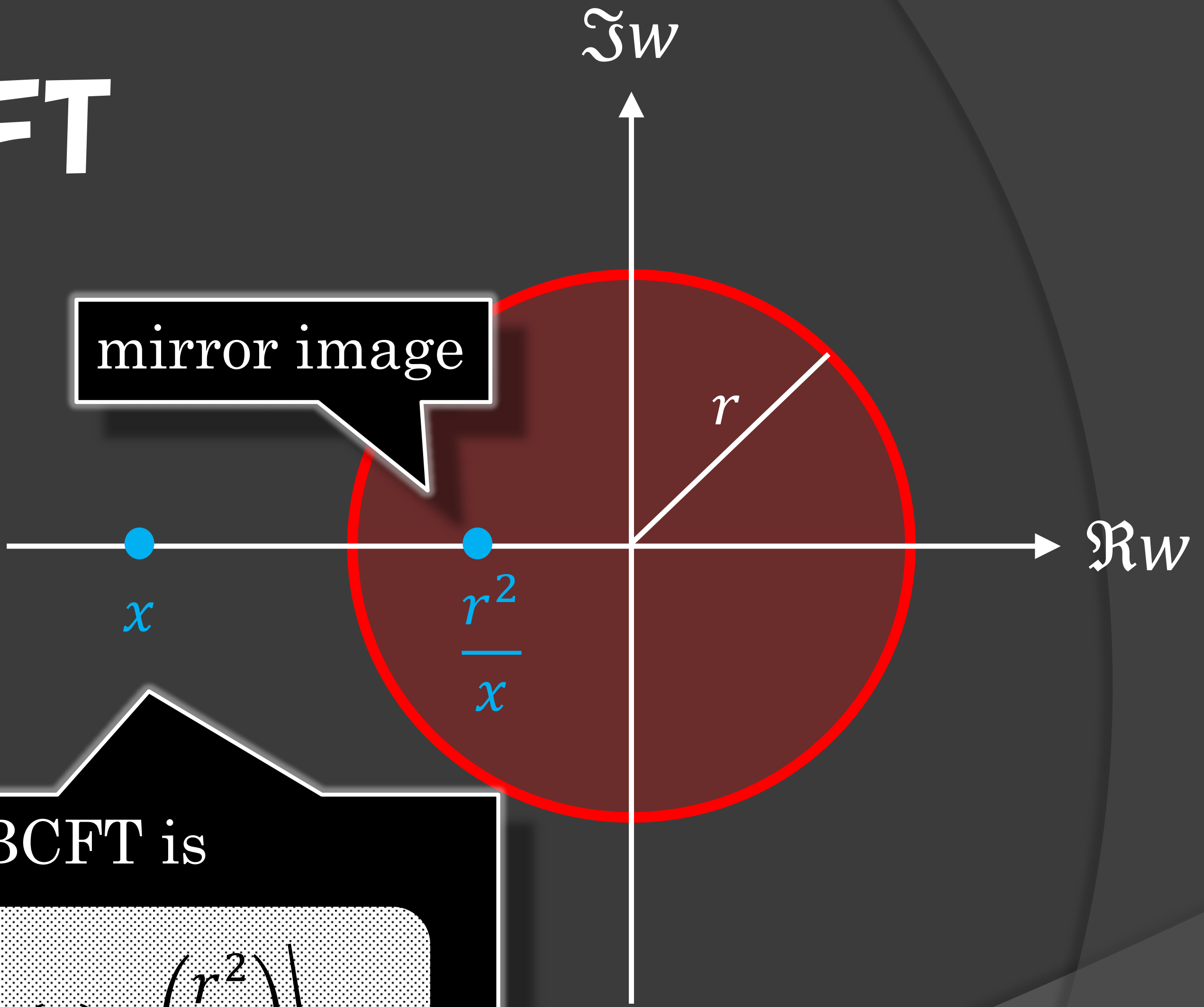
$T > 1/R$ implies

$$r^2 = \beta^2 - \alpha^2 < 0$$

Imaginary radius is problematic, but in gravity picture, this analytic continuation is not so problematic.
We formally define **imaginary BCFT**.

EE in BCFT

Let us recall that mirror method can completely fix 1-pt.correlator.



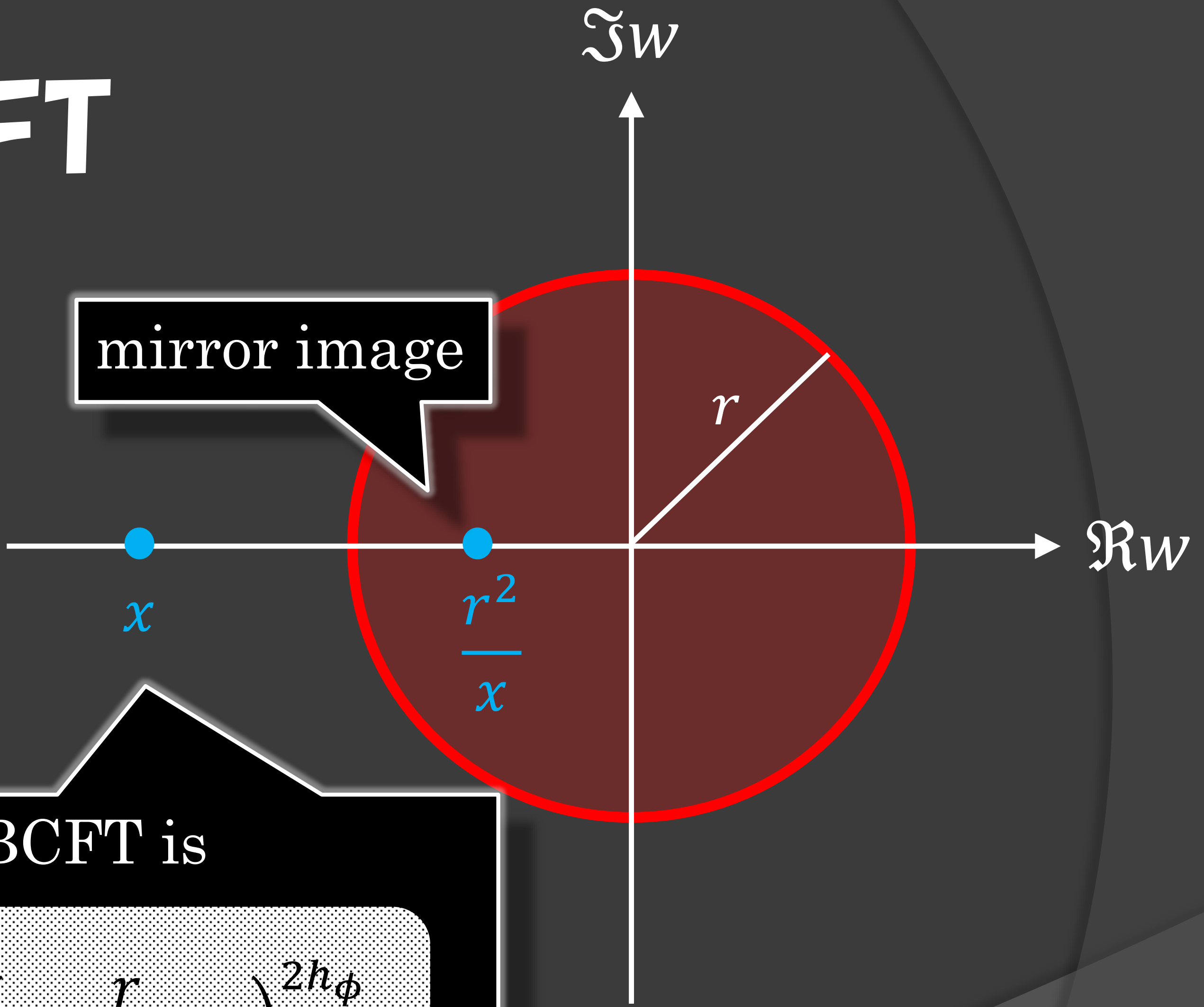
1-pt. correlator in BCFT is

$$\langle \phi(x) \rangle_{disk} \simeq \left\langle \phi(x)\phi\left(\frac{r^2}{x}\right)\right\rangle$$

up to conformal prefactor.

EE in BCFT

Let us recall that mirror method can completely fix 1-pt.correlator.



1-pt. correlator in BCFT is

$$\langle \phi(x) \rangle_{disk} = g_B \left(\frac{r}{|w|^2 - r^2} \right)^{2h_\phi}$$

EE in BCFT

For example, EE in the right setup can be evaluated by

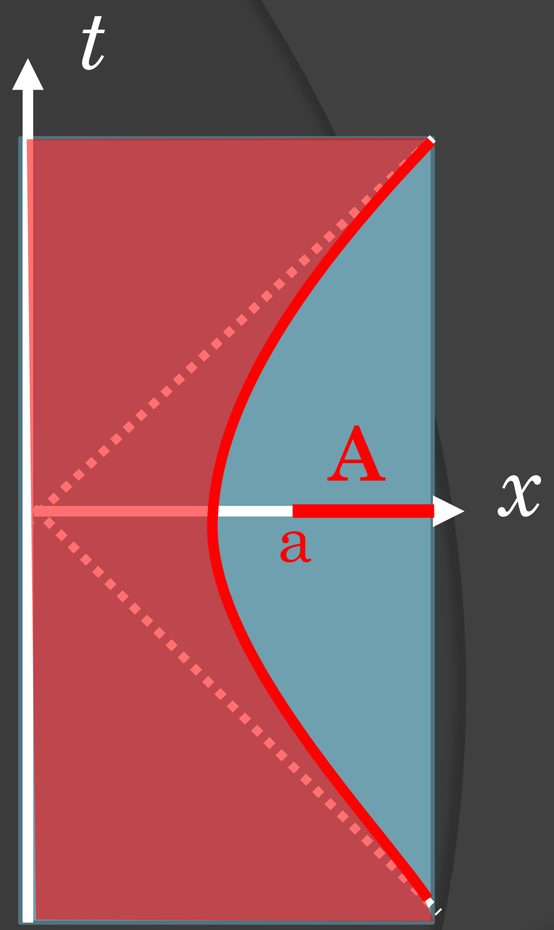
$$\langle \sigma_n(\tau + ia) \rangle_{disk} = g_B \left(\frac{r}{\tau^2 + a^2 - r^2} \right)^{2h_{\sigma_n}}$$

where σ_n is twist op. with $h_{\sigma_n} = \frac{c}{24}\left(n - \frac{1}{n}\right)$

After analytic continuation $\tau \to it$,

$$S_A(t) = \lim_{n \to 1} \frac{1}{1-n} \log \langle \sigma_n(\tau + ia) \rangle_{disk}$$
$$= \frac{c}{6} \log \frac{a^2 - t^2 - r^2}{\epsilon r} + S_{bdy}$$

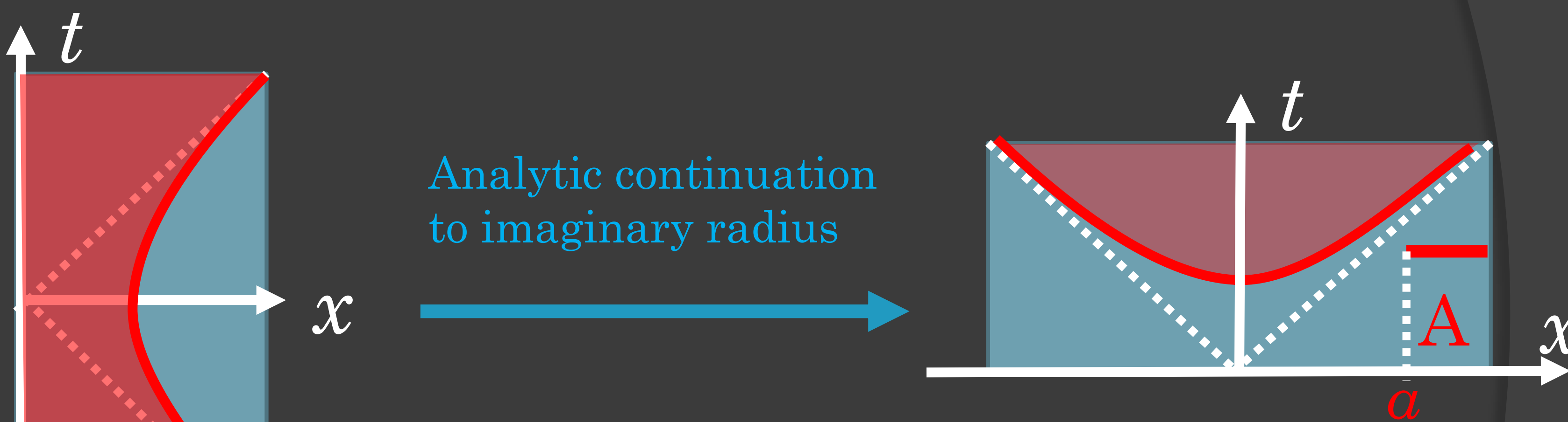
where $S_{bdy} = \log g_B$ is boundary entropy



Lorentzian ver. of Euclidean disk

EE in Im-BCFT

Let us consider imaginary BCFT in physical setup



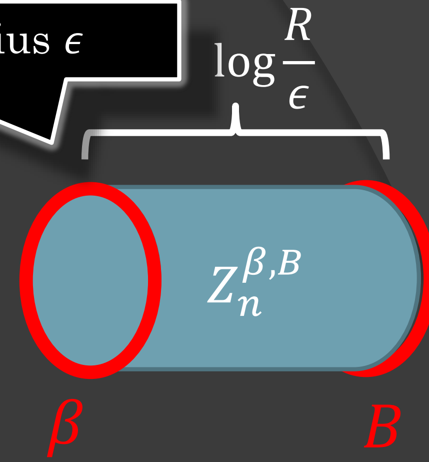
creation of universe setup

Naïve analytic continuation is imaginary.
To be more careful.

$$S_A(t) = \frac{c}{6}\log\frac{a^2 - t^2 + r^2}{i\epsilon r} + S_{bdy}$$

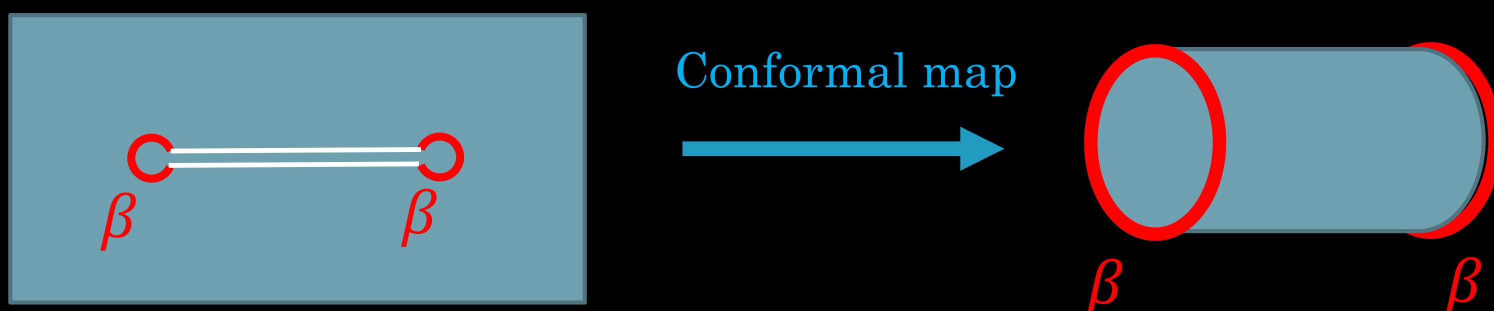
EE in Im-BCFT

σ_n = hole with radius ϵ



$$g_B = \langle \sigma_n | B \rangle = \frac{Z_n^{\beta,B}}{\left(Z_1^{\beta,B}\right)^n}$$

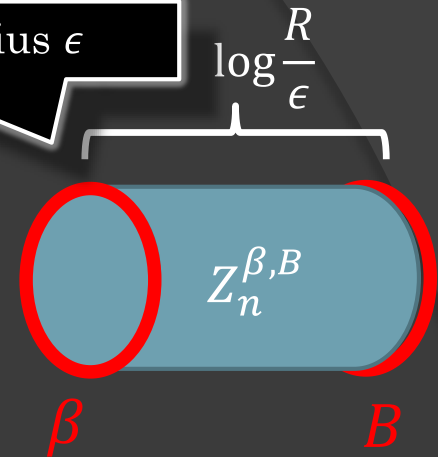
where β is boundary state related to σ_n (which can be absorbed in regularization ϵ)



Cylinder with circumstance $2\pi n$

EE in Im-BCFT

σ_n = hole with radius ϵ



$$g_B = \langle \sigma_n | B \rangle = \frac{Z_n^{\beta,B}}{\left(Z_1^{\beta,B}\right)^n}$$

where β is boundary state related to σ_n (which can be absorbed in regularization ϵ)

$$Z_n^{\beta,B} = \langle \beta | e^{-\frac{1}{2\pi n}\log\frac{R}{\epsilon}H} | B \rangle \underset{\epsilon\to 0}{\longrightarrow} e^{-\frac{1}{2\pi n}\log\frac{R}{\epsilon}E_0} \langle \beta | 0 \rangle \langle 0 | B \rangle$$

R is formal radius $R = 1$ ($\rightarrow i$ later). As a result,

$$\langle \sigma_n | B \rangle = \left(\frac{R}{\epsilon}\right)^{-2h_{\sigma_n}} \left(\langle \beta | 0 \rangle \langle 0 | B \rangle\right)^{1-n}$$

Analytic continuation ($R \rightarrow iR$) leads to

important point

$$\langle \sigma_n | Im\text{-}B \rangle = i^{-2h_{\sigma_n}} \langle \sigma_n | B \rangle$$

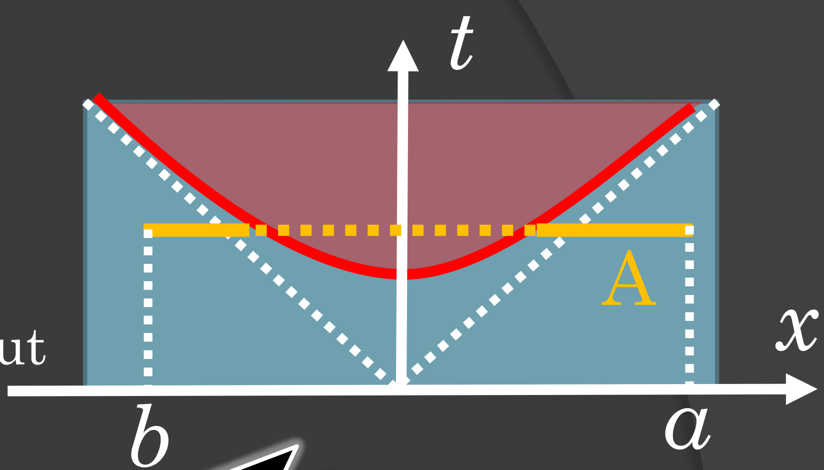
cancel imaginary part

$$\boxed{S_A(t) = \frac{c}{6}\log\frac{a^2 - t^2 + r^2}{\epsilon r} + S_{bdy}}$$

EE in Im-BCFT

CFT calculation is non-trivial but gravity calculation is trivial (just a geodesic in a cut AdS).

⇒**Consistency check** from gravity calculation is possible.



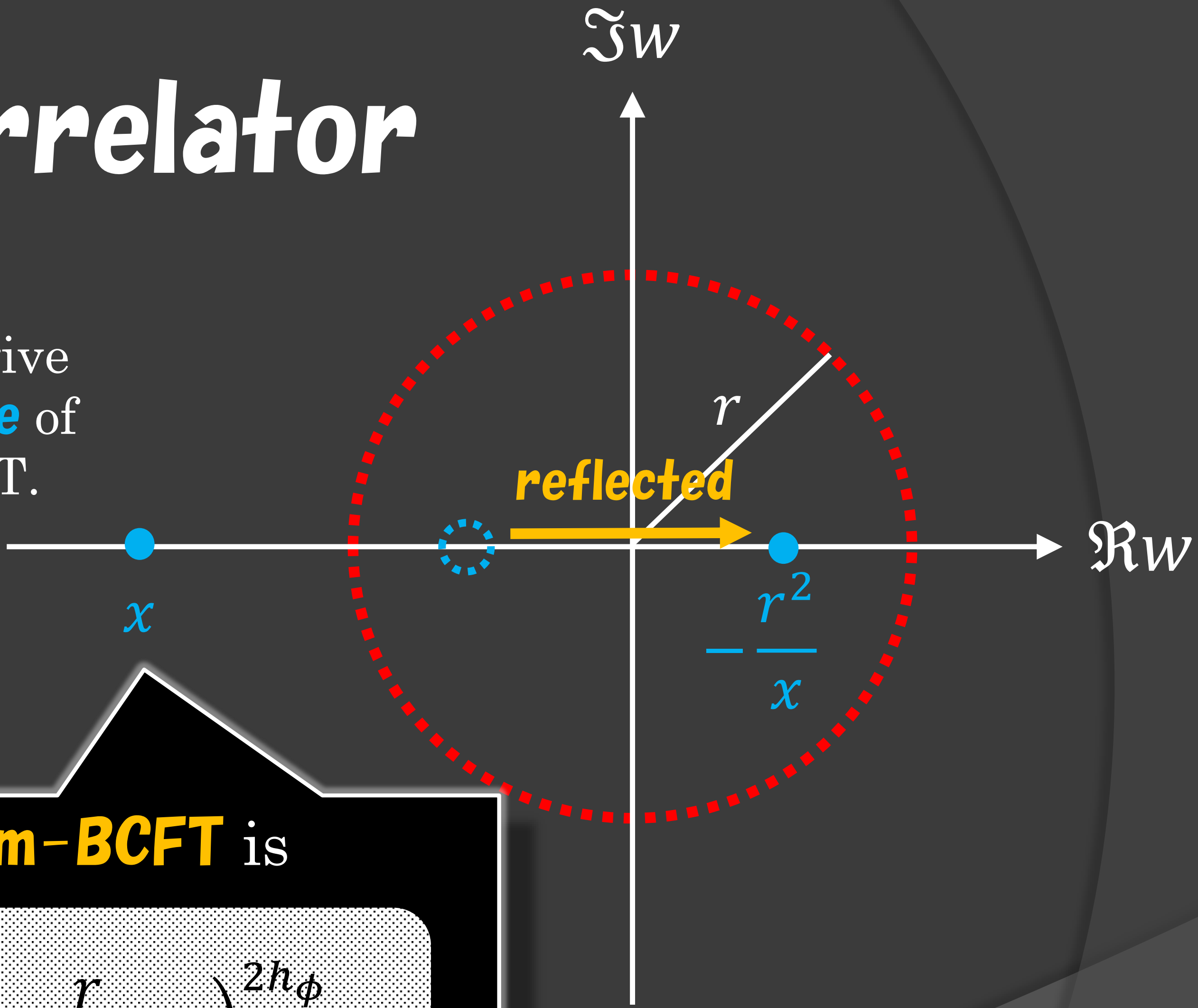
$$S_A = \lim_{n\to 1}\frac{1}{1-n}\log\langle\sigma_n(a,t)\overline{\sigma_n}(b,t)\rangle_{Im-disk}$$

$$= \begin{cases} \frac{c}{6}\log\frac{a^2+r^2-t^2}{r\epsilon}+\frac{c}{6}\log\frac{b^2+r^2-t^2}{r\epsilon}+2g_B & \text{: disconnected} \\ \frac{c}{3}\log\frac{b-a}{\epsilon} & \text{: connected} \end{cases}$$

CFT calculation based on our prescription completely matches with the gravity calculation!

BCFT Correlator

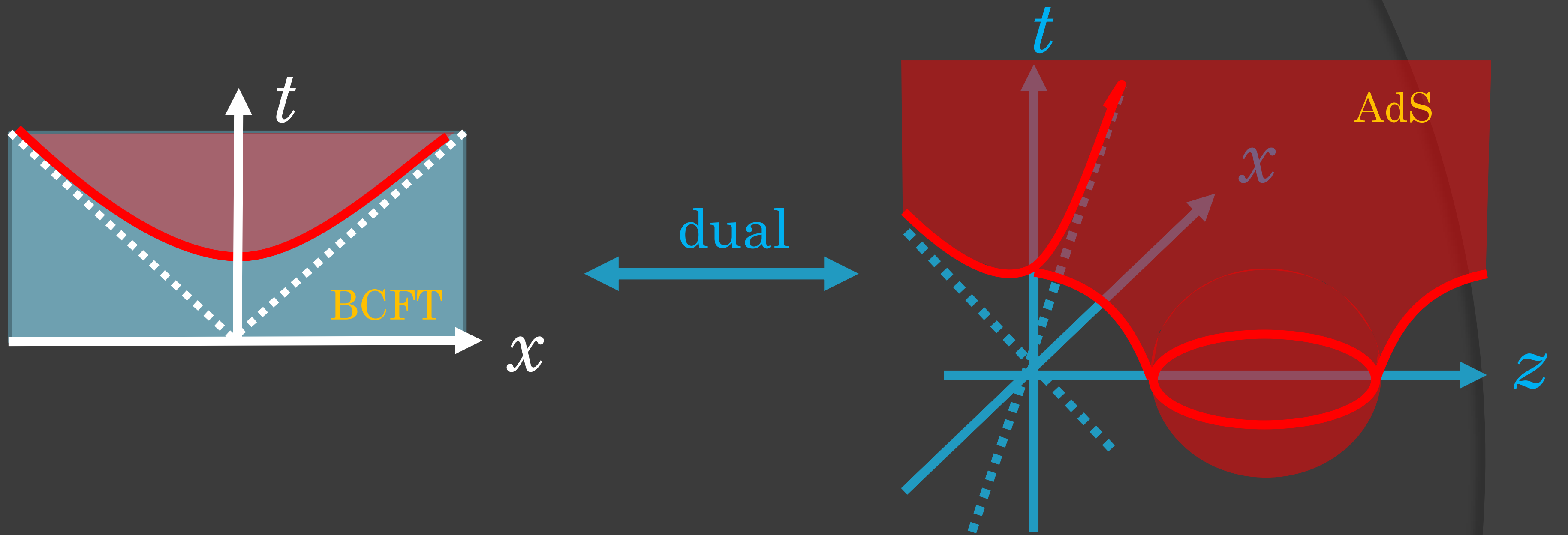
Instead of technical consideration, we can give a ***simple calculation rule*** of a correlator in Im-BCFT.



1-pt. correlator in **Im-BCFT** is

$$\langle \phi(x) \rangle_{disk} = \left(\frac{r}{|w|^2 + r^2} \right)^{2h_\phi}$$

Lorentzian Im-BCFT



Analytic continuation to Lorentzian signature provides time-like boundary in CFT.

This setup is interesting for two reasons:

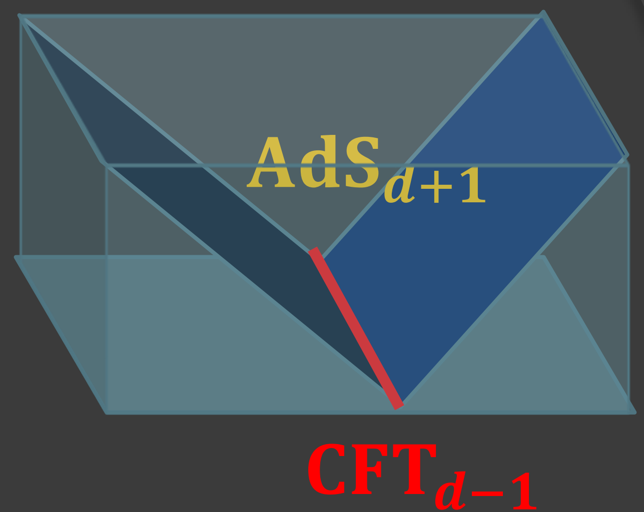
- ***universe creation setup***
- ***dS braneworld*** (as mentioned before) by CFT description as boundary state

Contents

Summary①

Wedge Holography

$\mathrm{AdS}_{d+1} = \mathrm{CFT}_{d-1}$



Justifications

- Brane world picture:

  Brane world holography + AdS/CFT (at large ρ_*)

- BCFT picture:

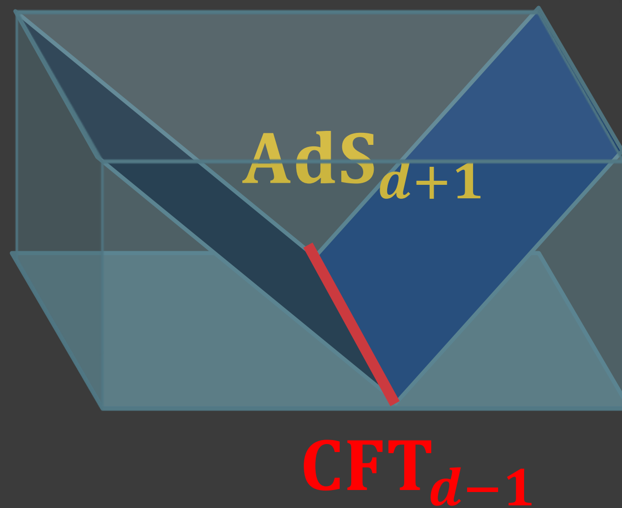
  CFTs on two boundaries with bulk interaction

- Matching of physical quantities

  explicit calculation of DoF and EE from both sides

Summary①

Wedge Holography
$\mathrm{AdS}_{d+1} = \mathrm{CFT}_{d-1}$

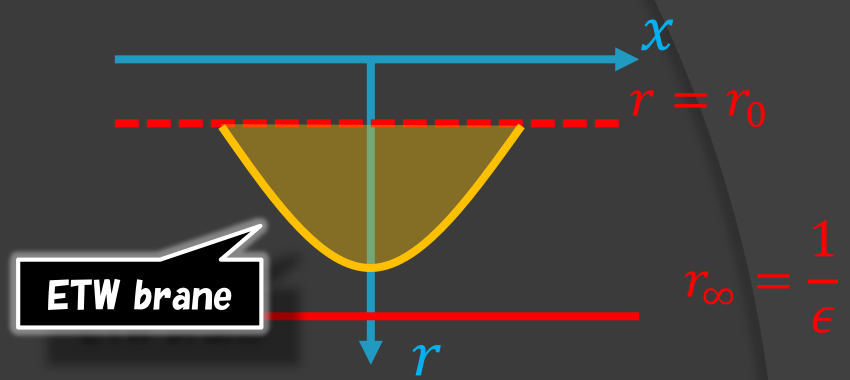


Future direction

- Employing this new laboratory
- Identifying universal properies of CFT_{d-1}
- Joining gravity from AdS_{d+1} and CFT_{d-1}
  $\Rightarrow$more general double holography setup
- More justification (e.g. realization via string theory)

Summary②

dS brane/Imaginary BCFT duality



$T > 1/R$
dS brane

Future direction

- Employing this new laboratory
- More understanding of dS braneworld holography
- Universe creation setup
- More justification